# WORKSHOP DAY 1

# CIRCUIT DIAMETERS AND AUGMENTATION

*May 15, 2023 9:00 a.m.-5:40 p.m. (MDT)*

Room 4125 Student Commons Building, University of Colorado Denver

| Time | Item |
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| 9:00 a.m. | *Meet & Greet, Welcome Round* |

9:30 a.m. **Sean Kafer**

An Introduction to the Circuits of Polyhedra: Basics, Diameters, and Optimization

This talk will serve as a ground-up introduction to the topic of circuits of polyhedra. The circuits are a set of directions associated with a given polyhedron which generalize the set of its edge-directions. As such, they are studied for their relevance to diameters of polyhedra, as well as to linear optimization.

We will discuss the fundamental properties of circuits that motivate their usefulness, and we will look at the circuits of the matching and fractional matching polytopes as a concrete example. We will then introduce the circuit diameter - a generalization of the combinatorial diameter - discuss the ways in which the circuit diameter differs substantially from the combinatorial diameter, and introduce some results on the topic. Finally, we will discuss some ways in which circuits have been applied to the topic of linear optimization.

10:30 a.m. *Break*

11:00 a.m. **Jesús De Loera**

From Circuits to Graver Bases: A Survey

Circuits have been shown to be key elements of every linear optimization problem.

There is a larger set of vectors, called Graver set, that generalize many of the lovely properties and applications of circuits to be applied in integer linear optimization problems. I will introduce some key facts about Graver bases and their complexity.

I will present some open questions and interesting research directions. This survey lecture will be a summary of 20 years of work by many people.

12:00 p.m. *Networking Lunch (room 4020)*

1:30 p.m. **Bento Natura**

Circuit imbalance measures and linear programming

We study properties and applications of various circuit imbalance measures associated with linear spaces. These measures describe possible ratios between nonzero entries of support-minimal nonzero vectors of the space. The fractional circuit imbalance measure turns out to be a crucial parameter in the context of linear programming, and two integer variants can be used to describe integrality properties of associated polyhedra. This is joint work with Farbod Ekbatani and László Végh.

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| 2:05 p.m. | **Cedric Koh** |

On Circuit Diameter Bounds via Circuit Imbalances

In this talk, we study the circuit diameter of polyhedra, introduced by Borgwardt, Finhold and Hemmecke (SIDMA 2015) as a relaxation of the combinatorial diameter. For a polyhedron in standard equality form with constraint matrix A, we prove an upper bound on the circuit diameter that is quadratic in the rank of A and logarithmic in the circuit imbalance measure of A. This yields a strongly polynomial circuit diameter bound e.g. if all entries of A have polynomially bounded encoding length in the size of A. We also give circuit augmentation algorithms for linear programming with similar iteration complexity.

Joint work with Daniel Dadush, Bento Natura and László Végh.

2:40 p.m. *Break*

3:00 p.m. **Tamon Stephen**

Klee-Walkup and Circuits

A key question in optimization is how the combinatorial diameter of a polyhedron relates number of facets f and dimension d of the polyhedron. In the seminal paper of Klee and Walkup (1967), the conjectured Hirsch bound of f-d, was shown to be equivalent to several seemingly simpler statements, and was disproved for unbounded polyhedra through the construction of a particular 4-dimensional polyhedron with 8 facets.

This talk looks at circuit analogous of these statements. Many of the Klee-Walkup results and techniques translate to the circuit setting. However, the 4-dimensional version of the d-step conjecture holds for unbounded polytopes, while it fails in the combinatorial case.

This is joint work with S. Borgwardt and T. Yusun.

3:35 p.m. **Alex Black**

On the Circuit Diameter Conjecture for Hirsch Counter-examples

In 2010, Santos disproved the Hirsch conjecture that stated the combinatorial diameter of a polytope is at most n - d for a polytope with n facets and d dimensions. However, the circuit diameter conjecture, which asks whether the circuit diameter of polytopes is at most n-d remains open. Since the combinatorial diameter of a polytope is an upper bound on the circuit diameter of a polytope, any counterexample to the circuit diameter conjecture must also be a counterexample to the Hirsch conjecture. In pursuit of this problem, we showed that some of Santos' counterexamples are, in fact, not counterexamples to the circuit diameter conjecture. We also showed that Todd's monotone Hirsch counterexample is not a counterexample to the monotone variant of the circuit diameter conjecture. I will discuss these results and describe new structural properties we discovered about these Hirsch counterexamples. This talk is based on joint work with Steffen Borgwardt and Matthias Brugger.

4:10 p.m. *Break*

4:30 p.m. **Sean Kafer**

It's not hard to solve LPs quickly with circuits

We will discuss the efficacy of various *circuit augmentation algorithms* for solving linear programs (LPs). Such algorithms generate a sequence of feasible solutions of improving objective value, and at

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| | each iteration choose the next feasible solution by moving maximally along a circuit direction. This is done until an optimal solution is reached or unboundedness is detected. Since edge-directions are always circuits, this framework is a generalization of the famous Simplex method for solving LPs. Like the Simplex method, any circuit augmentation algorithm requires a circuit-pivot rule for deciding which circuit direction to move along at each iteration. Pivot rules for the Simplex method are usually easy to compute, but no known Simplex pivot rule is known to solve LPs with only a polynomial number of pivots. In the circuit setting, it is known, for example, that the Greatest Improvement circuit-pivot rule requires only a polynomial number of pivots to solve an LP and that the Steepest Descent circuit-pivot rule can be computed in polynomial time. Unfortunately, no polynomial bound is known on the number of pivots required by the Steepest Descent circuit-pivot rule, and we show that the Greatest Improvement circuit-pivot rule is NP-hard to compute. This suggests a natural question: *Does there exist a circuit-pivot rule that is computable in polynomial time and requires only a polynomial number of pivots to solve an LP?*<br><br>We show that there does exist such a rule. We propose two variants on the previously-studied Steepest Descent circuit-pivot rule. We show that if the LP is defined over a 0/1-polytope - i.e., one whose vertices have coordinates in {0,1} - then the first variant is computable in polynomial time and requires only a strongly-polynomial number of pivots. The second, which we call Asymmetric Steepest Descent, is inspired by the work of Schultz and Weismantel. We show that for *any* LP, it requires only a polynomial number of pivots and can be computed in polynomial time.<br><br>Joint work with Laura Sanità and Jesús De Loera. |
| 5:05 p.m. | **Matthias Brugger**<br>Circuits in Extended Formulations<br><br>An extended formulation of a polyhedron P is a linear description of a polyhedron Q such that Q linearly projects onto P. Extended formulations are particularly useful in combinatorial optimization for obtaining compact linear programming formulations. In my talk, I will explore the relationship between the circuits of such pairs of polyhedra P and Q: are all circuits of P projections of circuits of Q? While this property is known to be true for the edge directions of P, it does not extend to the full set of circuits in general, not even if P or the projection map from Q to P is fixed. Projecting polyhedra may create many „new“ circuits in the image, even for relevant examples in combinatorial optimization.<br><br>This is joint work with Steffen Borgwardt. |

# WORKSHOP DAY 2
# CIRCUIT DIAMETERS AND AUGMENTATION

*May 16, 2023 9:00 a.m.-5:40 p.m. (MDT)*

Room 4125 Student Commons Building, University of Colorado Denver

| Time | Item |
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| 9:00 a.m. | **Laura Sanità**<br>Circuits, diameters, and matching games<br>Several fundamental algorithmic game theory problems involve the structure of matchings in graphs (e.g. stable matchings, cooperative matchings, network bargaining games). In this talk, I will show algorithms to stabilize instances of some matching games, employing the polyhedral concept of circuits. I will also discuss some diameter results for the associated polytopes. |
| 10:00 a.m. | *Break* |
| 10:30 a.m. | **Jon Lee**<br>Recent algorithmic advances for maximum-entropy sampling<br>The maximum-entropy sampling problem (MESP) is to select a subset, of given size s, from a set of random variables, so as to maximize the "differential entropy". If C is the covariance matrix and we are in the Gaussian case, then we are simply seeking to maximize the determinant of an order-s principal submatrix of C. A key application is for the reconfiguration of an environmental-monitoring network. MESP sits at the intersection of optimization, data science and information theory, and so it has attracted a lot of recent attention. The problem is NP-hard, and there have been algorithmic attacks aimed at exact solution of moderate-sized instance for over three decades. It is a fascinating problem from the perspective of integer nonlinear optimization, as it does not fit within a framework that is successfully attacked via available general-purpose paradigms. I will give a broad overview of algorithmic work, concentrating on the many useful techniques related to various convex relaxations. |
| 11:30 p.m. | *Group Photos* |
| 11:45 p.m. | *Lunch Break* |
| 1:30 p.m. | **Daphne Skipper**<br>Blending Models in Gas Network Optimization<br>In the European Union, gas network regulations emphasize the quality of the product reaching demand nodes from a variety of sources. However, modeling the calorific value of blended gas through the network requires adding nonlinear and nonsmooth pooling constraints to an already highly complex model of the physical and chemical properties of the gas network. In 2019, Hante and Schmidt proposed a complementarity-based NLP formulation of the pooling model, which improved on an existing MINLP model. Continuing in this vein and inspired by recent work comparing piecewise-linear reformulations of bilinear terms by Barmann, et. al. (2022), we propose and test MIP relaxations of the pooling model. Using AMPL and the NEOS Server, we test our models, as well as our reproduction of Hante and Schmidt's NLP, on realistic instances in the GasLib repository.<br>This is work with: Geonhee Kim, Jon Lee, Chris Lourenco, and Luze Xu |

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**2:05 p.m.** — **Stephan Patterson**

Programming Models for Computing Barycenters

A barycenter is, informally, a weighted average of a set of probability measures. Finding barycenters is of interest in a variety of fields, including machine learning, materials science, economics, and more; however, the barycenter problem has recently been shown to be NP-hard. Barycenters can be computed using linear programming, but designing practical exact and approximate algorithms remains an active area of research. In this talk, I will describe the linear programming models that form the basis of the computations, and our latest computational innovation towards a more practical algorithm: a mixed-integer program for pricing potential support points of the barycenter.

**2:40 p.m.** — *Break*

**3:00 p.m.** — **Amy Wiebe**

The Slack Realization Space of a Polytope

The realization space of a polytope is the set of all geometric instances of a particular combinatorial type (face lattice). In this talk we discuss a recent model for studying the realization space of a polytope, which represents a polytope by its slack matrix and is called the slack realization space.

This model was first developed to study PSD rank of polytopes, and provides an algebraic approach to answering realization space questions. We will see how this algebraic structure encodes certain polytopal properties, providing a new way to classify polytopes. We will also see how this model is connected to more classical models, and how these connections can be exploited to improve computational efficiency of the model.

**3:35 p.m.** — **Michael Wigal**

Hardness and Approximations of Submodular Minimum Linear Ordering Problems

The minimum linear ordering problem (MLOP) asks to minimize the aggregated cost of a set function $f$ with respect to some ordering $\sigma$ of the base set. That is, MLOP asks to find a permutation $\sigma$ that minimizes the sum $\sum_{i=1}^{|E|}f(\{e \in E: \sigma(e) \le i\})$. We will cover some results on hardness and better approximations for various instances of submodular MLOP. We will also end with some interesting open questions.

Joint work with Majid Farhardi, Swati Gupta, Shengding Sun, and Prasad Tetali.

**4:10 p.m.** — *Break*

**4:30 p.m.** — **Weston Grewe**

On the Combinatorial Diameters of Parallel and Series Connections

The investigation of combinatorial diameters of polyhedra is a classical topic in linear programming due to its connection with the possibility of an efficient pivot rule for the simplex method. We are interested in the diameters of polyhedra formed from the so-called parallel or series connection of oriented matroids.

We prove that, for polyhedra whose combinatorial diameter satisfies the Hirsch-conjecture bound regardless of the right-hand sides in a standard-form description, the diameters of their parallel or series connections remain small in the Hirsch-conjecture bound. These results are a substantial step toward devising a diameter bound for all polyhedra defined through totally-unimodular matrices based on Seymour’s famous decomposition theorem. Our proof techniques and results exhibit a

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| | number of interesting features. While the parallel connection leads to a bound that adds just a constant, for the series connection one has to linearly take into account the maximal value in a specific coordinate of any vertex. This is joint work with Steffen Borgwardt and Jon Lee. |
| 5:05 p.m. | **Youngho Yoo**<br>Path odd-covers of graphs<br><br>We study the minimum number of paths needed to express the edge set of a given graph as the symmetric difference of the edge sets of the paths. This can be seen as a weakening of Gallai’s path decomposition problem, and a variant of the “odd cover” problem of Babai and Frankl which asks for the minimum number of complete bipartite graphs whose symmetric difference gives the complete graph. We relate this “path odd-cover” number to other known graph parameters and prove upper bounds with an adaptation of the techniques that were used to bound the diameter of the partition polytope. Joint work with Steffen Borgwardt, Calum Buchanan, Eric Culver, Bryce Frederickson, and Puck Rombach. |