MATHEMATICAL PROGRAMMING MODELS FOR ENVIRONMENTAL QUALITY CONTROL

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This paper surveys the use of mathematical programming models for controlling environmental quality. The scope includes air, water, and land quality, stemming from the first works in the 1960s. It also includes integrated models, generally that are economic equilibrium models which have an equivalent mathematical program or use mathematical programming to compute a fixed point. A primary goal of this survey is to identify interesting research avenues for people in mathematical programming with an interest in applying it to help control our environment with as little economic sacrifice as possible.

The United States spends more than 2% of its gross domestic product on pollution control, and this is more than any other country (Carlin 1990). There is an economic imperative to establish policies, both government and private, that control the environmental quality as cost effectively as possible. The purpose of this survey is to present a comprehensive bibliographic tour of mathematical programming models built for environmental quality control.

Since the 1960s, mathematical programming began to be applied to certain problems of environmental quality control. The first was in 1962, by Lynn, Logan and Charnes, which was a linear programming model for wastewater treatment plant design. Mathematical programming models for other environmental control problems then began to appear; this survey will put these models into a mathematical programming perspective.

In some cases, the model is used for resource management, and the mathematical program is designed to prescribe decisions for operations and planning to minimize cost subject to quality standard constraints. In other cases, the model is used for policy analysis, and the mathematical program is designed to describe economic and environmental impacts. The management models tend to be detailed representations of an area, like a portion of one stream or an airshed covering one city. The policy models tend to be aggregate representations of countries.

The rest of this paper is organized as follows. Section 1 presents some basic terms and concepts plus caveats concerning the scope of this survey. Sections 2–4 summarize the literature on mathematical programming models for air, land, and water quality control, respectively. I include models that seek economic equilibria which either are equivalent to mathematical programs or use mathematical programming to compute a solution. Section 5 describes the literature of integrated models that represent pollution in connection with the economy, not specific to air, land, or water.

Section 6 offers a guide to the periodicals that were used in this study, including some for which there are no citations. In addition, there is a table of how different mathematical programming models statistically distribute in the literature (according to this survey) over the past three decades with respect to each part of the environment. The last section presents some conclusions.

The main contribution of this survey is the annotated bibliography, which contains 355 citations. Of these, 224 are articles, 18 are reports or theses, 32 are books or monographs that contain original results, 36 are chapters in books with original results, and 11 are textbooks. (The others are relevant, but not directly about environmental control or without a mathematical programming model.) The citations are given alphabetically by author(s), so no special reference is given in the text and the annotations when the author(s) and year have been specified.

One caveat is that all citations are items I could obtain. In particular, this means I did not include old technical reports or theses that are no longer available.

1. TERMS AND CONCEPTS

1.1. The Environment

We will suppose that our environment is made up of air, land, and water. Other parts of the environment, such as life and related ecological concerns, are not included in

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this study. When we speak of environmental quality, we mean how free of pollutants are these fundamental parts.

The primary issue in environmental control is how to maintain high quality with as little economic sacrifice as possible. Other issues will be described, but the focus of this survey is the economic tradeoffs to achieve environmental quality. (Measuring quality by the presence of pollutants is imperfect, but that is what we will consider because that is what most models represent.)

One form of air pollution is the presence of undesirable chemicals, like carbon monoxide (CO), hydrochloric acid (HCl), nitrogen dioxide (NO_2), and sulfur dioxide (SO_2). These are caused by automobile emissions, electricity generation, industrial emissions, and other sources. Another form of air pollution is noise, which is not included in this survey. Here we consider only economic tradeoffs of chemical emission controls.

Water pollution is also the presence of undesirable chemicals, and a key measure of pollution is by the concentration of dissolved oxygen (DO). Models refer to the DO *deficit* as the difference from what is needed to support life, such as fish and plants, and to provide safe water for drinking and recreation. A related measure is the biochemical oxygen demand (BOD), which is the amount of oxygen necessary to stabilize a given waste by microbial action. Objectives and decision variables are designed to increase the DO concentration to overcome the deficit or lower the BOD (which, in turn, increases DO concentration).

Another form of water pollution is thermal, caused by the use of water in cooling, then discharging the warm water back into the stream. Only a few models explicitly considered thermal pollution in conjunction with BOD and DO concentrations (Dysart 1970, Nicholson, Pyatt and Moreau 1970, Hwang et al. 1973, Bayer 1976).

In this survey, groundwater contamination is also considered, but there are two kinds of models. One class is similar to surface water quality control. Similar equations arise (notably, the hydrological and mass transport), giving similar optimal control models. Furthermore, the groundwater in such cases is a supply used for drinking, so the quality issues are the same, or similar, as for surface water.

A second kind of groundwater quality model pertains more to land. Toxic waste, for example, might contaminate the land covering the groundwater. Here the models deal not so much with the flows, but with the damage in the immediate area, such as impacts on irrigation. Pesticides used by farmers, for example, are direct sources of land pollution. Soil erosion is another consideration that relates to water quality, but those models are classified here as land quality control.

More generally, chemical land pollution occurs from purposeful injection (like pesticides) and the storage of polluting materials, such as hazardous waste. Storage can be underground (including landfill) or in tanks (which have the potential to leak). The associated control problems are usually classified as hazardous waste or solid waste in the environmental control literature. Land quality is also affected by landscape disturbance, such as by strip mining coal and by garbage that must be collected. Other environmental problems pertaining to land, such as those that arise in forest and wildlife management, are not covered in this survey.

1.2. Mathematical Programming Models

The mathematical programming model is defined by four ingredients:

- A set, X, of finite dimensions, whose members are called *decision variables*;
- A *constraint function*, g, that maps X into $\mathfrak{R}^m$ ($m = 0$ means there is no constraint function);
- *Bounds* on variables: $L \leq (x, y) \leq U$, where $y = g(x)$; a bound can be logical, such as nonnegativity, or data dependent, such as a capacity limit or demand requirement (infinite values are admitted to allow no explicit bound on some of the variables);
- An *objective function*, f, that maps X into $\mathfrak{R}$.

Then, the usual notation is: *optimize* $\{f(x)\colon x \in X, y = g(x)$, and $L \leq (x, y) \leq U\}$, where *optimize* can be either *minimize* or *maximize*.

A *family* of mathematical programs is defined by extending the ingredients to include a parameter space, Θ, which is augmented to the domain of the basic ingredients:

$$\text{Optimize}\{f(x;\ \theta)\colon x \in X(\theta),\ y = g(x;\ \theta),\ \text{and } L(\theta) \leq (x, y) \leq U(\theta)\} \quad \text{for } \theta \in \Theta.$$

A point (x) is called *feasible* if it satisfies all constraints. It is called *optimal* if it is feasible and no other feasible solution has a better objective value. Many of the models, especially for water quality management, are multiobjective. This means we seek to optimize several functions at once. Since this generally cannot be achieved, we settle for solutions that are called *Pareto optimal*: No other feasible solution exists that improves one objective without worsening some other. One way to obtain a Pareto optimal solution is to optimize a weighted sum of the objectives. There are other ways to obtain a Pareto optimum, and some analysts rely on interactive computation to understand the tradeoffs among competing objectives.

Objectives can be explicit, such as the cost to operate a treatment plant. They can also be *utility*, or *net benefit*, functions, and a source of multiplicity is the different agents in the market model. Agents could be polluters at different locations, perhaps in different states. Such a linear programming model was described by Dorfman and Jacoby in 1972.

Classes of mathematical programs are defined by the structures of the basic ingredients. Suppose that $g = Ax - b$ for some matrix A and vector b, and $f = cx$ for some vector c. Furthermore, let $X = \mathfrak{R}^{n+} \equiv$

$\{x \in \Re^n: x \geq 0\}$. Then, we have a linear programming model, denoted LP. The standard form is $\text{Min}\{cx: y = Ax,\ L \leq (x, y) \leq U\}$. Typically, $L = 0$ for the x variables. A prevalent class of equations are material balances, where $L = U = 0$ for y variables. For example, if x_{ij} is the level of flow from i to j, a material balance equation is $y_i = \sum_j x_{ji} - \sum_j x_{ij} = 0$, which requires that the flow into i must equal the flow out of i.

Another class of mathematical programming models is the integer program, where X restricts the variables to have integer values. If the mathematical program is otherwise linear, and only some of the variables are required to be integer, this is called a mixed integer program, denoted MIP.

One type of integer restriction is that a variable must be binary-valued, 0 or 1. Any model that has capacity constraints, such as for treatment plants, can be extended to include capacity expansion by adding a binary decision variable for each expansion option. For example, consider the constraint $\sum_j a_j x_j \leq b$, where a_j is the rate of using capacity for the activity whose level is x_j, and b is the total available capacity. To extend this to allow capacity expansion, let u be a 0-1 variable such that $u = 0$ means no capacity is added, and $u = 1$ means K units of capacity are added. The constraint becomes: $\sum_j a_j x_j - uK \leq b$. Then, if some solution has $u = 0$, the original capacity limit applies. If another solution has $u = 1$, the constraint becomes $\sum_j a_j x_j \leq b + K$, allowing K units of additional capacity to be used by the x variables. The binary variable can also appear in the total cost with the term Cu. This adds a fixed charge of C dollars to the cost if $u = 1$ and nothing if $u = 0$. Another prevalent use of binary decision variables is when a particular process is either not used at all ($x_j = 0$), or it has both lower and upper bounds, say $L_j \leq x_j \leq U_j$. A 0-1 variable can be introduced, say u_j, with the constraints, $L_j u_j \leq x_j \leq U_j u_j$. Then, $u_j = 0$ forces $x_j = 0$, and $u_j = 1$ forces $L_j \leq x_j \leq U_j$.

When binary variables are added to extend the scope of a model, constraints can also be added to restrict their relative values. For example, a constraint of the form $\alpha \leq \sum_j u_j \leq \beta$ means that the number of positive binary decisions must be at least α and at most β. A budget constraint has the form $\sum_j C_j u_j \leq \gamma$, where C_j is the fixed charge of the jth (binary) option, and γ is the total budget.

When a mathematical program has uncertain parameters, it is called a stochastic program. One approach is a recourse model that considers all scenarios and the effect that decisions at one time have on later options. Another approach, more common in the environmental control literature, is the use of a *chance constraint*. If the original constraint is $g(x;\ \theta) \leq 0$, and θ is a random variable, it is replaced by the chance constraint $P\{g(x;\ \theta) \leq 0\} \geq \alpha$, where α is a new parameter that specifies an acceptable level of probability of not violating the constraint.

If there is only one random variable and the constraint is linear, it can be reformulated as a linear constraint $P\{ax \leq b\} \geq \alpha \Longleftrightarrow \text{ax} \leq \text{F}^{-1}(\alpha)$, where F is the cumulative distribution function whose inverse is assumed to exist (e.g., F is continuous and strictly increasing). For some distributions, like the normal, this can be expressed in terms of the mean (μ) and standard deviation (σ), $ax \leq \mu + \nu\sigma$, where ν depends on α. In either case, the reformulation of the chance constraint, which is linear in this case, is called the *certainty equivalent.*

With joint linear chance constraints, where A is a matrix whose elements are random variables, the chance constraint, $P\{Ax \leq b\} \geq \alpha$, is more complex. The certainty equivalent is, under certain assumptions, of the form $E[A]x + \nu(x^t Vx)^{1/2} \leq \beta$, where $E[A]$ is the expected value of A, V is a variance-covariance matrix, and ν and β depend on α and the distribution parameters.

A dynamic program, denoted DP, has the added dimension of time, and the addition of *state variables* $s(t) \in S(t)$. The decision variables are indexed by time, and their admissible values are dependent upon the state $x(t) \in X(t, s(t-1))$. The initial state, $s(0)$, is given, and the state equations are given by a *state transition function* $s(t) = T(t, s(t-1), x(t))$ for $s(t-1) \in S(t-1)$ and $x(t) \in X(t, s(t-1))$. A *policy* is the specification of a *decision rule*, $x^*(t, s) \in X(t, s)$ for $s \in S(t)$. The dynamic programming model is:

$$\text{Optimize}\Big\{ \sum_t f(t, x(t), s(t-1)): \\ s(t) = T(t, s(t-1), x(t)) \in S(t) \text{ and} \\ x(t) \in X(t, s(t-1))\Big\}.$$

(The summation of each time period's return function is somewhat arbitrary. Other operators apply in general, but this is the most common form in the environmental control literature.)

The *fundamental recursion* of a DP is:

$$F(t, s) = \text{Opt}\ \{f(t, x, s) + F(t+1, s'): \\ x \in X(t, s),\ s' = T(t, s, x)\} \\ \text{for } t = 1, \ldots, N,$$

where N is the number of time periods, called the *horizon*, and $F(N+1, s) \equiv 0$. The function F gives the optimal value upon entering time period t in state s (this is a forward recursion; there also can be a backward recursion). Inherent in the DP approach is the assumption of perfect information about the future (over the horizon), which is a form of clairvoyance.

A stochastic dynamic programming model is where the state transition function is stochastic, as when its domain is extended to depend on a random variable. There are different ways to deal with this uncertainty, and some have been applied to environmental control (for example, Yaron 1983, Whiffen and Shoemaker 1993). The fundamental recursion replaces $F(t+1, s')$ with its expected value $\sum_{s'} F(t+1, s')P(s, s'; t, x)$, where $P(s, s'; t, x)$ is the (known) probability of the transition from state s to state s' upon making decision x in period t. In this case, the decision rule, $x^*(t, s)$, defines what to do when entering period t in state s. The effect of that decision is uncertain, and it is chosen to optimize the expected value, taken over all possible state transitions.

We have assumed discrete time in the DP, but the concepts and methods apply to continuous time (optimal control) models. Often these are discretized, using the fundamental recursion to compute an optimal decision rule. In general, the horizon could be infinite, but all of the models in this survey have finite horizons. (If the model is purely optimal control theory, with no mathematical programming used for analysis or algorithm design, it is not included in this survey.)

Dynamic programming should not be confused with other dynamical systems that use optimization at each period. The model might be a simulation of how agents respond to system controls. A rational behavior assumption can be consistent with a myopic optimization rule in each time period, without the clairvoyance assumed in dynamic programming. That is, a decision made by an agent might be modeled as an optimal response to the state, but without a lookahead to the future consequences of that decision. We classify this use of mathematical programming as linear (LP), mixed integer (MIP), or nonlinear (NLP), according to the form of the period optimization model, but not as dynamic programming (DP). The DP classification assumes the clairvoyance in its definition.

We also classify a paper as using dynamic programming when time is only implicit, but the model uses a multistage form. For example, Mhaisalkar et al. (1993) defined the stages to be a sequence of processes, implying an underlying temporal order, but the time index is actually a process index. Thus, this survey takes the view that DP is a technique, rather than just a dynamic model. If the fundamental recursion is used, whether time is explicitly modeled or not, the paper is classified as DP. If this recursion is not used, even if the model is dynamic, the paper is not classified as DP.

Contrary to the impression that mathematical programming is normative, the use of mathematical programming could be the way the arithmetic is done, not the economic modeling. Instead, a family of mathematical programs is defined by a parameter vector, θ, whose initial value is specified. At iteration k, x^{k+1} is obtained as a solution to

$$\text{Opt}\{f(x;\ \theta^k):\ x \in X(\theta^k),\quad L(\theta^k) \leq (x, g(x;\ \theta^k)) \leq U(\theta^k)\},$$

along with Lagrange multipliers, π^{k+1}. Then, a rule is applied to obtain new a parameter vector $\theta^{k+1} = F(x^{k+1}, \pi^{k+1}, \theta^k)$, to complete an iteration. A fixed point is reached when $x^{k+1} = x^k$ and $\pi^{k+1} = \pi^k$.

For example, suppose that $\theta = (x, \pi)$ and we reach a fixed point, (x^*, π^*), by solving primal and dual linear programs. Then, we have:

$$x^* \in \text{argmin}\{c(x^*, \pi^*)x:\ x \geq 0,\quad A(x^*, \pi^*)x \geq b(x^*, \pi^*)\}$$
$$\pi^* \in \text{argmax}\{b(x^*, \pi^*)\pi:\ \pi \geq 0,\quad \pi A(x^*, \pi^*) \leq c(x^*, \pi^*)\}.$$

This is *not* the same as:

$$x^* \in \text{argmin}\{c(x, \pi^*)x:\ x \geq 0,\quad A(x, \pi^*)x \geq b(x, \pi^*)\}$$
$$\pi^* \in \text{argmax}\{b(x^*, \pi)\pi:\ \pi \geq 0,\quad \pi A(x^*, \pi) \leq c(x^*, \pi)\}.$$

Neither x^* nor π^* needs to be an optimum in the above mathematical programs!

Moreover, although it is natural to think of a mathematical programming model as prescribing what to do for optimal management, it also is used for efficient computation of a fixed point with some underlying optimizing behavioral model. As such, it can also be considered a simulation that describes what will happen with certain policies. In this use of mathematical programming the objective could be something designed to aid convergence to an economic equilibrium, rather than some utility or something prescriptive. The objective could also contain behavioral assumptions about market agents, such as maximizing their surplus revenues. In this case, the objective has meaning, but the model is still not prescriptive because it is designed to simulate how optimizing agents behave, not how they should behave.

1.3. Mathematical Programming and Economic Equilibria

An economic equilibrium is described by equations and inequalities that represent behavioral assumptions about market agents. Prices and quantities are such that no agent can improve his welfare by changing the variables under his control. The presence of multiple agents is sometimes considered a distinction from a mathematical program that is generally regarded as a single decision-making agent. This is really a matter of interpretation, however, not a matter of mathematics. For example, there is no mathematical distinction if an index j represents an agent in the variable x_j, or if j indexes some choice of abatement process for a single firm.

A classical approach to representing an economic equilibrium is based on duality theory for convex programs; for example, in 1974 Mäler applied this to environmental control. More recently, in 1992 Manne and Richels used this principle in their Global 2100 model that combines energy market behavior with carbon emissions (a part of air quality). Nordhaus (1992, 1993) used a related, but different, approach in his DICE model, with economic relations for Ramsey growth and Cobb–Douglas production. These are also the basis for Duraiappah's (1993) model.

A process model can be extended to represent macroeconomic interactions. One such case is MARKAL, reported by Abilock and Fishbone (1979), which is an LP that represents energy processes and includes some of the chemical emissions. In 1992, Manne and Wene linked this with ETA-MACRO to form MARKAL-MACRO, which is an NLP (also see Ahn 1992, Hamilton et al., 1992).

In general, the mathematical programming approach to determine economic equilibria presumes free market conditions, such as perfect competition. This is consistent with representing environmental controls as constraints on emissions or as taxes, because the agents are still allowed to behave as in a free market. More generally, Greenberg and Murphy (1980, 1985) showed how to incorporate complex regulatory structures into a mathematical programming framework. Although their development is for an energy model, the approach generalizes and applies to equilibrium modeling for environmental impacts analysis.

Because some of the references present economic equilibrium models, the term *mathematical programming* might be absent from their title, or even in the contents. These models are included in this survey if they are equivalent to a mathematical program. (Not all economic equilibrium models are equivalent to optimization models.) We also include the reference if mathematical programming is used to compute the economic equilibrium, as a fixed-point computation described earlier.

Related to economic equilibria is the notion of a *game*. This is well suited to some of the regulatory concerns, and there is an explicit optimization base for such models. This has been done for air quality control: Some use LP (for example, Okada and Mikami 1992); most use NLP (for example, Bird and Kortanek 1974, Carbone et al., 1978). Giglio and Wrightington (1972) used a simple LP game model for water quality control. Those cited here are fundamentally based on mathematical programming; other papers that use game theory, but are not so based, are not included.

In the models that are mentioned in the following sections, there is an *equity issue* that arises when using mathematical programming for control. For example, using dual prices (Lagrange multipliers) as taxes is not necessarily what is economically or socially best. Similarly, requiring the same percentage reductions of polluting emissions or discharges by all companies in an area is not necessarily best. The first to address this for water quality control policies was Liebman, in 1968. This was followed by Loehman, Pingry and Whinston (1974), Herzog (1976), Brill, Liebman and ReVelle (1976), and Lohani and Thanh (1978). The first to address this for air quality control policies were Carbone and Sweigart (1976). This was followed by Carbone et al. (1978). Many of the policy models for air and water quality control that are built from welfare economics implicitly represent equity issues by the market relations. Current approaches to address the equity issue directly use particular economic functions, rather than mathematical programming.

2. AIR

Although there were some economic approaches to air quality control in the early 1960s (see Wolozin 1966), the first application of mathematical programming for air quality control was the linear program developed by Teller in 1968. This was implemented for the U.S. Environmental Protection Agency in 1972 by Chilton et al. (also see Gass 1972). In 1969, Kohn did a Ph.D. thesis that used principles of welfare economics to develop a detailed LP model and applied it to particular airsheds (see Kohn 1978).

The structure of the early LP models is first to have process activities, notably for electricity generation, then augment *quality constraints* of the form: $\sum_j e_{ij} x_j \leq Q_i \equiv$ Max allowable level of ith pollutant. The coefficient, e_{ij}, is the rate of emission (net of transport) of the ith pollutant by the jth activity.

A simple LP was used as a starting point for representing uncertainty in the transfer coefficients:

$$\text{Minimize } cx\text{: } \sum_j (1 - x_j)E_j \tau_{ji} \leq b_i, \quad L \leq x \leq U,$$

where $L_j \geq 0$ and $U_j \leq 1$. Here x_j is the portion of reduction from source j, E_j is its emissions without reduction, and τ_{ji} is the *transfer coefficient* that describes the rate at which the pollutant moves from source j to receptor i. The right-hand side, b_i, is a limit on total emissions. (The indexes can be extended to include more than one pollutant.) The principle uncertainty in this model are the transfer coefficients, which are obtained from a diffusion model averaged over some time period (e.g., a month or a year). Most approaches use a chance constraint model and consider different certainty equivalents, some are linear and some are nonlinear.

In 1971, Kohn published two articles about his model, and his 1978 book gives the detailed LP and its applications for policy analysis in particular airsheds. In 1971, Seinfeld and Kyan extended Kohn's model to an NLP. In 1972, Seinfeld considered the problem of locating monitoring stations, giving an NLP model to minimize measurement error. Also in 1972, Kortanek and Gorr, Blumstein, et al. and Gorr, Gustafson and Kortanek applied semi-infinite linear programming, based on a diffusion model equation for a single source. In 1973, Gustafson and Kortanek presented another nonlinear programming model, based on a similar diffusion model to account for uncertain weather variations (also see the survey by Burton, Pechan and Sanjour in the same book). In 1974, Dathe applied an LP simplification of the

Gorr–Kortanek model, and Darby, Ossenbruggen and Gregory showed how to formulate a nonlinear system as a (linear) network optimization problem to locate air sampling stations. Also in 1974, de Haven presented several NLP models to control automobile emissions; Trijonis used a decomposition strategy to incorporate a particular nonlinearity; Werczberger developed a mixed integer programming model that deals jointly with air quality and land use; and Tietenberg showed how the Baumol–Oates theorem, which was about taxation for water quality control, extends to air quality control. In 1975, Singpurwalla presented an LP model to minimize the total cost of fuel used by sources, subject to each source's energy requirements, and a total air quality limit at each of several receptors. In 1976, Atkinson and Lewis used separable programming as an extension of the early LP models, and Mathur applied NLP with several welfare economic models. Also in 1976, Houghland and Stephens showed how MIP applies to choose locations of monitoring stations, and Carbone and Sweigart used an NLP to address the equity issue. In 1977, Guldmann and Shefer published a simple MIP model to locate plants and choose pollution abatement processes, which they applied to the Haifa area. Also in 1977, Schlottmann published a very detailed LP model for emissions from coal (along with other environmental effects, such as the effect of strip mining on the land). In 1978, Guldmann published a follow-up paper; Carbone et al. used NLP to solve a cooperative game approach to the equity issue; and Hamlen formalized aspects of the models by Baumol and Oates, and Tietenberg. In 1979, Abilock and Fishbone reported a user's guide for MARKAL. They were part of the Brookhaven National Laboratory team to develop this LP model of energy markets, including environmental constraints.

The 1970s was a decade of development that set a foundation for the application of mathematical programming models for air quality control. The next decade brought added sophistication in several ways. In 1980, further integration with energy and the economy appears in the model presented by Lakshmanan and Ratick, which is an early version of EPA's Strategic Environmental Assessment System (SEAS). Also in 1980, Guldmann and Shefer published a book that describes not only extensions of their own model, but also provides a succinct review of other models. In 1982, Miller, Violette and Lent described what was then EPA's Air Quality Model. Brookhaven National Labs continued to develop and apply MARKAL (see Fishbone and Abilock 1981, Rowe and Hill 1989). In the same year, Anderson used nonlinear programming to determine how a cost-minimizing company that emits pollutants would respond to government control, and what the effect of the control would be on the cost and on the amount of pollutants emitted. Also in 1982, Gustafson and Kortanek used duality of a convex programming model for the economic analysis of satisfying a total air quality requirement in a space that has uncertainties due to weather. In 1983, Fortin and McBean used chance constraints to represent uncertainty in the transfer coefficients for the simple LP model of acid rain abatement. In 1984, Fronza and Melli introduced a distribution approach to deal with uncertainty in the LP introduced by Atkinson and Lewis 1976 (they also discussed a chance-constraint model, such as that of Fortin and McBean). In 1985, Morrison and Rubin presented an LP model, called OMEGA, for acid rain control policies (through SO_2 reductions). In 1985 and 1986, Ellis, McBean and Farquhar extended their earlier representation of uncertainty in the transfer coefficients of an LP with chance constraints. In 1986, Guldmann presented two certainty equivalents for the same chance constraint model, which he extended in 1988 to a dynamic model. In 1989, Batterman used LP to select sites for monitoring acid rain.

Ellis continued to show how stochastic programming can be effective for the policy debate on acid rain. In 1988 and 1990, he introduced how to incorporate estimates from different long-range transport models into one multiobjective stochastic program. In 1994, Ellis and Bowman applied this model to Maryland for the 1990 Clean Air Act. In 1991 and 1992, in a pair of companion papers, Trujillo-Ventura and Ellis considered a nonconvex nonlinear program for designing monitoring networks; in 1993, a pair of companion papers by Watanabe and Ellis considered five stochastic programming models and introduced a method to compare their results. In 1990, Alcamo, Shaw and Hordijk published the RAINS model, which contains a description of the modeling system plus some studies on acid rain in Europe. In 1991, Lehmann described how the RAINS model represents uncertainty with LP.

In 1991, Boyd and Uri used NLP to analyze President Bush's Clean Air Plan. The 1992 report by Cohan et al. gave an overview of the GEMINI modeling system, built by Decision Focus, Inc. for the EPA. In the same year, Manne and Wene reported the link of MARKAL with ETA-MACRO to create MARKAL-MACRO (also see Hamilton et al. 1992), which is an NLP. Manne and Richels developed Global 2100, which is an integrated model of the (macro) economy, electricity generation, nonelectric energy supplies, international oil trade, and carbon emissions. Peck and Teisberg extended Global 2100 by adding dependence of CO_2 concentration on CO_2 emissions and global mean temperature, plus a damage function that depends on the global mean temperature, which represents associated costs. They call their system CETA (Carbon Emissions Trajectory Assessment). The DICE model (Dynamic Integrated Climate-Economy), by Nordhaus, uses nonlinear programming to determine a dynamic, economic equilibrium that maximizes a discounted utility function of per-capita consumption and population.

In 1993, Falk and Mendelsohn applied nonlinear programming to determine an optimal level of abatement, trading off cost with damage over time. Altman and Ruszczyński presented a mean-variance model to represent uncertainty. Felder and Rutherford applied the Global 2100 model to consider the effects of international oil trade. Manne and Rutherford extended Global 2100 to obtain a general equilibrium by removing exogenous oil prices and import/export limits to address certain global issues. Peck and Teisberg applied CETA to learn more about the sensitivity of equilibrium solutions to model assumptions, like the dependence of the damage function on the rate of global temperature change, rather than on the level. Duraiappah published his holistic model, which is also a welfare equilibrium using NLP (it differs from Global 2100, DICE, and MARKAL-MACRO).

The recent models differ from the early models in that they are highly aggregate and deal with global issues, such as the greenhouse effect. The early models were detailed and dealt with the quality impact of emissions within particular airsheds.

3. LAND

The earliest paper cited here is 1970, by Edwards, Langham and Headley, who used a welfare economic approach to develop a linear programming model, which they applied to Dade County, Florida. This is one of the land quality control models that is about the agricultural sector.

Early agricultural models used mathematical programming to consider the effects of controls on pesticides and soil erosion. In 1974, Hueth and Regev presented a DP model (but used NLP for analysis). In 1977, Taylor and Frohberg presented an LP model, which they applied to the Corn Belt to analyze impacts of several pollution controls: bans on certain pesticides, soil erosion limits, and soil erosion taxes. In 1978, Taylor, Frohberg and Seitz published an analysis of soil erosion, using the LP model published in 1979, by Seitz et al.

A particularly clear presentation of such LP models for soil erosion was given in a collection of papers edited by Heady and Vocke in 1992. Based on those works, here is a generic LP for land use, where the environmental impact is soil erosion and chemical contamination. It can be extended to include, for example, storage of crops and livestock growth, over a planning horizon.

Basic Dimensions

Regions

i = producers; j = markets;

(there could be others, e.g., water supply in Nicol and Heady 1992).

Classes

k = methods of production (e.g., tilling);
s = soils;
h = chemicals (including pesticides and fertilizers);
p = products (crops and livestock commodities).

Activities

Production

X_{ipk} allocates land in region i to make product p by method k;

Distribution

T_{pij} transports product p from region i to market j.

Equations

Cost

$Z = \sum_{i,p,k} (CX)_{ipk} X_{ipk} + \sum_{p,i,j} (CT_{pij}) T_{pij}$;
$(CX)_{ipk}$ = production cost, which could include taxes;
$(CT)_{pij}$ = the transportation cost, which could include taxes.

Land Use

$L_i = \sum_{p,k} X_{ipk}$.

Balance

$\sum_k R_{ipk} X_{ipk} - \sum_j T_{pij} = 0$,
R_{ipk} = the rate of product p produced per acre in region i using method k.

Demand

$\sum_i T_{pij} \geq d_{pj}$,
d_{pj} = the demand for product p in market j.

Damage

$D_i = \sum_{p,s,k} a_{psk} \alpha_{is} X_{ipk}$;
a_{psk} = the rate of soil damage when producing p with soil class s;
α_{is} = 1 if region i has soil class s (else, $\alpha_{is} = 0$);
$C_{ih} = \sum_{p,k} b_{phk} X_{ipk}$;
b_{phk} = chemical h used by, or produced from, method k to make p ($b_{phk} < 0$ if used, such as a pesticide; $b_{phk} > 0$ if produced, such as nitrogen in cow manure, which can then be used as fertilizer for a crop).

Any of the activities can have bounds, and the distribution network can be sparsely linked, which limits the set of distribution activities. Any equation variable can be constrained, such as land use: $L_i \leq$ available land in producer region i. Typically, the objective is to minimize Z, subject to constraints on the other variables. The environmental variables could be constrained: $D_i \leq$ soil loss limit, and/or $C_{ih} \leq$ contamination level. Alternatively, they could be in the objective (purely, or with a tax), or goals could be established for their levels, which allow violations, but with minimum total (weighted) violation.

In 1980, Yaron and Tapiero presented a collection of papers that includes applications of mathematical programming to agriculture and some connections with water resources, notably for irrigation. Very few deal directly with environmental quality control, but some give useful background and relevant modeling frameworks.

Outside the agricultural sector, in 1972, Plourde used NLP with a welfare economics model to determine optimal waste control, such as garbage. In 1973, Clark gave a very good introduction to the solid waste problems and how mathematical programming applies. Also in 1973, Kühner and Heiler published a literature review, which cites the few LP and MIP models that had been published by that time. Liebman gave another review in 1975.

These early models have similar characteristics. The constraints are mass balance equations, capacity bounds, and disposal requirements. The fundamental model is an LP, which extends naturally to MIP to allow capacity expansion. In extending this to NLP, the main source of nonlinearity is in the cost function. Dynamic models can sometimes use DP, depending upon certain dimensions. In most cases, it is computationally more efficient to use LP, MIP, or NLP, rather than DP.

In 1977, Schlottmann gave a detailed LP model for coal allocation, and he used duality to analyze some environmental and economic impacts. In addition to air pollution (by sulfur emissions), effects on the land were considered, such as by strip mining. The LP has reclamation costs and explicit constraints, which can be analyzed with parametric LP. The 1978 LP text by Greenberg contains a chapter on solid waste management. The simplest model is a transshipment network with sources, intermediate treatment plants, and final disposal sites. This is extended in several ways, for example, using MIP to represent capacity expansion. In 1987, Turnquist considered the problem of finding a route in a network to transport a hazardous material. There are multiple objectives, such as cost, population exposed, and probability of an accident, which are uncertain and vary with the time of day.

In 1990, Stavins addressed environmental concerns of the depletion of forested wetlands. In 1991, ReVelle, Cohon and Shobrys presented a multiple objective MIP model for siting and routing in disposal of hazardous wastes. In 1993, Chang, Schuler and Shoemaker extended solid waste management models by augmenting the effect of recycling. Jenkins gave an economic model that applied nonlinear programming theory, namely the use of the Lagrangian for marginal analysis, to explain the behavior of households and firms. Querner used a risk analysis approach to the economic analysis of severe industrial hazards, and his book contains a chapter (IV) that uses NLP (just Lagrangian conditions for cost minimization, not algorithmic).

4. WATER

In 1962, Lynn, Logan and Charnes published the first LP formulation to minimize the cost of sewage treatment. The governing balance equations were from first principles: input = output. The dominant class of water quality control models, however, pertains to stream pollution. A key to this class of models is the description of BOD and DO by differential equations, and the most used is due to Streeter and Phelps (originally published in 1925, it underwent some modifications by the 1960s). With the (modified) Streeter–Phelps equations as a starting point, Thomann developed a systems model, which he and Sobel used in the first LP model in 1964. Whereas the Thomann–Sobel approach is suggestive of a variety of models, Deininger gave the first detailed LP in his 1965 thesis, using the Streeter–Phelps equations. (In the same year, Sobel presented several LP formulations using Thomann's equations.)

Although the underlying flow equations can differ, the LP structure of these models is the same. The LP is to minimize total cost subject to the flow equations and DO reductions at each segment of a stream, called a *reach*. The essential structure consists of balance equations, like inventories, except the reach is the ordered index, instead of time. Figure 1 illustrates this, where there is a tributary inflow (I_i), wastewater discharge (D_i), and treatment (T_i).

The following comprise the balance equations for the LP model (simplified for this introduction).
Total flow at end of reach i:

$$Q_i = Q_{i-1} + I_i + D_i.$$

Concentration (BOD and/or DO) at end of reach i:

$$C_i Q_i = \alpha_i C_{i-1} Q_{i-1} + \lambda_i I_i + \delta_i D_i + \tau_i T_i.$$

λ_i = concentration in tributary;
δ_i = concentration in wastewater;
τ_i = concentration in treatment;
T_i = level of treatment.

(Parameters α_i, λ_i, δ_i, and τ_i depend upon stream characteristics, like the rate of flow.)

Quality constraints are simple bounds: $\text{BOD}_i \le b_i$ limits the level of BOD at the end of reach i; and $\text{DO}_{ip} \le d_p$ limits the DO deficit at each observation point p that is downstream from reach i (DO_{ip} = DO deficit at p caused by effluent from i).

These works were quickly followed by Kerri (1966, 1967), Johnson (1967), Loucks, ReVelle and Lynn (1967, 1968), and Graves, Hatfield and Whinston (1969). In 1966, Liebman and Lynn published the first DP model of this same problem, allowing nonlinearities, notably in the

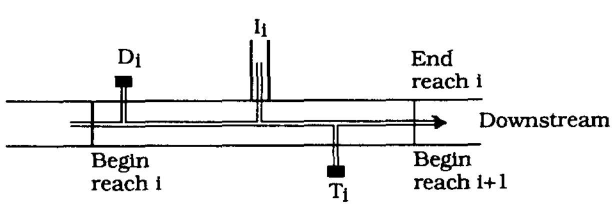

Figure 1. Flows in a stream.

cost function. Also in 1966, a similar model using NLP was presented by Goodman and Dobbins along with a FORTRAN library that contains not only the model equations, but also an implementation of steepest descent to compute an optimum. In 1968, the basic LP model was central to the systems approach by Anderson and Day. Also in 1968, Clough and Bayer extended the LP to an NLP model, and Matalas used NLP to optimize the location of gaging stations. In 1969, Shih and Krishnan published a DP model for wastewater treatment design.

The first book that includes mathematical programming models for water quality control was by Thomann, in 1972. A few years later, the basic model began to appear in LP textbooks, such as by Greenberg (1978), and in water management textbooks, such as those by Loucks, Stedinger and Haith (1981), and by Haith (1982).

The 1970s brought a variety of extensions in surface water quality control. In 1970, Dysart and Hines considered interaction effects of pollutants, Horowitz extended the use of NLP to the problem of minimizing treatment cost, and Jaworski, Weber, Jr. and Deininger gave a different kind of DP model. Also in 1970, Hass used a variant of the early LP models to find appropriate taxes levied on polluters that gave them the economic incentive to meet the quality standards. In 1971, Ecker and McNamara gave a geometric programming model for the design of waste treatment plants; Haimes developed a multilevel approach, which is an application of the Generalized Lagrange Multiplier Method to decompose the water resource system model. In 1972, Dorfman and Jacoby, and Loucks and Jacoby began to tie in the early LP allocation models with Pareto optimality and explicitly represent political power as an element of environmental decision making. Also in 1972, Chi presented an NLP model to determine where and when to build tertiary plants as part of the pipeline design; and Giglio and Wrightington presented a game model to address the equity issue. In 1973, Hwang et al. used NLP to consider several measures of water quality at once, rather than just BOD removal or increased DO concentration alone. Chang and Yeh presented a DP model to allocate aeration capacity to each of a series of aerators. Also in 1973, Miller and Byers presented a public investment MIP model and used parametric programming to show frontier functions of dollar benefit and level of sediment. In 1975, Ecker published a geometric programming model for the DO allocation problem with a more accurate (than linear) approximation of basic relations and cost functions, extended the following year by McNamara (also see Ecker and McNamara, 1971). Also in 1975, Arbabi and Elzinga presented a general NLP model to minimize total treatment cost under a variety of conditions. In 1976, Alley, Aguado and Remson used LP to select pumping rates to minimize cost by approximating steady-state flow conditions with finite differencing. In the same year, Futagami, Tamai and Yatsuzuka used LP to choose discharge rates that maximize water quality, combined with a finite element method to solve the flow equations. In 1976, Brill, Liebman and ReVelle presented several LP models to address the equity issue. In 1978, Lohani and Thanh extended the early DP model to address the question of tax equity among polluters, and the following year they used a chance constraint to represent uncertainty in the stream flows.

All of the citations in the 1960s are for surface water quality control. Groundwater systems seem to have developed about a decade later, even though the transport equations are essentially the same. The earliest paper given here is by Aguado et al. in 1974.

The way LP comes into play is by using a discrete approximation to the differential equations that describe the flows. To illustrate, consider just one aquifer. The differential equation describing steady-state flow is:

$$d^2h/dx^2 = W/T \quad \text{for } 0 < x < L,$$

where h = groundwater head above datum; x = spatial coordinate (say horizontal, with h vertical); W = discharge/recharge rate from/to aquifer; and T = transmissivity (constant). The boundary conditions are $h(0) = h_0$ and $h(L) = h_4$ if we use a grid of four points and apply finite differencing:

$$\begin{aligned} -2h_1 + h_2 - W_1(\Delta x)^2/T &= -h_0 \\ h_1 - 2h_2 + h_3 - W_2(\Delta x)^2/T &= 0 \\ h_2 + 2h_3 - W_3(\Delta x)^2/T &= -h_4. \end{aligned}$$

Also, $W \geq 0$ and $h_0 \leq h_1 \leq h_2 \leq h_3 \leq h_4$.

This gives a system of linear equations and inequalities that comprise the hydrologic constraints in an LP. Other constraints can be added, such as a range on total aquifer production: $L \leq W_1 + W_2 + W_3 \leq U$. There are various objective functions, depending on the intended use of the model. One management goal is head maintenance, which is formulated as maximizing $h_1 + h_2 + h_3$.

Quality is measured at the *head values* (h_i), which can appear as a constraint and/or in the objective. The finite differencing method leads to an LP formulation, but it must assume constant transmissivity, which can vary markedly, except on small areas. More generally, this LP is not the model of choice, considering the NLP approaches—for example, Gorelick, Remson and Cottle (1979), Ahlfeld et al. (1988), Gorelick (1990), and Ahlfeld (1990).

In 1976, Willis applied mixed integer programming to consider wastewater treatment in conjunction with reservoir supply in the selection of process units in the system design. In 1979, Willis presented an LP planning model with multiple objectives, and Gorelick, Remson and Cottle applied parametric linear programming to answer such questions as: What river concentration would be permitted if the most restrictive local groundwater quality limit were removed?

In the 1980s the mathematical programming models for surface and groundwater quality began to come together in the sense that we could see some of the same people working on these problems. We could also see models that apply to both surface water and groundwater quality control.

In 1982, Gorelick and Remson presented an LP model for groundwater quality control (Gorelick separately showed how the dual LP has some computational advantages), and an MIP model to locate waste disposal facilities. Also in 1982, Yakowitz gave a taxonomy for applying DP for water quality control; and Fiacco and Ghaemi provided a thorough sensitivity analysis of Ecker's geometric programming model (also see Fiacco 1983). In 1983, Gorelick gave a timely review; and Fishelson used DP to maximize the present value of water quality. In 1984, Colarullo, Heidari and Maddock presented a quadratic programming model to determine discharge rates that minimize total cost, which was a basis for more general NLP models in the later 1980s.

Surface water quality models began to consider uncertainty in 1985 (Burn and McBean). In 1986, Tung used an LP model, which he extended to deal with uncertainty by a chance constraint. In the same year, Pintér and Somlyódy presented an integer programming model for monitoring water quality (the decision variables are sample sizes, and the model could apply to monitoring air quality). Ahlfeld studied groundwater quality remediation extensively with Mulvey (1987) and Pinder (1986) and Wood (1988). In 1986 and 1987, Fujiwara, Gnanendran and Ohgaki used chance constraints to represent uncertainty in the downstream impacts of BOD removal, which comprise the quality constraints of the early models. In 1987, Clark and Adams presented an MIP model for granular activated carbon (GAC) treatment of surface or ground water. Also in 1987, Wagner and Gorelick presented a method to deal with parameter uncertainty in the hydraulic equations. In 1988, Meyer and Brill used integer programming (specifically, maximal location covering) iteratively with simulation to determine optimal well locations (the simulation dealt with uncertainty in whether wells can detect contamination at each of several locations). In 1987 and 1989, Ellis considered uncertainty with a DP approach. In 1989, Esogbue gave another taxonomy for applying DP for water quality control.

In 1990, Andricevic and Kitanidis used differential dynamic programming for real-time adaptive control of aquifer management in the presence of uncertain parameters. In the same year, Gorelick reviewed the methodology to combine NLP with simulation equations. In 1991, Lee and Kitanidis presented an adaptive control model that responds to real-time measurements of uncertain parameters, like transmissivities. In 1992, Burn and Lence compared LP formulations that varied by the choice of objective function.

Despite the large amount of research activity applying mathematical programming to water quality control, the American Society of Civil Engineers (ASCE) conference in 1989 (Harris) had only one paper on the subject. The most recent conference in 1993 (Hon) had only two.

In 1993, there were many papers, compared to previous years, but most dealt with the computational aspect of optimization; only a few extended the models in new ways. Cardwell and Ellis extended the 1966 DP model by Liebman and Lynn by considering a stochastic state transition function (with known probability distribution). Berkemer, Makowski and Watkins presented a decision support system based on a multiobjective mixed integer programming framework. Ruszczyński gave a succinct review of all types of mathematical programming models under steady-state conditions. Ostfeld and Shamir considered the removal of the assumption of steady-state conditions. Culver and Shoemaker continued their approach to apply differential dynamic programming for optimal control of groundwater remediation. Whiffen and Shoemaker extended earlier models by considering uncertainty and the effect of two types of errors: bias, such as misestimating the average hydraulic conductivity, and scatter, which is error in the node values for the mesh. Georgakakos and Yao presented a theory of state set control that applies both to streams and groundwater. Hudak and Loaiciga applied the Generalized Lagrange Multiplier Method to decompose an MIP into independent 0-1 knapsack problems, one for each hydrostratigraphic interval in each of two classes of sites. Jemaa and Mariño applied DP to minimize total square deviation from target values, where there is a feedback control mechanism. Marryott, Dougherty and Stollar applied simulated annealing to solve the basic NLP for groundwater remediation. Mhaisalkar et al. applied DP to select process units for the design of a wastewater treatment plant. Shafer and Varljen used the penalty function method of NLP to models that had been published.

One of the newest developments is the innovative design of ESIS (Environmentally Sensitive Investment System), by Pintèr et al. (1993). This is a sophisticated system to assist both industry and government in policy analysis. It incorporates artificial intelligence, data base technology, and visualization tools with economic models and operations research techniques. The core of ESIS is a generic nonlinear program, which can be complex (but need not be convex). The system has been applied to the pulp and paper industry in Canada, and its conceptual foundations, built on mathematical programming, have the potential to apply more generally to understanding the economic impacts of environmental controls. In 1994, Ahlfeld and Heidari gave a current account of optimal groundwater remediation, showing how LP applies under simplifying assumptions about the transport equations. Chan used a Monte Carlo method to solve a chance-constrained LP model for aquifer management.

5. INTEGRATED MODELS

Over the three decades that people have been developing the applications of mathematical programming for environmental control, there have appeared what I call *integrated models*. These pertain to chemical pollution, mostly in air and water, and do not deal with some of the details that are included in the environment-specific models. Their aim is to extend and/or apply economic theory to account explicitly for control of damage to the environment.

A (simplified) generic model has the form:

$$\text{Maximize } u(f(x), d(y)): e(x) = 0, g(y) = 0,$$
$$h(x, y) = 0, L \leq (x, y) \leq U,$$

where x is a vector of economic variables (e.g., income), and y is a vector of environmental variables (e.g., emissions from production). The objective is a utility function, u, whose arguments are a benefit function, f, and a damage function, d; typically, $u(f, d) = f - d$. Each set of variables can have its own constraints, and there is a system of coupling equations that relates economic and environmental variables.

From 1968–1979, Kneese and Bower launched a series of monographs. In 1970, Kneese, Ayres and d'Arge used LP, and the term *residuals management* emerged, where a 'residual' is defined as what remains after inputs are converted to outputs in a production or consumption industry (see Russell 1973, Russell and Vaughan 1974). This could take the form of solid waste or chemicals that enter the environment. A basic residuals management system is defined in terms of activities, like production and consumption, and receptors: people, animals, plants, and inanimate objects. Other early LP models were presented in this context, for example, Bohm and Kneese (1971). In 1972, d'Arge, and Russell and Spofford, provided frameworks for residuals management, using mathematical programming models, which Spofford extended the following year. In 1974, Cumberland presented an integrated LP model, using input-output relations among economic variables (like production and income), augmented with pollution emissions into the air and water, whose totals are limited by a quality standard.

Also from the early works of Kneese and Bower, *environmental economics* emerged as a subdiscipline of welfare economics. In 1974, Maler published the first comprehensive presentation of environmental economics that is based on mathematical programming, notably on Lagrangian duality. (This was also the inaugural year of the *Journal of Environmental Economics and Management*.) In 1975, Baumol and Oates used a resource allocation approach, with Lagrangian analysis to determine optimal pricing of exhaustible resources. In 1976, the welfare economic foundations were extended by Pearce, notably by his greater focus on Pigovian taxes to abate pollution. The same year Parvin and Grammas extended the use of input-output economic systems with a quadratic program that seeks to minimize total damage cost, and Adar and Griffin analyzed the effects of uncertainty. In 1977, Nijkamp presented one of the first textbooks on environmental economics, explicitly using mathematical programming formulations and analysis techniques. In 1978, James, Jansen and Opschoor published a broader-based book that describes a little mathematical programming in the context of general equilibria. In 1979, Field and Willis published an extensive annotated bibliography on environmental economics with insights into its evolution (those that use mathematical programming are included here).

Nijkamp extended his work in 1980, and the 1982 book by Dasgupta gives an elementary introduction to some of the game-theoretic foundation, with specific attention to air and water models that stem from the integrated approach. The 1983 book edited by Lakshmanan and Nijkamp contains papers that use multiobjective programming with a focus on the linkage among energy, environment, and the economy. In 1984, Hafkamp introduced a "multilayer" approach. The 1987 book by Johansson applied some NLP (though less mathematically than Mäler) and considered its inappropriateness for binary variables (though he did not apply MIP).

In 1992, Siebert revised his earlier (1981) book, and gave a detailed development of environmental economics from the approach of optimal resource allocation. In 1993, van Ierland published a monograph that begins with some background on the development of environmental economics and the uses of taxation and regulation in models that seek Pareto optima for minimizing abatement and damage costs. (Although written in a more generic context, the detailed LP presented in Chapter 7 is specifically for air quality control.) There have been literally thousands of papers on environmental economics, some of which use mathematical programming (at least Lagrangian duality). An excellent entrance into this literature is given by Hoagland and Stavins (1992).

6. SOME STATISTICS ABOUT THE LITERATURE

Tables I–III give a list of journals cited in this survey for air, land, and water quality control, respectively, showing the number of citations, the earliest and the newest. (There could be other citations in the same year, but only one is given in the tables.) Table IV gives the same information for what I call integrated models, which are welfare economic models that mostly use NLP techniques.

Here are some other journals whose titles suggest they might have published relevant papers, but I was unable to find any that uses mathematical programming:

Environmental and Resource Economics
EPA Journal
Environment
Journal of Energy Engineering
Journal of Energy Resources Technology
Journal of Industrial Engineering

Table I
Journals Cited in this Survey for Air Quality Control

Journal	Number	Earliest	Newest
American Economic Review	1	1974 (Tietenberg)	
Applied Mathematical Modeling	1	1993 (Wanatabe and Ellis)	
ASCE Journal of Environmental Engineering	5	1974 (Darby et al.)	1994 (Ellis and Bowman)
Atmospheric Environment	7	1972 (Seinfeld)	1992 (Trujillo-Ventura & Ellis)
Computers and Operations Research	1	1993 (Wanatabe and Ellis)	
Econometrica	1	1971 (Kohn)	
Energy Research	1	1981 (Fishbone and Abilock)	
Engineering Optimization	1	1992 (Ellis)	
Environment and Planning[a]	1	1972 (Gorr et al.)	
Environmental Science and Technology	2	1974 (Trijonis)	1988 (Ellis)
European Journal of Operational Research	1	1990 (Ellis)	
Geographical Analysis	1	1986 (Guldmann)	
Journal of the Air Pollution Control Association	3	1977 (Ott)	1985 (Morrison and Rubin)
Journal of Environmental Economics and Management	6	1976 (Atkinson and Lewis)	1993 (Welsch)
Journal of Resource Management and Technology	1	1993 (Chang et al.)	
Management Science	2	1971 (Kohn)	1976 (Carbone and Sweigart)
Operations Research	2	1972 (Blumstein et al.)	1973 (Kohn)
Papers of the Regional Science Association	1	1974 (Werczberger)	
Socio-Economic Planning Sciences	4	1971 (Seinfeld and Kyan)	1978 (Guldmann)
The Energy Journal	1	1992 (Peck and Teisberg)	
Water Resources Bulletin	1	1992 (Okada and Mikami)	

[a]In 1974, this split into parts A and B (part B is planning and design, which has no mathematical programming models for environmental quality control).

Natural Resource Modeling
Resource and Energy Economics
The Annals of Regional Science
The Journal of Energy and Development

Table V gives a distribution of publications that are cited here, except it does not include textbooks that report previously published results or just general background. Combinatorial optimization models are pure integer programs, but they are counted as MIP. Some books and reports are cited for the relevant background they provide, but do not get counted in the table because they do not present a specific mathematical programming model for environmental quality control.

The table also excludes 34 citations for efforts that were made to build an integrated framework for economic analysis. As stated earlier, these works attempt to build economic theories of the environment, which use NLP analysis equivalent to equilibrium theory. They partly include air and water pollution, but they do not include some of the other issues, such as effects on land.

In some cases, the classification could be ambiguous. For example, some of the DP models are solved by NLP

Table II
Journals Cited in this Survey for Land Quality Control

Journal	Number	Earliest	Newest
American Journal of Agricultural Economics	3	1974 (Hueth and Regev)	1977 (Taylor and Frohberg)
Canadian Journal of Economics	1	1972 (Plourde)	
Journal of Environmental Economics and Management	1	1990 (Stavins)	
Journal of Resource Management and Technology	1	1993 (Chang et al.)	
Journal of Soil and Water Conservation	1	1978 (Taylor et al.)	
Land Economics	1	1979 (Seitz et al.)	
Natural Resources Journal	1	1970 (Edwards et al.)	
Transportation Science	1	1991 (ReVelle et al.)	

Table III
Journals Cited in this Survey for Water Quality Control

Journal	Number	Earliest	Newest
Advances in Water Resources	1	1986 (Ahlfeld et al.)	
Annals of Operations Research	1	1991 (Pinter)	
ASCE Journal of Environmental Engineering	14	1977 (Grady)	1993 (Mhaisalkar et al.)
ASCE Journal of Hydraulics	3	1974 (Aguado and Remson)	1976 (Futagami et al.)
ASCE Journal of Sanitary Engineering	8	1966 (Goodman and Dobbins)	1971 (Bishop and Hendricks)
ASCE Journal of Water Resources Planning and Management	7	1986 (Tung)	1994 (Chan)
Biotechnology and Bioengineering	1	1974 (Middleton and Lawrence)	
Canadian Operational Research Society Journal	1	1968 (Clough and Bayer)	
CRC Critical Reviews in Environmental Control	1	1977 (Tyteca et al)	
Ground Water	1	1974 (Remson et al)	
IEEE Transactions on Systems Science and Cybernetics	1	1970 (Dysart and Hines)	
International Journal of Water Resource Development	1	1983 (Lohani and Lee)	
Journal of Environmental Economics and Management	2	1974 (Russell and Vaughan)	1976 (Herzog)
Journal of the Water Pollution Control Federation[a]	12	1962 (Lynn et al.)	1976 (Middleton and Lawrence)
Management Science	3	1967 (Loucks et al.)	1975 (Ecker)
Mathematical Programming	1	1990 (Gorelick)	
Operations Research	3	1978 (Jarvis et al.)	1982 (Fiacco and Ghaemi)
Water Resources Bulletin	9	1970 (Keegan and Leeds)	1984 (Colarullo et al.)
Water Research	1	1971 (Fan et al.)	
Water Resources Research	35	1967 (Johnson)	1993 (Whiffen and Shoemaker)

[a]In 1989, this split into *Water Environment & Technology* and *Research Journal of the Water Pollution Control Federation* (also called *Water Environmental Research*).

Table IV
Journals Cited in this Survey for Integrated Quality Control

Journal	Number	Earliest	Newest
American Economic Review	1	1973 (Russell)	
Environment and Planning[a]	1	1973 (Tihansky)	
Journal of Economic Literature	1	1976 (Fisher and Peterson)	
Journal of Economic Theory	1	1971 (Keeler et al.)	
Journal of Environmental Economics and Management	4	1974 (Tietenberg)	1976 (Parvin and Grammas)
Journal of Environmental Systems	1	1978 (Nayayan and Bishop)	
Management Science	1	1973 (Ferrar)	
Papers of the Regional Science Association	1	1972 (Mathur and Yamada)	
Socio-Economic Planning Sciences	1	1973 (Muller)	

[a]In 1974, this split into parts A and B (part B is planning and design, which has no mathematical programming models for environmental quality control).

Table V
Distribution of Publications

	Air					Land					Water				
	LP	MIP	NLP	DP	Total	LP	MIP	NLP	DP	Total	LP	MIP	NLP	DP	Total
1962–69	1	0	0	0	1	0	0	0	0	0	13	2	4	3	22
1970–79	13	5	17	0	35	7	3	2	0	12	24	7	21	12	64
1980–89	17	1	8	0	26	2	1	1	0	4	7	7	22	6	42
1990–94	8	1	17	0	26	7	2	5	0	14	5	3	8	10	26
Total	39	7	42	0	88	16	6	8	0	30	49	19	55	31	154

(LP = Linear Programming, MIP = Mixed Integer Programming, NLP = Nonlinear Programming, DP = Dynamic Programming) methodology. By this I mean that there is no use of the fundamental DP recursion. Instead, an NLP method, like gradient projection, is applied to the total model at once. For this reason, the classification was entered as NLP. Conversely, a paper that uses DP to solve a model is classified as DP even if the model is static, that is, it does not explicitly have a time index.

Thus, the statistics must be used with care in drawing inferences. With this caveat in mind, here are some observations.

- LP tends to be the mathematical programming model of choice when first addressing a problem with many decision variables and relations.
- MIP is used, as an extension of LP models, to represent capacity expansion (e.g., treatment plants) or location decisions (e.g., wells). Other MIP models pertain to hard combinatorial optimization problems, such as finding routes for complex transport problems.
- NLP is used to improve a model's validity, or accuracy. One source of nonlinearity is the cost function. Another source is the approximation of the differential equations that describe hydraulic and aerodynamic phenomena.
- Most of the integrated modeling and analysis, which come from welfare economics, use NLP. Lagrangian duality applies when benefit and damage functions are presumed strictly concave and strictly convex, respectively. Without the strong duality, Lagrangian analysis still applies to derive necessary conditions about the structure of an economic equilibrium.
- Uncertainty is often represented by chance constraints, which retains an LP structure under assumptions of independence. With joint chance constraints, the assumptions are such that a certainty equivalent is represented by a quadratic constraint, which is sometimes presumed convex (erroneously). Other approaches have been considered, leading to complex (nonconvex) NLP models. Other models that deal with uncertainty also introduce nonlinearities by seeking a minimum variance and/or violation penalty.
- Multiple objectives, as in Pareto optima, are typically reformulated as a weighted sum. Although multiple objectives were considered periodically since 1973 (Cohon and Marks), they have only recently become recognized as crucial in modeling environmental control. Formulating, solving, and analyzing multiobjective mathematical programs is regarded by leading researchers in environmental control as an important frontier.
- DP is used for computational efficiency when the state space can be defined appropriately. Related optimal control techniques, especially the more recent methods of differential dynamic programming, are effective in representing feedback mechanisms for adaptive control. However, it appears that DP has been used predominately for water quality control, not for air or land. Dynamic models, other than for water quality control, use LP or NLP for solution computation and analysis.
- Environmental economics has emerged as a branch of welfare economics, and this has complemented the engineering approaches to environmental control. Many of these economists use NLP analysis techniques, notably Lagrangian duality, whereas engineers tend to focus on algorithms to solve design and operational problems.
- Besides the environmental economics approach, there are opportunities for integrating approaches, across environmental control. For example, although it is not clear how acid rain relates to global climate changes, control policies could be designed to address both simultaneously.
- Decomposition strategies have been used to formulate and manage large-scale models. In addition to better model management, this generally results in more efficient computation. In some cases, the decomposition separates primary controls (like discharge rates) from their effects (obtained by solving a system of differential equations). In some cases, a model is mostly linear, and decomposition is used to separate this portion from the much smaller nonlinear portion. In other cases, the decomposition paradigm is to partition the model into modules that separate economic variables (like income) from physical variables (like emissions). The insightful 1994 report by Murphy puts decomposition into perspective and its effect on convergence.
- Most of the research to date has been on water quality control. Recent trends are more air quality modeling, particularly in conjunction with energy modeling, using welfare economic models. Recent research in water quality control has been primarily algorithm improvements.
- One problem that has received limited attention is monitoring. Although some mathematical programming models have been presented for air and water separately, there is an opportunity to develop a general model, separating the mathematical statistics from the optimization. The decision variables are the location of sampling points, sample sizes, and frequencies. In its general form, the mathematical program is dynamic, nonlinear, integer, stochastic, and has multiple objectives.

7. SUMMARY AND CONCLUSIONS

For more than three decades, researchers have developed the applications of mathematical programming models for environmental control, beginning with water quality. Most of the results, especially for the past decade, have been reported by civil engineers and economists, usually separately from each other.

Air quality control has undergone recent advances, particularly its integration with energy and the rest of the economy. Further modeling developments have occurred in only this decade: DICE, Duraiappah's model, Global 2100 (and its variants), MARKAL-MACRO, and RAINS. Applications of mathematical programming for land quality control have been in the agricultural sector, pertaining to soil erosion and contamination, and outside the agricultural sector for solid and hazardous waste transport. (Recall that wildlife and related ecological issues have been ignored in this study.) The recent papers on water quality control have been primarily on computing solutions. The main advances in modeling occurred during the first two decades.

While some environment-specific models, like the recent air quality models, use an integrated modeling approach, I use the term *integrated modeling* here to mean something more generic, not environment-specific, which has come to be known as *environmental economics*. The central idea is to have a damage function included in the net benefit function, and use welfare economic theory of production and consumption to represent relations. Some of the models that result from this approach use mathematical programming, mostly NLP analysis techniques. Other integrated approaches are sparse.

For entrance into this field, I suggest starting with the elementary models presented by Haith in 1982. I found, however, that it was useful going to original references. For air quality, Kohn's 1978 book is definitive, and the contemporary books by Manne and Richels, Nordhaus, Duraippah, and Alcamo, Shaw and Hordijk offer additional insights. The 1980 book by Guldmann and Shefer offers a succinct introduction that includes diffusion models. For land quality, the 1992 collection of papers in Heady and Vocke is definitive for the agricultural sector, and Clark's 1973 review is a good introduction to nonagricultural models. For water quality, Deininger's 1965 thesis and the 1972 book by Thomann are good starting points. (A quicker introduction is the 1967 article by Loucks, ReVelle and Lynn.) The 1981 book by Loucks et al, the 1987 book by Willis and Yeh, and the 1982 survey by Yakowitz offer good introductions. For integrated models, look at the 1974 monograph by Mäler, and the 1992 report by Hoagland and Stavins.

Current trends emphasize dynamic, multiobjective mathematical programs under uncertainty. Damage functions and representations of transport continue to be a modeling concern. Beyond the mathematics, there are implementation considerations; in particular, there is a need for visualization tools. The most comprehensive state-of-the-art is given by Jones (1994).

In conclusion, according to this survey, fully integrated frameworks, based on both engineering and economic principles, have been sparse. This reveals opportunities for social progress in effective environmental control through the use of mathematical programming models. Unfortunately, there appears to be a cultural gap between those who could provide mathematical programming models, analysis techniques, and algorithms and those who could use them. Fortunately, for those seeking a socially important research arena, this gap poses an opportunity to use mathematical programming for environmental control.

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REFERENCES

ABILOCK, H., AND L. B. FISHBONE. 1979. *User's Guide for MARKAL (BNL Version)*. Brookhaven National Laboratory, Upton, N. Y.

This is an LP that represents energy supply, conversion and demand. Air quality control constraints are included, referring to "environmental residual" as any polluting emission, such as CO_2 during electricity generation. (Also see Fishbone and Abilock, 1981.)

ADAR, Z., AND J. M. GRIFFIN. 1976. Uncertainty and the Choice of Pollution Control Instruments. *J. Envir. Econ. and Mgmt.* **3**(3), 178–188.

This allows uncertainty in the net benefit function in a generic welfare model, and defines the objective to be its expected value. The "instruments" are taxes, regulation constraints, and auctions (allowing polluters to trade rights). The authors conclude that uncertainty in the damage function has "absolutely no effect" on the choice of policy instrument.

AGUADO, E., AND I. REMSON. 1974. Groundwater Hydraulics in Aquifer Management, *ASCE J. Hydraul.* **100**(1), 103–118.

This does not model groundwater quality specifically, but it gives a good introduction to formulating LP models from the hydraulic equations.

AGUADO, E., I. REMSON, M. F. PIKUL AND W. A. THOMAS. 1974. Optimal Pumping for Aquifer Dewatering. *ASCE J. Hydraul.* **100**(7), 869–877.

Using a finite difference approximation of the Streeter–Phelps equations, this is an LP to determine the number of wells, their locations and pumping rates to minimize cost.

AHLFELD, D. P. 1990. Two-Stage Ground-Water Remediation Design. *ASCE J. Water Res. Plan. and Mgmt.* **116**(4), 517–529.

The two stages refer to containment and maintenance. Pumping strategies, which apply to the containment stage, are determined in the design process. Maintenance costs include remediation, which is affected by the pumping strategy chosen in the design. This paper considers the two-stage model for determining an optimal pumping strategy. The model is similar to Ahlfeld and Mulvey (1987).

AHLFELD, D. P., AND M. HEIDARI. 1994. Applications of Optimal Hydraulic Control to Groundwater Systems. *ASCE J. Water Res. Plan. and Mgmt.* **120**(3), 350–365.

The problem is to find a least-cost solution for withdrawal and recharge of groundwater. Although the objective function is linear in the pumping variables, the mathematical program is complicated by the need to solve a dynamical system of equations, generally done by simulation, for any particular pumping policy. The simulation results give "head values" at specified points. The head values must satisfy linear constraints, including simple bounds, so a particular pumping policy can be infeasible. In the less complicated case, the head values are assumed to be linear functions of the withdrawal and recharge rates, in which case the complete model is a linear program.

AHLFELD, D. P., AND J. M. MULVEY. 1987. Optimal Groundwater Quality Remediation: An Overview. In Lev et al., 121–135.

The problem is one of hydraulic control. The variables are recharge (q_j^+) and withdrawal (q_j^-) rates at specified pump locations (j): Minimize $\sum_j (q_j^+ + q_j^-)$ subject to $0 \leq q_j^{\pm} \leq U_j^{\pm}$ and $c_{iT}(q_j^+ - q_j^-) \leq b_i$, where c_{iT} is the concentration of contamination at the last time period (T) at the ith observation point for a given pumping policy. A deterministic simulation (i.e., stepping through the dynamical system of hydraulic equations) is used to evaluate c_{iT}. (This could incorporate uncertainty, where c_{iT} is then an expected value.) In this survey, this is classified as an NLP model, complicated by the use of simulation to evaluate the contamination constraint function, c_{iT}.

AHLFELD, D. P., J. M. MULVEY AND G. F. PINDER. 1986. Designing Optimal Strategies for Contaminated Groundwater Remediation. *Adv. in Water Resour.* **9**(2), 77–84.

This is an NLP model to locate pumps and set their rates of discharge, using deterministic simulation (i.e., stepping through the dynamical system of hydraulic equations) to evaluate each policy. This paper extends earlier models by considering plume stabilization and removal.

AHLFELD, D. P., J. M. MULVEY, G. F. PINDER AND E. F. WOOD. 1988. Contaminated Groundwater Remediation Design Using Simulation, Optimization, and Sensitivity Theory 1. Model Development. *Water Resour. Res.* **24**(3), 431–441.

This defines the NLP model by Ahlfeld and Mulvey (1987) as "the regulatory constraint optimization formulation." An alternative model replaces the contamination concentration constraints ($c_{iT}(q_j^+ - q_j^-) \leq b_i$) with the objective to minimize $\sum_i V_i R_i c_{iT}(q_j^+ - q_j^-)$, where V_i is the volume of the aquifer at node i, and R_i is the "retardation coefficient" that is defined in the transport equations used for the simulation. These can be combined by using the Lagrangian: Min $(\sum_j (q_j^+ + q_j^-) + \sum_i \lambda_i c_{iT}(q_j^+ - q_j^-)) + \lambda b$. This approach is used for a unified model theory and for sensitivity analysis, but there is no convexity theory to ensure its correctness.

AHLFELD, D. P., J. M. MULVEY, G. F. PINDER AND E. F. WOOD. 1988. Contaminated Groundwater Remediation Design Using Simulation, Optimization, and Sensitivity Theory 2. Analysis of a Field Site. *Water Resour. Res.* **24**(3), 443–452.

This applies the two models developed in Ahlfeld et al. (1988), contaminant minimization, and regulatory constraint optimization, to an aquifer in Woburn, Massachusetts.

AHN, S. J. 1992. MARKAL-MACRO/2—An Energy-Environmental Modeling System. Engineer's Thesis, Department of Operations Research, Stanford University, Stanford, Calif.

This is the all-GAMS version of MARKAL-MACRO, which is an NLP to represent economic and air quality impacts of policies associated with energy processes. (See Abilock and Fishbone (1979), Hamilton et al. (1992), Manne and Wene (1992), Rowe and Hill (1989).

ALCAMO, J., R. SHAW AND L. HORDIJK (EDS.). 1990. *The RAINS Model of Acidification*. Kluwer, The Netherlands.

This reports the results of IIASA's Acid Rain Project from the period 1983–1988, which was stimulated by concerns for long-distance effects of polluting emitters throughout Europe, primarily sulfur and nitrogen deposition. Chapter 1 gives an overview of the RAINS (Regional Acidification INformation and Simulation) model, and other chapters present studies that use RAINS. There are three primary modules: pollution generation and control, atmospheric transport and deposition, and environmental impacts. Each module has submodels, and optimization can be used to determine control strategies that satisfy goals and constraints on the environmental impacts. (There is another use of RAINS, going the other way, where the user enters the control strategies and finds the environmental impacts.) Subject to these constraints and goals, plus internally defined transport equations, a cost-minimizing strategy is found. Chapter 9 gives more information about the underlying LP used for optimization.

ALLEY, W. M., E. AGUADO AND I. REMSON. 1976. Aquifer Management Under Transient and Steady-State Conditions. *Water Resour. Bull.* **12**(5), 963–972.

This is an LP model to select pumping rates to minimize cost. Finite differencing is used to approximate steady-state flow conditions with linear equations.

ALTMAN, A., AND A. RUSZCZYNSKI. 1993. Cost-Effective Sulphur Emission Reduction Under Uncertainty. Working Paper WP-93-62, IIASA, Laxenburg, Austria.

This is a (convex) quadratic programming model whose objective is a weighted sum of the mean and variance of violating air quality standards. The variance represents risk, and the quadratic is transformed to an equivalent separable form.

ANDERSON, M. W., AND H. J. DAY. 1968. Regional Management of Water Quality—A Systems Approach. *J. Water Poll. Control Fed.* **40**(10), 1679–1687.

This applies the LP model of Deininger (1965) to the Miami River Basin in Southwestern Ohio.

ANDERSON, JR., R. J. 1982. Using Mathematical Programming Models for Cost-Effective Management of Air Quality. In Fronza and Melli, 59–74.

The mathematical programming models, described fully, are aimed at determining how a cost-minimizing company that emits pollutants would respond to government control, and what the effect of the control is on the cost and on the amount of pollutants emitted. The cost of a single receptor is a function of the level of emission and a random variable that represents uncertainty. A second random variable, presumed independent of the cost uncertainty, affects the rate of emissions. The first model is simply to minimize the expected value of the cost subject to a constraint on the expected value of total emissions. The decision variable is the level of control used to reduce pollution. The second model is similar, except a function is introduced that maps a management control variable to a level of emission. The third model allows the receptor to purchase a permit in competition with other receptors. With total pollution constrained, the awarding agency limits the awards and allows bidding.

ANDRICEVIC, R., AND P. K. KITANIDIS. 1990. Optimization of the Pumping Schedule in Aquifer Remediation Under Uncertainty. *Water Resour. Res.* **26**(5), 875–885.

This seeks to minimize expected cost, where the decision variables are the pumping rates of an aquifer system. The cost function is expressed as the sum of a deterministic function of initial flow rates and final states plus another function times the variance (the mean error is assumed to be zero). The state transition is uncertain, and this method uses differential dynamic programming to obtain a best estimate. Then, a first-order expansion is applied to derive a linear control system. The authors suggest that this method performs better than other approaches that use a deterministic equivalent, notably those that substitute best estimates. This claim is supported by using each solution in a test model that enumerates all possible realizations of the uncertainties.

ARBABI, M., AND J. ELZINGA. 1975. A General Linear Approach to Stream Water Quality Modeling. *Water Resour. Res.* **11**(2), 191–196.

This begins with what has become a standard NLP formulation of obtaining a least-cost treatment policy. The decision variables are levels (x_j) of BOD concentrations discharged by each of several plants (j). The levels are bounded, $L \leq x \leq U$, and discrete values can be imposed. Treatment costs, $C_j(x_j)$, comprise the domain of an arbitrary cost function, $F(C)$. DO concentration at the beginning and end of each reach of each plant are functions, $B_j(x)$ and $E_j(x)$, respectively, which have lower bound constraints. The DO concentration levels are obtained from (linear) flow balance equations that have become standard in stream modeling.

ARTHUR, L. M. 1988. The Greenhouse Effect and the Canadian Prairies: Simulation of Future Economic Impacts. Chapter 11 in Johnston et al., 226–243.

This is an elementary LP model, based on an economic input-output equation.

ATKINSON, S. E., AND D. H. LEWIS. 1976. Determination and Implementation of Optimal Air Quality Standards. *J. Environ. Econ. and Mgmt.* **3,** 363–380.

This uses separable NLP to represent each of two kinds of cost minimization strategies: achieve air quality improvements, and achieve regional emissions reductions. The former requires knowledge of ambient air properties, like pollution dispersion. The latter is what most models consider, especially those using LP (i.e., Teller (1968) and Kohn (1978)). The authors relate both strategies to State Implementation Plans.

BATTERMAN, S. A. 1989. Selection of Receptor Sites for Optimized Acid Rain Control Strategies. *ASCE J. Environ. Engin.* **115**(5), 1046–1058.

This uses a linear programming model for the entitled problem. The decision variables are regional emission reductions by category of abatement method. The objective is to minimize total cost, and the constraints are simple bounds and air quality standards. The solution is used to decide if a receptor is "inactive" versus "influential" by thresholding. (The problem is a mixed integer program, but MIP is not used to solve it.)

BAUMOL, W. J., AND W. E. OATES. 1975. *The Theory of Environmental Policy.* Cambridge University Press, Cambridge, U.K. (Also see 2nd ed., 1988.)

This shows the use of Lagrangian duality to analyze nonlinear programs that represent a welfare economic model. A resource allocation approach is used to determine optimal pricing of exhaustible resources, and this serves as background for an analysis of environmental policy design. Policy approaches include emission permits, taxes and subsidies.

BAWA, V. S. 1975. On Optimal Pollution Control Policies. *Mgmt. Sci.* **21,** 1397–1404.

The first part of the model development is a derivation of cumulative distribution functions for air and water, which both depend upon a level of taxation that is stationary. The air model is a random multiple of the pollutant level in the previous period plus the level of new emissions as a function of the tax rate. The water model is the sum of the previous period's pollutant level, the new emissions (also a function of tax rate) and a random variable that represents natural reduction, such as evaporation. The two recursions differ in form, but the resulting distribution functions are both shown to be increasing functions of the tax rate. This monotonicity is what makes it possible to treat the optimal control problem in a uniform manner, that is, the effluents could be into the air or water. The resulting analysis of the optimal control problem (which uses NLP) gives a recursive equation for the cumulative distribution function of the (optimal) equilibrium pollutant level.

BAYER, M. B. 1972. A Non-Linear Mathematical Programming Model for Water Quality Management. In Biswas, Vol. 2, 1972, 341–351.

This model minimizes total construction cost to build wastewater treatment plants, storage dams and reservoirs designed to control water quality and supply. The equations include the method of the early LP models, except nonlinear functions relate DO deficit and flow variables. The cost functions are also nonlinear (but separable), resulting in an NLP model. Using the same data, the model is applied to the Willamette River in Oregon and compared with the earlier LP and DP solutions.

BAYER, M. B. 1976. A Water Quality Optimization Model for Non-Serial River Systems. In Brebbia, 253–267.

This is a quadratic programming model, where the primary decision variables are levels of waste treatment for each of several plants. The (linear) constraints are the same as the early LP models plus limits on water temperature.

BEN-JEMAA, F., AND M. A. MARINO. 1993. Optimal Strategy for Aquifer Remediation. In Hon, 585–588.

This model is a dynamic program that is similar to earlier models, except that the control is a feedback mechanism. The objective is different from the other models in two respects. First, it seeks to minimize the total square deviation from target state values (rather than cost). Second, this approach employs DP, following the standard recursion, rather than an NLP technique.

BERKEMER, R., M. MAKOWSKI AND D. WATKINS. 1993. A Prototype of a Decision Support System for River Basin Water Quality Management in Central and Eastern Europe. Working Paper WP-93-049, IIASA, Laxenburg, Austria.

The core of the DSS is the steady state, as in the early LP models, plus binary variables to allow flexibility in capacity and abatement options. Multiple objectives are allowed, and a Pareto optimum is sought. The user must enter weights for the objectives, and the system allows users to enter "aspiration levels," which restrict the solution to satisfy these goals, if possible. As illustrated with their application, uncertainty is handled by scenario analysis.

BIRD, C. G., AND K. O. KORTANEK. 1974. Game Theoretic Approaches to Some Air Pollution Regulation Problems. *Socio-Econ. Plan. Sci.* **8,** 141–147.

This uses an n-person cooperative game model to gain insight into the formulation of regulations that seek to minimize total cost. One of the novelties is a new weighting scheme of various coalitions involving the preferences of the population of polluters. NLP is used to obtain the core of the game.

BISHOP, A. B., AND D. W. HENDRICKS. 1971. Water Reuse Systems Analysis. *ASCE J. Sanit. Engin.* **97**(1), 41–57.

This presents a transportation model to allocate supply to sectors at minimum cost. Treatment plants can be added, resulting in a transshipment model.

BISWAS, A. K. (ED.). 1972. *Proceedings of the International Symposium on Mathematical Modelling Techniques in Water Resource Systems*, Ottawa, Canada.

This has three volumes. The papers relevant to this survey are Bayer and Deininger.

BISWAS, A. K. (ED.). 1976. *Systems Approach to Water Management*. McGraw-Hill, New York, N. Y.

This collection of papers was compiled to provide a text written by experts in different aspects of water management (not necessarily focused on water quality). The only one directly relevant to this survey is Loucks.

BLUMSTEIN, A., R. G. CASSIDY, W. L. GORR AND A. S. WALTERS. 1972. Optional Specifications of Air-Pollution-Emission Regulations Including Reliability Requirements. *Opns. Res.* **20,** 752–763.

This is a semi-infinite LP that determines minimum cost proportions of pollutant decreases from several sources subject to an infinite number of constraints corresponding to quality limits everywhere in a (bounded) space. A simple dominance argument brings it back to ordinary LP, which is extended with a chance-constraint model to represent random breakdowns of pollution control devices. This is where reliability enters the formulation.

BOGGESS, W. G., AND E. O. HEADY. 1992. A Separable Programming Analysis of Alternative Income and Soil Conservation Policies for U.S. Agriculture, Chapter 10 in Heady and Vocke, 234–250.

This uses the constraint structure in the LP by Meister and Heady (1992), but the demands use a Cobb–Douglas function of price. This leads to an objective that is a quadratic, separable function of producer and consumer surpluses. Three soil conservation policies are analyzed. A conclusion is that net farm income increases due to the interaction between rising production costs and inelastic commodity demands, while soil erosion levels vary markedly.

BOHM, P., AND A. V. KNEESE (EDS.). 1971. *The Economics of Environment*. MacMillan Press, London, U.K. (Essays reprinted from *The Swedish Journal of Economics* **73**(1), 1971.)

This is a collection of related papers, and the background paper by Kneese (pp. 1–24) mentions a generic LP for global air and water quality for economic analysis, based on a residuals management view. Others relevant to this study are Mäler and Russell.

BOON, J. G., J. PINTER AND L. SOMLYODY. 1989. A New Stochastic Approach for Controlling Point Source River Pollution. Publications of the International Association of Hydrologic Societies No. 80. Proceedings of the Baltimore Symposium, May 1989, on *Closing the Gap Between Theory and Practice*, 141–149.

This is a joint chance-constraint model of the early LP models that constrain the levels of BOD and DO concentration. Unlike similar models, the deterministic equivalent is not simplified, resulting in a difficult nonlinear program. The authors solve it with a global optimization technique, combined with Monte Carlo simulation to evaluate each sequentially generated decision variant.

BOYD, R., AND N. D. URI. 1991. The Cost of Improving the Quality of the Environment. *Environ. and Plan.* **A23,** 1163–1182.

This uses a general equilibrium model to analyze President Bush's Clean Air Plan, which is solved by a sequence of linear complementarity problems.

BREBBIA, C. A. (ED.). 1976. *Mathematical Models for Environmental Control*. John Wiley, New York.

This is the Proceedings of the International Conference held at the University of Southampton, U.K., September 8–12, 1975. The papers relevant to this survey are Bayer; Escudero; Gustafson and Kortanek; and Orth and Ahrens.

BRILL, JR., E. D., J. C. LIEBMAN AND C. S. REVELLE. 1976. Equity Measures for Exploring Water Quality Management Alternatives. *Water Resour. Res.* **12**(5), 845–850.

A standard LP model for N dischargers into a stream is Min cx: $L \leq x \leq U$, $Ax \geq b$, where x_j = the waste removal efficiency of the jth discharger ($L \leq 0$ and $U \leq 1$), A_{ij} = the rate of water quality improvement at the ith checkpoint per unit of x_j, and b_i = the required water quality improvement at the ith checkpoint. The equity issue is how dischargers are required to behave. In particular, equal dischargers (say j and k) should be required to behave equally ($x_j = x_k$), and conversely. This paper proposes some modifications to the LP to address this equity issue. One model is an elastic program that minimizes total deviation from the average ($\sum_j x_j/N$), putting cost as a budget constraint. Other models minimize the range of efficiency ($x_{max} - x_{min}$) or just the maximum (x_{max}).

BUNDGAARD-NIELSEN, M., AND C. L. HWANG. 1976. A Review of Decision Models in Economics of Regional Water Quality Management. *Water Resour. Bull.* **12**(3), 461–480.

This points to the early literature for model elements: various cost functions for wastewater treatment plants, benefit functions that had appeared in the environmental economics literature, and equations expressing the transformation between waste discharge and accruing water quality, stemming from the Streeter–Phelps equations for BOD and DO.

BURN, D. H., AND B. J. LENCE. 1992. Comparison of Optimization Formulations for Waste-Load Allocations. *ASCE J. Environ. Engin.* **118**(4), 597–612.

The "waste-load allocation" is the level of treatment for BOD removal at each of a collection of point sources along a stream. The formulations are cited as those of Burn and McBean (1985) and Ellis (1987). Uncertainty in the transport impacts is modeled by simultaneously including scenarios that represent hydrologic, meteorologic, and pollutant loading design conditions. This paper presents four LP models, using the same transport equations, differing by the objective function: 1) minimize maximum violation, 2) minimize maximum regret, 3) minimize total violations, 4) minimize total regret. The approach is applied to the Willamette River in Oregon.

BURN, D. H., AND E. A. MCBEAN. 1985. Optimization Modeling of Water Quality in an Uncertain Environment. *Water Resour. Res.* **21**(7), 934–940.

This begins with the early LP: min cx subject to $L \leqslant x \leqslant 1$ and $Ax \geqslant b$, where A_{ij} is the transfer rate at which plant j's pollution removal reaches stream location i, and b_i is the quality requirement, net of uncontrolled levels. The decision variables (x_j) are the fractions of pollutants removed by the plants, and c is the vector of costs. To deal with uncertainty in the requirements (b), a chance-constraint model is formulated with independent quality requirements. Since the chance constraints are not joint, the certainty equivalent is an LP. Then, uncertainty in the transfer rates is analyzed with a case study of the Speed River in Ontario.

BURTON, E. S., E. H. PECHAN, III AND W. SANJOUR. 1973. A Survey of Air Pollution Control Models. Chapter 11 in Deininger, 219–235.

This outlines the framework and cites the only two linear programs that had been developed by that time (Teller 1968, Kohn 1971).

CAMPBELL, J. C., AND E. O. HEADY. 1992. A Study of Sediment-Control Policies for U.S. Agriculture Under Low and High Export Levels. Chapter 7 in Heady and Vocke, 162–172.

This applies the LP by Meister and Heady (1992) to the entitled problem, except some of the dimensions are aggregated. Two sediment control instruments are considered: a limit and a tax. A conclusion is that under low export levels, both control policies greatly reduce the sediment load at a relatively low cost. Under high imports, the cost can increase dramatically.

CARBONE, R., W. L. GORR, K. O. KORTANEK AND J. R. SWEIGART. 1978. A Bargaining Resolution of the Efficiency Versus Equity Conflict in Energy and Air Pollution Regulation. 1978. *TIMS Studies Mgmt. Sci.* **10,** 95–108.

Efficiency, as used here, is a min-cost solution for a region. This can result in an inequity as to how much each polluter is required to reduce their emissions. Resolution of this conflict is with a cooperative game model that uses NLP.

CARBONE, R., AND J. R. SWEIGART. 1976. Equity and Selective Pollution Abatement Procedures. *Mgmt. Sci.* **23,** 361–370.

The equity issue is a control policy that requires all polluters in a region to reduce their emissions by the same percentage. An NLP model is presented that finds necessary reductions in emissions that minimize total cost. This allows coalitions to form among polluters, so they need not reduce their emissions at the same rate.

CARDWELL, H., AND H. ELLIS. 1993. Stochastic Dynamic Programming Models for Water Quality Management. *Water Resour. Res.* **29**(4), 803–813.

This reviews the extensions of the early DP model by Liebman and Lynn (1966), where the state transition function is stochastic with known probability distribution. The review includes a succinct description of the issues associated with water quality management problems and why DP is well suited, especially compared with LP. Alternatives to the chance-constraint model are also described, notably frequency-based regret. They present some comparative results for the Schuylkill River near Reading, Pennsylvania.

CARLIN, A. 1990. Environmental Investments: The Cost of a Clean Environment. Technical Report EPA-230-12-90-084, U.S. Environmental Protection Agency, Office of Policy, Planning and Evaluation (PM-221), Washington, D.C.

This gives some basic facts and how EPA views the different aspects of environmental control, according to the laws. Numbers quoted in the text that cite this reference are 1993 updates obtained by phone from Alan Carlin.

CHAN, N. 1994. Partial Infeasibility Method for Chance-Constrained Aquifer Management. *ASCE J. Water Resour. Plan. and Mgmt.* **120**(1), 70–89.

This begins with a review of the literature on using LP and NLP to solve the entitled problem. An infeasibility is the (linear) amount of violation. The rest of the paper is about the method, which uses Monte Carlo simulation in lieu of the standard approach to deal with uncertainty in the LP model.

CHANG, L-C., C. A. SHOEMAKER AND P. L-F. LIU. 1992. Optimal Time-Varying Pumping Rates for Groundwater Remediation: Application of a Constrained Optimal Control Problem. *Water Resour. Res.* **28**(12), 3157–3173.

The state variable in the DP model is the vector of hydraulic heads and contaminant concentration. The control variables are the pumping rates, which can vary over time. This model addresses complexities of structure (in particular, the nonconvexities in the transport equations) and size (in particular, a large number of wells and observation points). Using differential dynamic programming, this paper focuses on the computational aspects with two differences from earlier works: using a penalty function method, and using a finite-element model to compute state transition values.

CHANG, N-B., R. E. SCHULER AND C. A. SHOEMAKER. 1993. Environmental and Economic Optimization of an Integrated Solid Waste Management System. *J. Resour. Mgmt. and Tech.* **21**(2), 87–100.

This extends solid waste management models, such as by Liebman (1975), by augmenting the effect of recycling. Air quality standards are represented by linear constraints on emissions. The paper includes a case study for Broome County, New York.

CHANG, S., AND W. YEH. 1973. Optimal Allocation of Artificial Aeration Along a Polluted Stream Using Dynamic Programming. *Water Resour. Bull.* **9**(5), 985–997.

The problem is to allocate aeration capacity to each of a series of aerators. The constraints include the standard mass balance equations in a polluted stream that requires treatment to reach a specified level of DO. The objective is a weighted sum of quadratic damage and aeration costs. To formulate this as a dynamic program, a Lagrange multiplier is used to put the total capacity constraint into the objective. The resulting model is then a multistage process (without feedback) whose state equations are mass transport. Everett's Generalized Lagrange Multiplier method governs the multiplier search.

CHI, T. 1972. Wastewater Conveyance Models. Chapter 8 in Dorfman et al., 312–361.

The problem is to determine where and when to build tertiary plants as part of the pipeline design. The model is a nonlinear program that minimizes the present value of total cost subject to what are now standard stream flow equations and quality constraints (DO levels) at specified points. The nonlinearity is only in the objective; constraints are linear equations and inequalities. The primary purpose of the model is to satisfy demands for water, and quality is considered only in the constraint requirements. The design problem, therefore, is about delivery, such as choosing the pipe's diameter, rather than about optimal treatment.

CHILTON, C. H., J. H. BROEHL, R. W. SULLIVAN AND A. W. LEMMON, JR. 1972. Task Report on EPA Energy Quality Model Exercises for 1975. Battelle, Columbus Laboratories, Columbus, Ohio.

This gives an overview of the EPA Energy Quality Model, developed by Teller (1968) (also see Gass 1972). It is a linear program that represents the relevant portion of the energy market over an aggregation of EPA's Air Quality Control Regions to about 50 to 100 regions. Supply limits and demands are fixed, and each energy-producing activity emits pollutants whose rates are estimated. Sample runs are included in this report.

CLARK, R. M. 1973. Solid Waste: Management and Models, Chapter 14 in Deininger, 269–305.

This is a very good introduction to the solid waste problems and how mathematical programming applies. The author identifies two kinds of problems: collection, storage, and transport; and disposal, including operation and location of treatment plants. Linear and mixed integer models are presented, and particular algorithms are reviewed. Many of the 38 references are general, such as for optimal location. Most of those that deal specifically with a solid waste problem are technical reports, which are generally not available anymore. This reflects the newness of applying mathematical programming to such environmental problems at that time.

CLARK, R. M., AND J. Q. ADAMS. 1987. Modeling and Operations Research for Drinking Water Systems. In Lev et al., 81–104.

Both surface and ground water can be treated by granular activated carbon (GAC) reactivation, raising questions of least-cost regional design and control. Costs include both capital and operation and maintenance for reactivation processes, and the cost function is assumed (or approximated) to be a piecewise linear convex function, which results in a MIP model. There are two classes of continuous-valued variables: the level (x_{ij}) of GAC at one site (i) reactivated at another site (j), and the amount (w_{rj}) of carbon to be reactivated in a furnace (r) at some site (j). There are two classes of 0-1 variables. First, $y_{rj} = 1$ allows reactivation alternative r at site j by the constraints: $L_{rj}y_{rj} \leq w_{rj} \leq U_{rj}y_{rj}$, where L_{rj} and U_{rj} are the least and greatest pounds of carbon that can be reactivated in furnace r, if that alternative is chosen. There is also an associated fixed charge. Second, q_{ij} requires that a water utility ships all or none of its GAC from site i to site j by constraint $x_{ij} = D_i q_{ij}$, where D_i is the pounds of GAC the utility at site i requires to be activated. Furthermore, some site must satisfy this due to the constraint $\sum_j x_{ij} = D_i$. The paper presents a particular application to the Ohio River Valley. For a typical scenario with three furnace alternatives, the MIP has 180 constraints and 560 variables, of which 80 are 0-1.

CLOUGH, D. J., AND M. B. BAYER. 1968. Optimal Waste Treatment and Pollution Abatement Benefits on a Closed River System. *Canadian Opnl. Res. Soc. J.* **6**, 153–170.

This extends the early LP models by using a logarithmic objective function that better represents cost as a function of BOD removal. The model was applied to river systems in Ontario.

COHAN, D., A. DIENER, M. DROZD, A. GJERDE, S. HAAS AND A. SMITH. 1992. Analyzing Strategies for Reducing Greenhouse Gas Emissions: The GEMINI Energy-Environmental Model. Decision Focus Incorporated, Mountain View, Calif.

This contains an overview of the GEMINI modeling system, developed for the EPA, and its application to an emissions study conducted by the Energy Modeling Forum (based at Stanford University). Emissions of most gases of interest (CO_2, N_2O, CH_4, CFCs, and HCFCs) from electricity generation and agricultural activities are represented in a market model that computes an equilibrium using mathematical programming. The document does not specify details about the equations, but it appears to be mostly linear with some nonlinear forms.

COHON, J. L., AND D. H. MARKS. 1973. Multiobjective Screening Models and Water Resource Investment. *Water Resour. Res.* **9**(4), 826–836.

Screening models seek optimal management of a complex water resource system. The decisions are resource allocations that affect supply and demand, with water quality treated as a constraint. This paper shows the use of multiobjective LP to find Pareto optimal solutions.

COLARULLO, S. J., M. HEIDARI AND T. MADDOCK III. 1984. Identification of an Optimal Groundwater Management Strategy in a Contaminated Aquifer. *Water Resour. Bull.* **20**(5), 747–760.

Pumping costs are minimized to operate a shallow aquifer, subject to localized contamination from surface waste disposal. The discharge rates for each well at each time period are determined by solving a quadratic program. (The constraints are all linear.) The quadratic term in the objective is the present value of the net discharge cost, which is the total of the products of average pumpage and associated discharge rates.

CONNER, J. R., AND E. LOEHMAN (EDS.). 1974. *Economics and Decision Making for Environmental Quality*. The University Presses of Florida, Gainesville, Fla.

This is an interesting collection of papers, but the only ones relevant to this survey are Cumberland and Loehman et al.

CULVER, T. B., AND C. A. SHOEMAKER. 1992. Dynamic Optimal Control for Groundwater Remediation With Flexible Management Periods. *Water Resour. Res.* **28**(3), 629–641.

This is an extensive review of applications of mathematical programming to groundwater quality control. It refers to earlier LP models by Willis (1976, 1979) and NLP models by Gorelick et al. (1984) and suggests that the control theory approach has computational advantages.

CULVER, T. B., AND C. A. SHOEMAKER. 1993. Optimal Control for Groundwater Remediation by Differential Dynamic Programming With Quasi-Newton Approximations. *Water Resour. Res.* **29**(4), 823–831.

This paper deals with the computational theory of models described by Gorelick et al. (1984) and Ahlfeld (1990).

CUMBERLAND, J. H. 1974. A Model of Economic-Environmental Relationships, in Conner and Loehman, 251–283.

This is an integrated model of air and water pollution with economic variables, like production and income. An input-output matrix is augmented with emissions whose total is constrained by an upper limit.

DAETZ, D., AND R. H. PANTELL. 1974. *Environmental Modelling: Analysis and Management.* Dowden, Hutchinson & Ross, Stroudsburg, Pa.

This is a collection of papers, most of which were already published elsewhere. The only one directly relevant to this survey is Seinfeld and Kyan, but this collection has some other papers of related interest.

DARBY, W. P., P. J. OSSENBRUGGEN AND C. J. GREGORY. 1974. Optimization of Urban Air Monitoring Networks. *ASCE J. Environ. Engin.* **100**(3), 577–591.

The objective is to maximize effectiveness, which is a nonlinear function of pollution concentration, exposure time, population exposed, and the age distribution of the exposed population. Constraints include regional sample sizes, fixed by EPA regulations, and a budget constraint. After some formulation tricks, the model is a min-cost network problem with binary variables $x_{ij} = 1$ if a sampler that measures pollutant i is located in region j.

D'ARGE, R. C. 1972. Economic Growth and the Natural Environment. In Kneese and Bower, 11–34.

This reviews earlier works and the materials balance approach to economic modeling of environmental quality. The dynamics represent a purely extraction-consumption-waste process. The objective of the optimization model separates into the difference between a function of per-capita income and one of waste density, each varying over time. This is multiplied by the population, which also changes over time. The usual convexity and monotonicity assumptions are used with Lagrangian duality analysis of price paths.

DASGUPTA, P. 1982. *The Control of Resources.* Basil Blackwell, Oxford, U.K.

This is a research monograph that applies to the economics of both air and water quality control. After introducing game-theoretic models with linear tax functions (Chapter 2), the author gives an elementary presentation of goals, constraints and prices (Chapter 3). He briefly discusses nonconvexities in the net benefit function (notably in the damage function) and multiple objectives.

DATHE, H. M. 1974. Decision Making for Environmental Planning. In Gottinger, 175–191.

This is an LP model that chooses SO_2 reductions by each of several polluters to achieve a total air quality standard to minimize the total cost. The model, which is a simplification of the Kortanek and Gorr (1972) model, has the form: Min cx: $Ax \leq q$, $L \leq x \leq U$, where $c_j x_j$ is the cost to reduce the jth polluter's level of emissions by x_j, and $A_{ij} x_j$ is the net effect on area i.

DAVIDSON, B., AND R. W. BRADSHAW. 1970. A Steady-State Optimal Design of Artificial Induced Aeration in Polluted Streams by the Use of Pontryagin's Minimum Principle. *Water Resour. Res.* **6**(2), 383–397.

This formulates an optimal control problem to minimize the integral of a quadratic function of the amount of re-oxygenation functional, subject to the Streeter–Phelps equations.

DE HAVEN, D. L. 1974. Systems Approach to Reduce Atmospheric Pollution by Controlling Automobile Emissions. In Gottinger, 129–173.

This presents several NLP models for the entitled problem. Decision variables include restrictions on fuel use in urban areas, type of fuels, and engine design specifications. Objectives include maximum reduction of emissions per total cost, maximum reduction of emissions for a fixed cost, and minimum cost for a fixed reduction of emissions. The models are part of a software system, called PROSE, that uses a Newton–Raphson method applied to the Lagrangian.

DEININGER, R. A. 1965. Water Quality Management: The Planning of Economically Optimal Pollution Control Systems. Ph.D. Thesis, Department of Civil Engineering, Northwestern University, Evanston, Ill.

This was the first detailed LP formulation of water quality control using the Streeter–Phelps equations. The thesis first considers only BOD removal, then combines it with DO deficit reduction in a larger LP (the appendix has a FORTRAN listing of a program to solve this). Because treatment decisions are discrete, the thesis considers the use of integer programming as well, but at that time, the computational state-of-the-art was very limited. The last chapter (III) considers the use of chance constraints to deal with uncertainty.

DEININGER, R. A. 1969. Linear Programming for Hydrologic Analyses. *Water Resour. Res.* **5**(5), 1105–1109.

This shows how to formulate an LP from the Streeter–Phelps equations, based on Deininger (1965).

DEININGER, R. A. 1972. Minimum Cost Regional Pollution Control Systems. In Biswas, Vol. 2, 352–361.

This reviews earlier works by the author and others and suggests an alternative algorithm to solve the LP.

DEININGER, R. A. (ED.). 1973. *Models for Environmental Pollution Control.* Ann Arbor Science, Ann Arbor, Mich.

This is a collection of papers, most of which were presented at a NATO Advanced Study Institute, December 1972, in Baiersbronn, Germany. Those relevant to this survey are Boon et al.; Clark; Gustafson and Kortanek; Hahn et al.; Kühner and Heiler; and Pingry and Whinston. The editor's introduction, "Systems Analysis for Environmental Pollution Control" (pp. 3–18), contains a fairly complete list of references through 1972.

DINKEL, J. J., G. B. KLEINDORFER, G. A. KOCHENBERGER AND S. N. WONG. 1976. Environmental Inspection Routes and the Constrained Travelling Salesman Problem. *Comput. and Opns. Res.* **3**(4), 269–283.

The problem is to find a route for an inspector to visit plants and return home in the least time. It is a traveling salesman problem with an added time constraint. The authors discuss their experience with heuristics and with data acquisition.

DORFMAN, R., AND H. D. JACOBY. 1972. An Illustrative Model of River Basin Pollution Control. Chapter 3 in Dorfman et al., 84–141.

Using data from particular streams, LP is used to obtain Pareto optimal BOD removal options. There is an assumed net benefit function for each of the participants, and a weighted sum is maximized subject to what became standard stream flow equations and flow quality requirements at specified points (see text).

DORFMAN, R., H. D. JACOBY AND H. A. THOMAS, JR. (EDS.). 1972. *Models for Managing Regional Water Quality*. Harvard University Press, Cambridge, Mass.

This contains a readable collection of papers. Those that contain mathematical programming models for environmental control are Chi; Dorfman and Jacoby; and Loucks and Jacoby.

DRAKE, A. W., R. L. KEENEY AND P. M. MORSE (EDS.). 1972. *Analysis of Public Systems*. MIT Press, Cambridge, Mass.

The only chapter that contains a mathematical programming model for environmental control is Marks.

DUBOIS, D. M. (ED.). 1981. *Progress in Ecological Engineering and Management by Mathematical Modelling*. Editions Cebedoc, Belgium.

This is the proceedings of a second international conference held in Liége, Belgium. The papers relevant to this study are Smeers; and Tyteca.

DURAIAPPAH, A. K. 1993. *Global Warming and Economic Development*. Kluwer, Dordrecht, The Netherlands.

This is a detailed description of an NLP model that focuses on CO_2 emissions, designed to address the following question: What is the optimal level of greenhouse gases emissions that does not perturb the climate as well as the economic system? The term *holistic model* is used to mean that all variables are endogenous, even if they belong to another system. In particular, climate variables, like temperature, are in the model, as well as economic variables, like measures of growth. An appendix gives the GAMS code, including data. The world is partitioned into two regions: developed and undeveloped. There are three sectors: agriculture, industry, and service, and there are three processes: abatement, intermediate, and intensive. Time periods are 5-year intervals from 1985 through 2035. The NLP represents a welfare economic equilibrium in three submodels: economic, carbon cycle, and temperature. The economic model determines CO_2 emissions, which is input to the carbon cycle model. This determines the effect of CO_2 fertilization on the agriculture sector of the economic model plus inputs to the temperature model, which gives the economic model the effect of a temperature rise on agriculture. The objective function is a quadratic that represents a weighted sum of multiple objectives. Constraints include material balances and limits on capital, land, and deforestation. A typical scenario has about 1,130 equations and 1,340 variables.

DYSART, III, B. C. 1970. Water Quality Planning in the Presence of Interacting Pollutants. *J. Water Pollut. Control Fed.* **42**(8) (Part 1), 1515–1529.

This extends earlier models to minimize abatement cost by considering the interaction of pollutants. (Previous models simply measured levels of BOD and DO concentrations without interaction effects.) The standard decomposition into reaches is used with state variables: temperature, BOD, and DO levels. This also assumes the state is constant throughout a reach, but the state transition function accounts for interaction effects.

DYSART, III, B. C., AND W. W. HINES. 1970. Control of Water Quality in a Complex Natural System. *IEEE Trans. Syst. Sci. and Cybern.* **SSC-6**(4), 322–329.

This extends the early DP model by Liebman and Lynn (1966) for water pollution control by adding the complexity of interaction effects of organic and thermal wastes. The model was applied to the Chattahoochee River around Atlanta.

ECKER, J. G. 1975. A Geometric Programming Model for Optimal Allocation of Stream Dissolved Oxygen. *Mgmt. Sci.* **21**(6), 658–668.

This is a geometric programming formulation of the stream treatment allocation problem, which previously used LP (Deininger 1965) and DP (Liebman and Lynn 1966). The nonlinearities in this formulation arise by the consideration of processes acting in series and by not approximating the cost function (as in the LP models). This paper also reports the use of the model to analyze abatement policies for the Upper Hudson River in New York.

ECKER, J. G., AND J. R. MCNAMARA. 1971. Geometric Programming and the Preliminary Design of Industrial Waste Treatment Plants. *Water Resour. Res.* **7**(1), 18–22.

This reformulates the Shih and Krishnan model (1969) as a geometric program.

EDWARDS, W. F., M. R. LANGHAM AND J. C. HEADLEY. 1970. Pesticide Residues and Environmental Economics. *Nat. Resour. J.* **10**(4), 719–741.

This is based on Edwards' 1969 Ph.D. Thesis at the University of Florida. It uses a welfare economic approach to develop an LP model, which is applied to Dade County, Florida. This paper does not present the mathematical formulation of the model, but its tables and references indicate it is a linear program. The decision variables are acres of land allocated to each of several crops. Each crop requires some chemical treatment; chlorinated hydrocarbons and organic phosphates are two that are cited in the Dade County study. The objective is to maximize a net benefit function, which includes damage caused by pesticide residues.

ELLIS, H., AND M. L. BOWMAN. 1994. Critical Loads and Development of Acid Rain Control Options. *ASCE J. Environ. Engin.* **120**(2), 273–290.

(Note that the first author has usually published under the name J. H. Ellis.) This is an LP similar to Ellis (1988), applied to meeting the 1990 Clean Air Act for Maryland.

ELLIS, J. H. 1987. Stochastic Water Quality Optimization Using Imbedded Chance Constraints. *Water Resour. Res.* **23**(12), 2227–2238.

This model minimizes the cost of BOD removals from waste treatment plants. It extends the early models by allowing uncertainty in not only stream flow, but also initial BOD level and DO deficit. The chance constraints become joint, and the deterministic equivalent is quadratic.

ELLIS, J. H. 1988. Acid Rain Control Strategies: Options Exist Despite Scientific Uncertainties. *Environ. Sci. and Tech.* **22**(11), 1248–1255.

This addresses the complex issue whether we have enough reliable information to implement abatement strategies. LP models represent a deposition-constrained approach; the author argues that the lack of perfect information in the transfer coefficients need not preclude its effective use. The first LP is to minimize required emissions reductions in eastern North America, which corresponds to minimizing the cost of SO_2 removal, subject to reduction constraints that correspond to air quality standards. The uncertainty in the net emissions coefficients can be handled by solving an LP for the range, giving optimistic and pessimistic solution values. The author first uses regret analysis to consider the different emission rates that come from different long-range transport (LRT) models. Then robust optimization is used to find a policy that deals simultaneously with seven LRT models. Other criteria are considered, notably minimizing the maximum violation, and minimizing the maximum regret.

ELLIS, J. H. 1989. A Variable State-Space Dynamic Programming Model for Optimizing Industrial Waste Treatment Sequences. In Esogbue, 276–285.

This model is to obtain a min-cost treatment sequence constrained to satisfy stream quality standards, taking into consideration the uncertainty of influent waste. Stochastic DP is used, where the stages correspond to the ordering of the sequence of processes, and the state transition function contains parameters that have random variation.

ELLIS, J. H. 1990. Integrating Multiple Long-range Transport Models into Optimization Methodologies for Acid Rain Policy Analysis. *Eur. J. Opnl. Res.* **46,** 313–321.

This begins with an LP to minimize the cost of reducing SO_2 emissions when emission rates are known. Then the author considers uncertainty in transport (see the ambient model by Atkinson and Lewis (1976). The uncertainty due to weather variability is separated from the uncertainty that reflects the difficulty in modeling the physics and chemistry of long-range air transport and transformation. This proposes a joint chance-constraint model and shows how to incorporate estimates from different long-range transport models.

ELLIS, J. H., E. A. MCBEAN AND G. J. FARQUHAR. 1985. Deterministic Linear Programming Model for Acid Rain Abatement. *ASCE J. Environ. Engin.* **111**(2), 119–140.

This represents least-cost control of SO_2 emissions with five different constraint formulations, motivated by specific applications in North America. Emission rates are assumed, which is what makes the model deterministic. All models seek to minimize the total cost of abatement, and the decision variables are levels of pollutant removal at each source. Following Atkinson and Lewis (1976), the first type of model is the emission least-cost model, and the second is the ambient least-cost model. They differ in that the former uses a uniform emissions rate (scalar) and requires an aggregate level of removal to satisfy a prescribed air quality standard. The latter uses emission rates that depend upon the source and receptor and upon air quality standard constraints at each receptor (rather than aggregate).

ELLIS, J. H., E. A. MCBEAN AND G. J. FARQUHAR. 1985. Chance-Constrained/Stochastic Linear Programming Model for Acid Rain Abatement—I. Complete Colinearity and Noncolinearity. *Atmos. Environ.* **19**(6), 925–937. This represents the least-cost control of SO_2 emissions. Uncertainty in the transfer coefficients are modeled by two-stage recourse and chance-constraint equivalents. The cases of complete collinearity and noncollinearity are the extreme cases of complete dependence and complete independence, respectively, of the transfer coefficients.

ELLIS, J. H., E. A. MCBEAN AND G. J. FARQUHAR. 1986. Chance-Constrained/Stochastic Linear Programming Model for Acid Rain Abatement—II. Limited Colinearity. *Atmos. Environ.* **20**(3), 501–511.

This is the same model as in Ellis, McBean and Farquhar (1985), except that the transfer coefficients can have some interdependencies.

ENGLISH, B. C., AND E. O. HEADY. 1992. *Analysis of Long-Term Agricultural Resource Use and Productivity Change for U.S. Agriculture*. Chapter 8 in Heady and Vocke, 175–203.

Using the LP by Meister and Heady (1992), this chapter examines 7 of the 69 alternative control programs conducted by the USDA in 1980, applied to the year 2030.

ERICKSON, L. E., G. K. C. CHEN AND L. T. FAN. 1968. Modeling and Optimization of Biological Waste Treatment Systems. *Chem. Engin. Prog. Symp. Series* **64,** 97–110.

This emphasizes the range of designs from using a sequence of aeration tanks connected in series to one completely mixed tank. After modeling flows, system optimization is defined by minimizing the total holding time (a related objective considered is minimizing the total volume of the biological growth chamber). About half the paper is devoted to analyzing results with specific data and giving insights into the model's sensitivity to key parameters.

ESCUDERO, L. F. 1976. The Air Pollution Abatement MASC-AP Model. In Brebbia, 173–181.

This is an MIP model whose primary decision variables are the levels of pollutant reduction in each grid covering a region, for each of several meteorological conditions. Associated binary variables are used to limit each reduction variable, based on a diffusion model. The diffusion equations are also used to limit the binary values, as a surrogate for a probability constraint. The (linear) objective is to minimize total emission reduction.

ESOGBUE, A. O. (ED.). 1989. *Dynamic Programming for Optimal Water Resources Systems Analysis*. Prentice-Hall, Englewood Cliffs, N. J.

The editor gives a review of DP and a taxonomy for its applications to water resource management, including water quality. The other papers that present DP models for water quality control are Ellis; and Sugiyama.

ESOGBUE, A. O. 1989. A Taxonomic Treatment of Dynamic Programming Models of Water Resources Systems. In Esogbue, 27–71.

This gives a review of DP and a taxonomy for its applications to water resource management. Water quality problems cited are allocation of aeration along a stream (as in

Chang and Yeh 1973; Sugiyama 1989) and determination of water treatment plant capacities.

EVENSON, D. E., G. T. ORLOB AND J. R. MONSER. 1969. Preliminary Selection of Waste Treatment Systems. *J. Water Pollut. Control Fed.* **41**(11) (Part 1), 1845–1858.

This model minimizes cost subject to a constraint on the amount of BOD removal. Using DP, the state variable is the level of BOD, and the stages correspond to treatment processes. The decision variable at a stage is the amount of BOD removal by the treatment process, which is simply bounded. Each stage's process cost can be a nonlinear function, and the decision variable can be restricted to discrete values. To simplify the state transition function, this model assumes that BOD concentration changes linearly with the process level.

FALK, I., AND R. MENDELSOHN. 1993. The Economics of Controlling Stock Pollutants: An Efficient Strategy for Greenhouse Gases. *J. Environ. Econ. and Mgmt.* **25**(1) (Part 1), 76–88.

If left alone, stored toxic waste will increase its pollutant concentration and cause damage. The damage function and the other relationships are presumed known, and the issue is when to spend money to reduce the level of damage. With an assumed cost function, standard control theory (using Lagrange multipliers) is applied to infer that the optimal level of abatement occurs when the marginal total abatement cost equals the marginal total damage cost. Particular cost functions are investigated to make stronger inferences about the optimal abatement trajectory.

FAN, L. T., R. S. NADKARNI AND L. E. ERICKSON. 1971. Management of Optimum Water Quality in a Stream. *Water Res.* **5**, 1005–1021.

This model considers the assumption made by earlier ones: Pollutants discharged into a stream affects the water quality downstream, but not upstream. This model removes this assumption, which then disables the DP approach. Although the balance equations are still linear, the cost and benefit functions are nonlinear. The authors use a penalty function approach to solve the NLP.

FELDER, S., AND T. F. RUTHERFORD. 1993. Unilateral CO_2 Reductions and Carbon Leakage: The Consequences of International Trade in Oil and Basic Materials. *J. Environ. Econ. and Mgmt.* **25**(2), 162–176.

This gives an overview of a general equilibrium model, based on the Global 2100 model (Manne and Richels 1992), and applies it to the entitled problem. The model is equivalent to a nonlinear program that represents energy markets and CO_2 emissions.

FERRAR, T. A. 1973. Nonlinear Effluent Charges. *Mgmt. Sci.* **20,** 169–178.

This uses NLP for an economic equilibrium model that applies to emissions of pollutants into the air or discharge of waste into water. The model uses an effluent tax to meet a specified level of quality. The tax function is the same for all polluters, and it is an increasing, convex function of the level of effluence. Using a penalty function argument, the author concludes that the same solution can be obtained with a linear tax function. They point out, however, that the nonlinear function assures interim satisfaction of standards that might change over time, whereas the linear function cannot assure this.

FIACCO, A. V. 1983. *Introduction to Sensitivity and Stability Analysis in Nonlinear Programming.* Academic Press, New York.

Most of this textbook is on the entitled subject, but Chapter 8 contains the essence of what was reported by Fiacco and Ghaemi (1982), and there are references to earlier technical reports.

FIACCO, A. V., AND A. GHAEMI. 1982. Sensitivity Analysis of a Nonlinear Water Pollution Control Model Using an Upper Hudson River Database. *Opns. Res.* **30,** 1–28.

This is an in-depth sensitivity analysis of the model developed by Ecker (1975). Among the conclusions is the counterintuitive deduction that sludge removal from the bottom of the Upper Hudson River would yield a greater treatment cost reduction than a corresponding (1%) increase in the allowable DO.

FIELD, B. C., AND C. E. WILLIS. 1979. *Environmental Economics: A Guide to Information Sources.* Gale Research Company, Detroit, Mich.

This is a comprehensive annotated bibliography, partitioned into conceptual foundations and empirical studies. Those that use mathematical programming are included.

FISHBONE, L. G., AND H. ABILOCK. 1981. MARKAL, A Linear Programming Model for Energy Systems Analysis: Technical Description of the BNL Version. *Energy Res.* **5,** 353–375.

This complements the MARKAL user's manual (Abilock and Fishbone 1979). The reference to the Brookhaven National Laboratory (BNL) version is to distinguish it from its development partner, KFA (Germany). The paper succinctly specifies the LP data, variables, and equations.

FISHELSON, G. 1983. Dynamic Aspects of Water Quality Control. Chapter 3 in Tolley et al., 43–60.

This forms the fundamental equations for the optimal control problem that maximizes the present value of water quality, which is a function of water pollution, quantity, and purchased inputs for water treatment. The system state is the water quality, and standard control analysis is applied. The Lagrange multiplier associated with the state equation is shown to be the optimal (Pigovian) tax on pollution. From this base, a regional model is formulated, using the standard discretization of time to obtain a dynamic program.

FISHER, A. C., AND F. M. PETERSON. 1976. The Environment in Economics: A Survey. *J. Econ. Lit.* **14** (March), 1–33.

This is a lengthy review, beginning with some history that dates back to the late nineteenth century and citing the emergence of environmental economics. There is an interesting section about the nonconvexity of damage functions, followed by a bibliographic tour of general equilibrium models (those using mathematical programming have been cited here).

FOELL, W. K., AND L. A. HERVEY (EDS.). 1983. *National Perspectives on Management of Energy/Environment Systems.* John Wiley, New York.

This is a collection of short papers that summarize models used for policy analysis in each of 12 countries. In most cases, LP is used iteratively, such as described by Greenberg and Murphy (1980). The primary use of the models is for energy policy analysis, and environmental impacts are measured by emissions, such as sulfur from burning coal to generate electricity. The papers are not listed separately

here because they do not give enough detail about their models, but they do give some references (also generally incomplete).

FORTIN, M., AND E. A. MCBEAN. 1983. A Management Model for Acid Rain Abatement. *Atmos. Environ.* **17**(11), 2331–2336.

This is an LP model to minimize the total cost of abatement subject to limits on total emissions from each source and a budget constraint. The emissions constraint for the ith receptor has the form $\sum_j (1 - x_j)E_j\tau_{ji} \leq b_i$, where j indexes the sources, x_j = removal rate (between 0 and 1), E_j is the source emissions before treatment over the (fixed) time period, and τ_{ji} is the transfer coefficient of pollutants from source j to receptor i. Additional constraints represent equity among sources by limiting pair-wise differences in the removal rates. Meteorologic uncertainty, manifest in transfer coefficients, uses the mean-variance representation $\sum_j (1 - x_j)E_j(\mu_{ji} + \alpha\sigma_{ji})$, where τ_{ji} has mean μ_{ji} and standard deviation σ_{ji}; α is a parameter that represents allowable risk (the same for each source and receptor).

FRONZA, G., AND P. MELLI (EDS.). 1982. *Mathematical Models for Planning and Controlling Air Quality*. Pergamon Press, New York.

Most of the models are statistical. Two that use NLP are Anderson, Jr.; and Gustafson and Kortanek.

FRONZA, G., AND P. MELLI. 1984. Assignment of Emission Abatement Levels by Stochastic Programming. *Atmos. Environ.* **18**(3), 531–535.

The problem is to minimize total abatement costs subject to air quality standards, as in Atkinson and Lewis (1974). The modeling issue is to deal with uncertainty in pollutant dispersion. In addition to the usual chance-constrained model, the authors introduce a "distribution approach." This first solves the LP for each meteorological state and obtains a collection of abatement policies. Then a reliability index of each policy is defined as the expected amount of violation (over all meteorological states). Using expected cost and reliability as two criteria, a Pareto optimum is found among the policies.

FUJIWARA, O., S. K. GNANENDRAN AND S. OHGAKI. 1986. River Quality Management Under Stochastic Streamflow. *ASCE J. Environ. Engin.* **112**(2), 185–198.

This begins with the linearly constrained model to minimize a separable, possibly nonlinear, cost. The constraints are the standard ones (from the early models) that require a specified (lower) bound of BOD removal at each of several stream locations from each of several plants. Uncertainties in the downstream impacts of BOD removal are modeled with chance constraints, but they are not joint, so the certainty equivalent also has linear constraints (see text).

FUJIWARA, O., S. K. GNANENDRAN AND S. OHGAKI. 1987. Chance Constrained Model for River Water Quality Management. *ASCE J. Environ. Engin.* **113**(5), 1018–1031.

This extends the early models by considering the effect of storm water and tributary flows into the main stream as random variables. Using the linear equations resulting from the Streeter–Phelps model (with the Camp–Dobbins modification), this model formulates independent chance constraints, which are known to have a linear certainty equivalent.

FUTAGAMI, T., N. TAMAI AND M. YATSUZUKA. 1976. FEM Coupled with LP for Water Pollution Control. *ASCE J. Hydraul.* **102**(7), 881–897.

This combines a finite element method (FEM) with LP in what had become a standard model. FEM is used to discretize the equilibrium flow equations, and LP is used to choose discharge rates that maximize water quality.

GASS, S. I. 1972. Technical Analysis of the Application of the Dantzig-Wolfe Decomposition Technique in the EPA Energy Quality Model (EQM). Technical Report, Mathematica, Inc., Bethesda, Maryland.

This gives a detailed description of the EQM model (also see Teller 1968, Chilton et al. 1972), then focuses on the use of the use of Dantzig–Wolfe decomposition to reduce the computation time.

GASS, S. I., AND R. L. SISSON (EDS.). 1975. *A Guide to Models in Governmental Planning and Operations*. Sauger Books, Potomac, Maryland.

The mathematical programming models for environmental control are Liebman; Marks; and Singpurwalla.

GEORGAKAKOS, A. P., AND H. YAO. 1993. New Control Concepts for Uncertain Water Resources Systems 1. Theory. *Water Resour. Res.* **29**(6), 1505–1516.

This presents a theory of control that applies both to streams and groundwater. Controls and states are abstract variables that can specialize to a variety of water pollution problems. The idea is to identify acceptable state sets and find controls that guarantee reaching an acceptable final state. The state equations contain a random variable, so the dynamic program is a pessimistic objective, aimed at avoiding catastrophes entirely—that is, not allowing any possibility of reaching an unacceptable state. While the model and primary solution method is DP, the method uses LP to seek supporting hyperplanes of polyhedra that comprise reduced state sets.

GIGLIO, R. J., AND R. WRIGHTINGTON. 1972. Methods for Apportioning Costs Among Participants in Regional Systems. *Water Resour. Res.* **8**(5), 1133–1144.

This uses game theory to model the equity issue by allowing coalitions and bargaining. The result is a simple LP.

GOODMAN, A. S., AND W. E. DOBBINS. 1966. Mathematical Model for Water Pollution Control Studies. *ASCE J. Sanit. Engin.* **92**(6), 1–19.

This describes a FORTRAN library of routines to solve the entitled problem, including a routine to perform steepest descent. Decision variables include investment costs, and the system contains equations relating economic and physical flow variables. The flow equations are the standard Streeter–Phelps approximations (see text).

GORDON, S. I. 1985. *Computer Models in Environmental Planning*. Van Nostrand-Reinhold, New York.

This is a good introductory text to pollution problems, but it does not contain mathematical programming models. For water quality, it presents the Streeter–Phelps DO flow equations. For storm water runoff, it presents the hydrologic cycle. For air pollution, it presents climatological dispersion. For hazardous waste, it discusses routing. In all areas, it references some of the software systems that were available.

GORELICK, S. M. 1982. A Model for Managing Sources of Groundwater Pollution. *Water Resour. Res.* **18**(4), 773–781.

This extends the model by Gorelick and Remson (1982) and demonstrates that solving the dual LP has computational advantages.

GORELICK, S. M. 1983. A Review of Distributed Parameter Groundwater Management Modeling Methods. *Water Resour. Res.* **19**(2), 305–319.

This contains quality problems as a part of a succinct review of broader problems. There is an interesting section on nonlinearities in groundwater quality management and the need for better numerical methods.

GORELICK, S. M. 1990. Large-Scale Nonlinear Deterministic and Stochastic Optimization: Formulations Involving Simulation of Subsurface Contamination. *Math. Prog.* **48**(1) (Series B), 19–39.

This is the modeling approach given by Gorelick et al. in 1984, but with more insight into its structure (also see Ahlfeld et al. 1988). This also reviews the method of Wagner and Gorelick (1987) to deal with parameter uncertainty.

GORELICK, S. M., AND I. REMSON. 1982. Optimal Dynamic Management of Groundwater Pollutant Sources. *Water Resour. Res.* **18**(1), 71–76.

This is an LP model to manage several groundwater pollutant sources over time by maximizing total discharge rates, subject to quality constraints, similar to Gorelick, Remson and Cottle (1979). Parametric programming is applied to analyze the sensitivity of the maximum to the injection rate of a particular well.

GORELICK, S. M., AND I. REMSON. 1982. Optimal Location and Management of Waste Disposal Facilities Affecting Groundwater Quality. *Water Resour. Bull.* **18**(1), 43–51.

This presents an LP model to maximize the total disposal of waste solutes, subject to quality standards that limit the amounts discharged in each time period. Transfer rates are obtained by simulation (i.e., solving the transport equations to obtain a matrix that gives concentration rates at each source). The LP is then extended to an MIP that allows control over which injection wells operate during each of the time periods.

GORELICK, S. M., I. REMSON AND R. W. COTTLE. 1979. Management Model of a Groundwater System With a Transient Pollutant Source. *Water Resour. Res.* **15**(5), 1243–1249.

The model is a linear program, and the paper uses parametric programming to show how this applies to such questions as: What river concentration would be permitted if the most restrictive local groundwater quality limit were removed?

GORELICK, S. M., C. I. VOSS, P. E. GILL, W. MURRAY, M. A. SAUNDERS AND M. H. WRIGHT. 1984. Aquifer Reclamation Design: The Use of Contaminant Transport Simulation Combined With Nonlinear Programming. *Water Resour. Res.* **20**(4), 415–427.

The simulation pertains to solving groundwater flow equations (they use a finite element method). They report on the use of MINOS to solve a nonlinear program that seeks to minimize contaminant concentration subject to the flow constraints. The full representation, which they call the "embedding approach," results in a large NLP that is computationally prohibitive. They propose another approach that iterates between the simulation, given the controls, and the NLP that uses simulation results to approximate flow relations linearly (used as a subroutine, the simulation gives functional and Jacobian evaluations for an iteration of the MINOS NLP method).

GORR, W. L., S.-Å. GUSTAFSON AND K. O. KORTANEK. 1972. Optimal Control Strategies for Air Quality Standards and Regulatory Policy. *Environ. and Plan.* **4**(2), 183–192.

This shows the connection between the semi-infinite model by Gustafson and Kortanek (1973) and a moment problem. Then the damage function is discussed, and the (possibly nonlinear) objective is defined to minimize total cost.

GOTTINGER, H. W. (ED.). 1974. *Systems Approaches and Environmental Problems.* Vandenhoeck & Ruprecht, Gottingen, Germany.

This is a proceedings for an international symposium at Schloβ Reisenburg, Germany, June 18–21, 1973. The papers relevant to this survey are Dathe; and de Haven.

GRADY, JR., C. P. L. 1977. Simplified Optimization of Activated Sludge Process. *ASCE J. Environ. Engin.* **103**(3), 413–429.

The author explains the details of computing a solution to a DP model for designing a sludge process treatment plant.

GRANTHAM, G., E. E. PYATT, J. P. HEANEY AND J. CARTER. 1970. Model for Flow Augmentation Analysis—An Overview. *ASCE J. Sanit. Engin.* **96**(5), 1045–1056.

The model has two parts: simulation (solving a dynamical system of equations) and optimization. The optimization component is an LP to determine the use of existing and planned wastewater treatment facilities to satisfy water quality standards, as described by Loucks, ReVelle and Lynn (1967).

GRAVES, G. W. 1972. Water Pollution Control. In *Techniques of Optimization*, A. V. Balakrishnan and L. W. Neustadt (eds.). Academic Press, New York, 499–509.

This extends the LP in Graves, Hatfield and Whinston (1972) with binary variables to allow plants and pipes to be open or closed.

GRAVES, G. W., G. B. HATFIELD AND A. WHINSTON. 1969. Water Pollution Control Using By-Pass Piping. *Water Resour. Res.* **5**(1), 13–47.

This reviews LP in general, and the early LP models for water quality control (see text).

GRAVES, G. W., G. B. HATFIELD AND A. B. WHINSTON. 1972. Mathematical Programming for Regional Water Quality Management. *Water Resour. Res.* **8**(2), 273–290.

This extends their 1969 paper to allow treatment at the source and the use of regional treatment plants.

GREENBERG, H. J., AND F. H. MURPHY. 1980. Modeling the National Energy Plan. *J. Opnl. Res. Soc.* **31**, 965–973.

This describes the use of the Project Independence Modeling System (PIES) to analyze U.S. energy policies. Although the model computes a partial equilibrium for the energy sector, the framework uses LP iteratively (until a fixed point is reached). Particular regulatory structures are represented by modifying the LP each iteration. The presentation is focused on the application to the U.S. National Energy Plan, proposed by President Carter in 1977.

GREENBERG, H. J., AND F. H. MURPHY. 1985. Computing Market Equilibria with Price Regulations Using Mathematical Programming. *Opns. Res.* **33**(5), 935–954.

This is the theoretical companion to their earlier paper (1980) that establishes a general framework to incorporate

certain regulatory structures into an LP model. For example, demands are variables, rather than fixed, determined by imputed prices. In the case of electricity, consumers respond to average, rather than marginal, prices due to regulated capital returns. The LP is modified iteratively to account for these nonlinearities. Other structures are described, and theorems are established to prove how iterative price adjustments can be used to obtain a regulated equilibrium.

GREENBERG, M. R. 1978. *Applied Linear Programming for the Socioeconomic and Environmental Sciences*. Academic Press, New York.

As the title suggests, this begins with a basic introduction to LP (Part 1), then has a chapter on each of several applications (Part 2). Chapter 6 gives some LP models for solid waste, and Chapter 7 gives some for water resource management. The LP for solid waste is to determine a least-cost transportation of waste from sources to destinations, which can be via a waste treatment plant. The disposal stations, which are the final destinations, as well as the (intermediate) waste treatment plants have limited capacities. A waste treatment plant reduces the quantity of waste, which is the incentive for using it due to limited capacity at disposal stations. This LP is extended to an MIP that allows construction of new treatment plants and disposal stations. Finally, the MIP is made dynamic with assumed demands over time. The water resource management models represent least-cost solutions to satisfy demands, including facility siting. One section is on water quality, which is a simplified presentation of the early LP models.

GUARISO, G., AND H. WERTHNER. 1989. *Environmental Decision Support Systems*. Ellis Horwood Limited, Chichester, England.

This shows how optimization can be integrated into a decision support system. Chapter 4 refers to some water quality control models as in Rinaldi et al. (1979) and Loucks et al. (1981), but most of the book describes data management, user interfaces, and other aspects of their DSS.

GULDMANN, J-M. 1978. Industrial Location, Air Pollution Control, and Meteorological Variability: A Dynamic Optimization Approach. *Socio-Econ. Plan. Sci.* **12**(4), 197–214.

This formulates an MIP model of plant location, where the binary variables are $x_{ki} = 1$ if plant k is located at site i. Flow variables are q_{kmt} = pollution flow from plant k treated by technique m during period t. The objective is to minimize total cost, and constraints include air quality standards for different time frames (hourly, daily, monthly, annually). A variety of special cases is considered, notably when fuel combustion is the source of pollution. The dual is analyzed and its use as taxes is considered. The model is applied to the Haifa area, where the major polluting plants are the power plant and the refineries.

GULDMANN, J-M. 1986. Interactions Between Weather Stochasticity and the Locations of Pollution Sources and Receptors in Air Quality Planning: A Chance-Constrained Approach. *Geo. Anal.* **18**(3), 198–214.

This first defines the simple LP: Max $\sum_j x_j$: $Ax \leqslant b$, $L \leqslant x \leqslant U$, where x_j is the level of emissions at the jth source, A_{ij} is the transfer rate of pollutants from the jth source to the ith receptor, and b_i is the acceptable level of total emissions at the ith receptor. The bounds represent a range of emissions, and the cost to reduce emissions is the same across sources. The issue is how to represent uncertainties in the transfer coefficients. The paper gives two certainty equivalent models for the chance constraint: $P(Ax \leqslant b) \geqslant \alpha$. The first is distribution-free, which requires the construction of solution sets. The second assumes a Normal distribution and uses the Tchebysheff inequality to derive the mean-variance form (see text). Mathematically, both models have the same nonlinear form $Ex + \gamma(xBx)^{1/2} \leqslant \beta$.

GULDMANN, J-M. 1988. Chance-Constrained Dynamic Model of Air Quality Management. *ASCE J. Environ. Engin.* **114**(5), 1116–1135.

This extends the author's 1986 NLP model by allowing time-varying transfer coefficients and controls. The resulting joint chance-constraint model is solved by SUMT for monthly and annual air quality constraints using data obtained from the Columbus, Ohio weather station.

GULDMANN, J-M., AND D. SHEFER. 1977. Optimal Plant Location and Air Quality Management Under Indivisibilities and Economies of Scale. *Socio-Econ. Plan. Sci.* **11**, 77–93.

This is a location problem that includes air quality constraints. The binary decision variables are $x_{ijk} = 1$ if plant k is located in sector i and uses pollution abatement process j. There are logical constraints (e.g., every plant must be assigned to a sector, and some plants already exist) and an acreage limit for each sector. The air quality constraint for residential zone r has the form $\sum_{ijk} e_{ijkr} x_{ijk} \leqslant q_r$, where e_{ijkr} is the net emissions, determined by a product of other data, including the portion of zone r affected by sector i. The authors apply their model to the Haifa area, divided into 7 industrial sectors and 6 residential zones.

GULDMANN, J-M., AND D. SHEFER. 1980. *Industrial Location and Air Quality Control*. John Wiley, New York.

This contains some optimization models for the entitled problem after presenting diffusion models and the effects of air pollutants on various receptors. Most of the optimization models are LPs that minimize cost, including pollution abatement from various technologies. Some models are simple (as described in text), and some are more complex, involving land-use planning under a variety of assumptions, which can have binary variables. The book provides a succinct introduction to the use of such diffusion models in mathematical programming.

GULDMANN, J-M., AND D. SHEFER. 1980. Air Quality Control, Industrial Siting, and Fuel Substitution: An Optimization Approach. In *Advances in Environmental Science and Technology*, Volume 10, J. N. Pitts, Jr., R. L. Metcalf and D. Grosjean (eds.). John Wiley, New York, 301–367.

A diffusion model is used to give a transfer coefficient between each emitter (j) and receptor (i), A_{ij}. Then the linear equation, $y = Aq$, describes the pollution concentration (y_i) at each receptor for a level of emission (q_j). These are some of the constraints in an LP model, and pollution is constrained by a bound, $y \leqslant U$. The remaining constraints require energy production to equal specified levels. The decision variables are plant levels of fuel use, like types of coal and oil. Each fuel has a known energy output, which is a constant rate, and the objective is to minimize total cost. The model is applied to the Haifa area, and most of the paper focuses on analyzing uncertain parameters, such those affected by variable air patterns.

GUSTAFSON, S.-Å., AND K. O. KORTANEK. 1973. Mathematical Models for Air Pollution Control: Determination of Optimum Abatement Policies. Chapter 13 in Deininger, 251–265.

This formulates a semi-infinite convex program, where the pollution transfer function is derived from a diffusion model. Several variations are considered and a theorem is shown to apply that allows the use of ordinary convex programming techniques (despite the infinite number of inequality constraints). The authors also consider sensitivity analysis and the influence of errors.

GUSTAFSON, S.-Å., AND K. O. KORTANEK. 1976. On the Calculation of Optimal Long-Term Air Pollution Abatement Strategies for Multiple-Source Areas. In Brebbia, 161–171.

This is a separable, convex program with linear constraints. The decision variables are the percentage of sulfur reduction in each of several regions, such that each region's total sulfur emissions is within a prescribed limit. Interregional emission rates are assumed to be constant, and variations of the basic model are used to consider questions of side payments and taxes.

GUSTAFSON, S.-Å., AND K. O. KORTANEK. 1982. A Comprehensive Approach to Air-Quality Planning: Abatement, Monitoring Networks, and Real-Time Interpolation. In Fronza and Melli, 75–89.

The models are for SO_2, but the formulation generalizes to represent other pollutants that obey a superposition principle (i.e., inert chemicals). The first model assumes a continuous-control variable for each of several sources, presumed to lie in a closed, finite interval. The objective is to minimize total cost, summed over the sources, where each source's cost is a convex function of the level of control. The level of control is assumed to reduce the level of pollution proportionally, so there is one linear constraint at each point in space that represents a total air quality requirement. The space is bounded, but not discretized, so the number of constraints is infinite. Duality is then applied to obtain the usual economic interpretations of the Lagrange multipliers as marginal prices. This basic convex program is extended to represent uncertainties, such as due to weather. The (finite) number of sampling points, and their locations, are determined by optimization.

HAFKAMP, W. A. 1984. *Economic-Environmental Modeling in a National-Regional System*. Elsevier, New York.

This is what the author calls a "multi-layer" approach to environmental modeling in connection with the economy. He defines three layers as interrelated submodels: economic, employment, and environmental quality. Using about 550 variables and 470 equations, the model assumes that air, water, and land quality are measured by pollutants emitted from industry and people. The level of pollution is determined by known functions. The discussion about the multiobjective, nonlinear program (Chapter 5) is abstract, except for an example that suggests variables include regional-sectorial production volumes, value added, and employment plus regional pollution levels. The equations that relate these variables in the example are linear.

HAHN, H. H., P. M. MEIER AND H. ORTH. 1973. Regional Wastewater Management Systems. Chapter 3 in Deininger, 41–60.

This is a transshipment model, except the cost function includes a fixed charge. The decision variables are flows of wastewater from specified stream locations to treatment plants.

HAIMES, Y. Y. 1971. Modelling and Control of the Pollution of Water Resources Systems via Multilevel Approach. *Water Resour. Bull.* 7(1), 93–101.

This considers the NLP formulation of minimizing treatment cost. The author's "multilevel" approach is introduced to deal with the high dimensionality resulting from many reaches and pollutants. He decomposes the water resource system (e.g., a river) into subsystems that are optimized independently, using Lagrange multipliers to put coupling relations into the objective that enables the decomposition. He then considers coupling variables at a second level, which changes the Lagrange multipliers. (This is the Generalized Lagrange Multiplier method, introduced in 1963 by Hugh Everett; it is similar to the use of Dantzig–Wolfe decomposition in LP.)

HAIMES, Y. Y. 1977. *Hierarchical Analyses of Water Resources Systems: Modeling and Optimization of Large-Scale Systems*. McGraw-Hill, New York.

This begins with a sufficient introduction to mathematical programming, then presents many models for water resource management. Water quality is treated both as a separate problem and as part of the larger management problem. Earlier models are included, and there are some new formulations that take advantage of the "hierarchical structure." For example, one 3-level model has level 1 = individual polluters, level 2 = regional treatment plant, and level 3 = regional authority. Mathematically, the hierarchical structure has a mathematical program for each level plus a small number of coupling constraints that involves the decision variables of each level. In a linear form, this corresponds to a block diagonal matrix augmented by a few linking rows.

HAITH, D. A. 1982. *Environmental Systems Optimization*. John Wiley, New York.

This is an excellent introductory text for applying mathematical programming to environmental quality control. It has small numerical problems to illustrate the elementary models that use LP, NLP, DP, and MIP.

HALVORSEN, R., AND M. G. RUBY. 1981. *Benefit-Cost Analysis of Air-Pollution Control*. Lexington Books, D. C. Heath and Company, Lexington, Mass.

This does not contain a mathematical programming model, but it describes costs and benefits in some detail, which can be useful in formulating an NLP or DP model.

HAMILTON, L. D., G. A. GOLDSTEIN, J. LEE, A. S. MANNE, W. MARCUSE, S. C. MORRIS AND C-O. WENE. 1992. MARKAL-MACRO: An Overview. Informal Report BNL-48377, Brookhaven National Laboratory, Upton, N.Y.

This is an overview of the NLP resulting from combining MARKAL, which is an LP process model of energy, with ETA-MACRO, which is an NLP. (This version is the one written in GAMS; see Ahn (1992) and Manne and Wene (1992).)

HAMLEN, JR., W. A. 1978. The Optimality and Feasibility of Uniform Air Pollution Controls. *J. Environ. Econ. and Mgmt.* 5, 301–312.

This applies NLP similar to Baumol and Oates (1975) and Tietenberg (1974), formalizing aspects of the underlying diffusion process. Standard Lagrange multiplier techniques are used to analyze optimality of uniform controls.

HARRIS, S. C. (ED.). 1989. Water Resources Planning and Management. In *Proceedings of the 16th Annual Conference*, ASCE, New York.

There is only one paper that mentions an optimization model for environmental control. This is by C. Shoemaker, L-C. Chang, L-Z. Liao and P. Liu (pp. 129–132), and it is about the use of a supercomputer to solve a DP model reported elsewhere, for example, Ahlfeld et al. (1988) and Gorelick (1990).

HASIT, Y., R. RAJAGOPAL AND P. A. VESILIND. 1981. Sludge Management Systems: Optimal Planning. *ASCE J. Environ. Engin.* **107**(3), 493–509.

This is a capacitated transshipment model with fixed charges on the operation of treatment plants that process different categories of sludge. The three types of nodes are sources, intermediate processes, and ultimate processes.

HASIT, Y., AND P. A. VESILIND. 1979. Regional Sludge Management. Chapter 12 in *Treatment and Disposal of Wastewater Sludges* (revised ed.). Ann Arbor Science, Ann Arbor, Mich., 291–313.

The book is authored by P. A. Vesilind, except for the co-authorship of this chapter, and serves as an engineering text. This is the last chapter, which first presents a transportation model to minimize the cost of transporting sludge from plants to disposal sites, which have capacity limits. They extend this to a transshipment model, similar to Liebman and Lynn (1966). This becomes part of a mixed integer program that also determines capacity expansion and new facility location.

HASS, J. E. 1970. Optimal Taxing for the Abatement of Water Pollution. *Water Resour. Res.* **6**(2), 353–365.

This applies Dantzig–Wolfe decomposition to find a convex combination of each polluter's treatment plans (and levels of flows) that minimizes the total cost to achieve specified BOD removals, or DO concentrations. The purpose is to find appropriate taxes levied on each polluter that gives them the economic incentive to meet the quality standards.

HAZEGHI, K., W. SCHMID AND P. PETALAS. 1978. Application of Operations Research Techniques for a Problem in Water Resources Management: Economic Appraisal of Changes in Water Use Induced by Investments into Navigable Rivers and Canals. In Vansteenkiste, 977–988.

This is an NLP model with a concave cost function and balance equations for both water quantity and water quality. The formulation is motivated by questions of impacts of building weirs and canals for navigation. The generalized network properties are exploited in the algorithm design.

HEADY, E. O., AND G. F. VOCKE (EDS.). 1992. *Economic Models of Agricultural Land Conservation and Environmental Improvement*. Iowa State University Press, Ames, Iowa.

This is a collection of papers in honor of E. O. Heady (recently deceased), who coauthored all of the chapters. All the papers in this book use mathematical programming for the entitled problem, so most of them are annotated separately: Boggess and Heady; Campbell and Heady; English and Heady; Langley, English and Heady; Meister and Heady; Nagadevara and Heady; Nicol and Heady; Olson et al.; Saygideger and Heady; and Wade and Heady. The editors authored Chapter 4: Analysis of Some Environmental Policies for American Agriculture (pp. 109–125). Their introduction (pp. 3–24) gives a succinct view of the models, which are mostly LP (two are quadratic programs).

HERBAY, J-P., Y. SMEERS AND D. TYTECA. 1983. Water Quality Management With Time Varying River Flow and Discharger Control. *Water Resour. Res.* **19**(6), 1481–1487.

The problem is to determine a minimum-cost combination of treatment levels of discharges into a stream. Quality standards are constraints on the stream's BOD removal rates. This relaxes the steady-state assumptions of earlier models concerning ambient conditions and operation of the treatment system.

HERZOG, JR., H. W. 1976. Economic Efficiency and Equity in Water Quality Control: Effluent Taxes and Information Requirements. *J. Environ. Econ. and Mgmt.* **2**(3), 170–184.

This uses the early models with variations on quality controls that vary by the form of taxation and regulations about the amount of reduction each polluter must achieve. The management programs are assessed for a water quality management simulation of the Patuxent River in Maryland.

HOAGLAND, P., AND R. N. STAVINS. 1992. Readings in the Field of Natural Resource and Environmental Economics. Technical Report, John F. Kennedy School of Government, Harvard University, Cambridge, Mass.

This is an excellent entrance into the entitled field. It begins with a detailed outline that includes categorization by methodology and by the part of the environment. It contains about 600 citations.

HON, K. 1993. Water Management in the '90s: A Time for Innovation. In *Proceedings of the 20th Anniversary Conference*, ASCE, New York.

Unlike the earlier proceedings (Harris 1989), this has several mathematical programming models. Most of them, however, pertain to water supply and delivery, rather than to water quality. The only ones relevant to this survey are Ben-Jemaa and Mariño; and Shafer and Varljen.

HORNER, G. L., AND D. J. DUDEK. 1980. An Analytical System for the Evaluation of Land Use and Water Quality Policy Impacts Upon Irrigated Agriculture. In Yaron and Tapiero, 537–568.

A generic math program model is given, whose constraints include inventory equations and limited total land. Crop yield depends upon water quality (used for irrigation), which can be controlled (e.g., concentration of nitrogen). The objective is a net benefit function, but most attention is given to its cost component. Although the generic model admits nonlinearities, the specific model presented is an LP.

HOROWITZ, A. J. 1970. Optimization of Water Quality Systems by Nonlinear Programming. M.S. Thesis, School of Engineering and Applied Science, University of California, Los Angeles.

This was an early NLP model to minimize total treatment cost, subject to flow equations (Streeter–Phelps and Camp–Dobbins) and DO concentration requirements. It extended the two nonlinear programs that had been developed by that time (Liebman and Lynn 1966, Clough and Bayer 1968).

HOUGLAND, E. S., AND N. T. STEPHENS. 1976. Air Pollutant Monitor Siting by Analytical Techniques. *J. Air Pollut. Control Assoc.* **26**(1), 51–53.

This presents a simple linear model with 0-1 decision variables to illustrate its applicability to siting. A distance function of coverage for each possible monitoring site is defined, and the objective is to maximize total coverage. The specific model is meant to illustrate this "analytical technique" to air quality control engineers; the actual siting problem involves more variables and constraints.

HUDAK, P. F., AND H. A. LOAICIGA. 1993. An Optimization Method for Monitoring Network Design in Multilayered Groundwater Flow Systems. *Water Resour. Res.* **29**(8), 2835–2845.

The problem is to locate wells for monitoring groundwater quality in a region that contains a contaminant. A network is defined by discretizing the region, calling each location a node. The "multilayered" property refers to hydrostratigraphic intervals (HSIs), defined as a layer within which hydraulic conductivity is assumed uniform. Weights (W_{ij}) are derived for each node (i) and each HSI (j). This is a combinatorial optimization model (classified as MIP), where $x_{ij} = 1$ if a well is installed at node i in HSI j (else, $x_{ij} = 0$). The objective is to maximize the weighted sum, where the weights reflect preferred locations (there is a negative sum as well to penalize not locating wells in the "upgradient zone," which is for background monitoring). Constraints are composed of a requirement for some wells to be located in each HSI, a fixed number to be located in the upgradient zone, and a fixed total number of wells. Removing this one last constraint on the total number of wells, the MIP decomposes into optimizing for each HSI independently. It further decomposes into sites in the upgradient zone and sites not in the upgradient zone. Each problem becomes a 0-1 knapsack problem, which can be solved parametrically to then consider the coupling constraint on the total number of wells.

HUETH, D., AND U. REGEV. 1974. Optimal Agricultural Pest Management With Increasing Pest Resistance. *Am. J. Agric. Econ.* **56**(Aug), 543–551.

This is a dynamic model whose horizon is one growing season, but the analysis uses NLP. There are three state variables: potential plant product, pest population density, and an index of stock of pest susceptibility. The decision variables are a nonpest control and a chemical pest control. The objective includes a concave benefit function and a convex cost function. The state transition functions are assumed to have the convexity and monotonicity properties that make the overall NLP a convex program, and Lagrangian analysis is used to infer solution properties.

HWANG, C. L., J. L. WILLIAMS, R. SHOJALASHKARI AND F. T. FAN. 1973. Regional Water Quality Management by the Generalized Reduced Gradient Method. *Water Resour. Bull.* **9**(6), 1159–1181.

This considers several measures of water quality at once: DO concentration, BOD concentration, temperature, and the rise in temperature. The NLP is solved with a generalized reduced gradient method, and a simple (less realistic) version is compared with a DP method.

IBM. 1968. *Proceedings of the IBM Scientific Computing Symposium on Water and Air Resource Management.* White Plains, N. Y.

This contains the first LP model for air quality control (by Teller). Among the 21 papers, the others relevant to this survey are Liebman; and Matalas.

VAN IERLAND, E. C. 1993. *Macroeconomic Analysis of Environmental Policy.* Elsevier, Amsterdam, The Netherlands.

This text begins with some background in environmental economics, using taxes and regulation in an NLP model that seeks a Pareto optimum (multiple objectives represent different decision makers). Chapter 7 gives an LP model that had been formulated by Mäler (1974).

JAMES, A. (ED.). 1978. *Mathematical Models in Water Pollution Control.* John Wiley, New York.

This collection of papers was presented at a conference on "The Use of Mathematical Modelling in Water Pollution Control," held at the University of Newcastle, 1973. Those relevant to this survey are Knapton; and Lindholm.

JAMES, D. E., H. M. A. JANSEN AND J. B. OPSCHOOR. 1978. *Economic Approaches to Environmental Problems.* Elsevier, Amsterdam, The Netherlands.

This is a broad-based book that briefly indicates how mathematical programming pertains to general equilibrium assessment models (Chapter 7).

JARVIS, J. J., R. L. RARDIN, V. E. UNGER, R. W. MOORE AND C. C. SCHIMPELER. 1978. Optimal Design of Regional Wastewater Systems: A Fixed-Charge Network Flow Model. *Opns. Res.* **26,** 538–550.

The entitled model offers a computational advantage over earlier models. One of the novel features is the use of population units, rather than effluents discharged, to measure performance of wastewater treatment plants. The cost is a piece-wise linear, concave function of population capacity, which is approximated by fixed-charge functions (jump at origin, then linear) over the linearity intervals. The network is then defined by population nodes and treatment plant sites, with arcs that represent gravity flows, called trunks. Then the levels of the flow variables are the number of population units served on each trunk, costed on a particular interval. Binary variables are introduced to keep track of which interval is used to cost a flow. The resulting model exploits the network structure iteratively.

JAWORSKI, N. A., W. J. WEBER, JR. AND R. A. DEININGER. 1970. Optimal Reservoir Releases for Water Quality Control. *ASCE J. Sanit. Engin.* **96**(3), 727–741.

This begins with a succinct definition of the management problem. A DP approach is taken with the downstream direction defining the temporal order, and the state is the level of flow. The BOD and DO concentrations are defined by functions of the flow and the regulated system used. Unlike other DP models, this considers two regulated systems, which define release sequences, so at each stage there are two states (one per system). The model is applied to the Potomac River Basin.

JEMAA, F. B., AND M. A. MARIÑO. 1993. Optimal Strategy for Aquifer Remediation. In Hon, 585–588.

This model is a dynamic program that is similar to earlier models, except that the control is a feedback mechanism. The objective is different from the other models in that it seeks to minimize the total square deviation from target state values. Also, this approach employs dynamic programming, following the standard recursion, rather than a nonlinear programming technique (used by the others).

JENKINS, R. R. 1993. *The Economics of Solid Waste Reduction*. Edward Elgar, Brookfield, Vermont.

The entitled problem area, which is classified here as land quality control, is approached by a variety of economic methods. Some are econometric, and some are nonmathematical. One chapter (3) uses Lagrange multipliers with a welfare economic model to explain the behavior of households and firms.

JOHANSSON, P-O. 1987. *The Economic Theory and Measurement of Environmental Benefits*. Cambridge University Press, Cambridge, U.K.

This is more elementary (mathematically) than Mäler (1974), but it takes the same welfare economic approach to give an integrated economic analysis of environmental quality control. One new aspect is attention to discrete decision variables (Chapter 8) and to representation of uncertainty (Chapter 10). By "discrete analysis," the author simply illustrates the inappropriateness of using shadow prices when a variable is binary (does not proceed to use MIP). Uncertainty is handled by expected values.

JOHNSON, E. L. 1967. A Study in the Economics of Water Quality Management. *Water Resour. Res.* **3**(2), 291–305.

This considers an LP formulation by Thomann and Sobel (1964). Structurally, it goes further than Deininger (1965) and Sobel (1965) in presenting four variations on methods of allocation: uniform percentage of waste removal, single DO concentration level (letting cost minimization determine discharges), uniform effluent charge, and zone effluent charge. Each method leads to a different (linear) constraint on the discharge variables.

JOHNSTON, G. M., D. FRESHWATER AND P. FAVERO (EDS.). 1988. *Natural Resource and Environmental Policy Analysis*. Westview Press, Boulder, Colorado.

This is a collection of papers, mostly about specific studies. Those that use mathematical (linear) programming are Arthur; and LeBlanc and Reilly.

JONES, C. V. 1994. Visualization in Mathematical Programming. *ORSA J. Comput.* **6**(3), 221–257.

This is not directly related to environmental control, but it is referenced in the text in the context of avenues for further research. This is a comprehensive state-of-the-art survey on the entitled topic, which is aimed at improving our ability to understand a mathematical programming model and scenario solutions.

KEEGAN, R. T., AND J. V. LEEDS, JR. 1970. Dynamic Programming and Estuarine Water Quality Control. *Water Resour. Bull.* **6**(2), 235–248.

A DP model is formulated to address such questions as: How much pollutant can be discharged such that total cost is minimized while satisfying water quality standards? From where and with what type of treatment? For a given budget, how can total pollutant discharged be minimized? The first and last questions become reciprocal with the DP approach, because it is inherently parametric. Suppose that $f(b)$ = min cost for level of pollutants $\leq b$, and $g(c)$ = min pollutant discharge for cost $\leq c$. Then DP finds f for all b within some target level, so $g(c) = \min \{b: f(b) \leq c\}$. Similarly, DP can find g for all c, and $f(b) = \mathrm{Min}\ \{c: g(c) \leq b\}$. In words, f and g each represent the same envelope function that describes cost as a function of pollutant discharge.

KEELER, E. M., E. M. SPENCE AND R. ZECKHAUSER. 1971. The Optimal Control of Pollution. *J. Econ. Theory* **4**(1), 19–34.

This is a welfare economic approach to pollution, regardless of whether it pertains to air, land, or water. Three simple equilibrium models are described. By assuming convexity, the standard Lagrangian results apply.

KERRI, K. D. 1966. An Economic Approach to Water Quality Control. *J. Water Pollut. Control Fed.* **38**(12), 1883–1897.

This is an early LP to minimize cost subject to DO deficit reduction. Although it uses the Streeter–Phelps equations, it is different from the LP of Deininger (1965). The LP was applied to the Willamette River in Oregon.

KERRI, K. D. 1967. A Dynamic Model for Water Quality Control. *J. Water Pollut. Control Fed.* **39**(5), 772–789.

This extends the author's previous (1966) use of LP by considering three dynamic conditions: expansion of municipalities, production expansion (new or existing firms), and increased effluent standards.

KNAPTON, J. 1978. Optimization and Its Application to a Unit Process Design Problem. Chapter 4 in James, 81–104.

This is mostly a brief tutorial on mathematical programming. A sewage system design problem is used to illustrate the application of DP and NLP.

KNEESE, A. V. 1977. *Economics and the Environment*. Penguin Books, New York.

This is a fairly nontechnical introduction to the entitled subject. Chapter 6, however, presents "some useful models," which includes Lagrangian analysis of an elementary NLP and the LP approach to resource management. Examples include air and water quality control, separately and integrated.

KNEESE, A. V., R. U. AYRES AND R. C. D'ARGE. 1970. *Economics and the Environment: A Materials Balance Approach*. Johns Hopkins Press, Baltimore, Maryland.

As the title suggests, this is a mostly LP model (in which the flow equations are linear, but there can be nonlinear production functions), using the activity analysis approach developed during the 1950s. Optimal taxes are defined by the Lagrange multipliers applied to each sector's resource profile.

KNEESE, A. V., AND B. T. BOWER. 1968. *Managing Water Quality: Economics, Technology, Institutions*. Johns Hopkins Press, Baltimore, Maryland.

This is an early, comprehensive introduction that includes specific cases. The LP models (Thomann and Sobel 1964) of the Delaware estuary are mentioned, but no mathematical programming model is described with any detail for any of the cases. Welfare economics is considered as a general approach to policy issues, which later led to NLP models (by others). For this reason, some authors cite this as though it was the beginning of environmental economics.

KNEESE, A. V., AND B. T. BOWER (EDS.). 1972. *Environmental Quality Analysis*. Johns Hopkins Press, Baltimore, Maryland.

This collection of papers is primarily from the economist's perspective about environmental quality. Those papers relevant to this survey are d'Arge; Langham, Headley and Edwards; and Russell and Spofford, Jr.

KNEESE, A. V., AND B. T. BOWER. 1979. *Environmental Quality and Residuals Management*. Johns Hopkins Press, Baltimore, Maryland.

Residuals management (see text) had already evolved, apart from environmental quality issues, using Lagrangian duality for general analysis and LP for particular models. This book casts the residuals management problem in the context of controlling environmental quality through constraints and taxes. (Contrary to the implied claim in this book, these are not equivalent. The dual price used as a tax in place of the constraint can produce a different solution.) Specific mathematical programming models are presented as appendices to some of the chapters.

KOHN, R. E. 1971. Application of Linear Programming to a Controversy on Air Pollution Control. *Mgmt. Sci.* **17,** B609–B621.

This presents part of the author's thesis with its application to the St. Louis airshed. The 'controversy' indicated in the title pertains to the 1967 Missouri Air Conservation Commission regulation that the sulfur content of coal burned in the St. Louis area could not exceed 2%.

KOHN, R. E. 1971. Optimal Air Quality Standards. *Econometrica* **39**(6), 983–995.

This presents another aspect of the author's thesis. Using LP, he generates a set of alternative air quality levels that have the same total cost. The frontier tradeoff is compared to a social indifference curve, based on medical considerations, for the St. Louis airshed. Both curves are concave in the same direction.

KOHN, R. E. 1973. Labor Displacement and Air Pollution Control. *Opns. Res.* **21,** 1063–1070.

This is an outgrowth of the author's thesis that uses lexicographic ordering of two objectives. The first objective is cost (as in the thesis), and the second is labor displacement. Applied to the St. Louis airshed, the author concludes negligible changes in displacement, but he notes that this could be serious for airsheds with less industrial diversity than St. Louis.

KOHN, R. E. 1975. *Air Pollution Control*. DC Heath and Company, Lexington, Mass.

This presents the basic theory and partly serves as background for the author's LP model (1978). Chapter 1 gives the fundamental connection between welfare economics and mathematical programming via activity analysis. Subsequent chapters then rely on optimization modeling to represent economic behavior.

KOHN, R. E. 1978. *A Linear Programming Model for Air Pollution Control*. MIT Press, Cambridge, Mass.

This gives a detailed description of the entitled model after a review of basic principles. The examples are extensive and use St. Louis airshed data for emissions of pollutants. The author analyzes the results using shadow prices, showing how the LP model can describe such things as marginal damage per capita. The LP model was first developed in 1969 as the author's Ph.D. Thesis, Economics Department, Washington University, St. Louis, Missouri.

KOLSTAD, C. D. 1987. Pollution Possibility Frontiers. In Lev et al., 273–287.

This presents an NLP approach as an economic theory of transforming chemical emissions, called residuals (see text), into ambient pollution. The net benefit function is the difference between production benefits and pollution damage. Due to multiple industries, Pareto solutions are sought, which comprise the "possibility frontier." Properties of the frontier are derived under assumptions of homogeneity, subadditivity, and monotonicity of the concentration measure.

KORTANEK, K. O., AND W. L. GORR. 1972. Numerical Aspects of Pollution Abatement Problems: Optimal Control Strategies for Air Quality Standards. In *Proceedings in Operations Research*. Physica-Verlag, Würzburg, Germany.

This model is described by Gorr, Gustafson and Kortanek (1972) (also see Gustafson and Kortanek 1973, 1976, 1982). This paper focuses on numerical solution techniques.

KOSOBUD, R. F., T. A. DALY AND K. G. QUINN. 1991. Tradeable Permits for Global Warming Control: Implications for Regional Economies and Public Utilities. Technical Report, University of Illinois at Chicago (Presented at the 53rd American Power Conference, Chicago, Ill.).

This uses LP to provide estimates of global and regional time paths of permit prices and energy use, and to trace gas emissions. In the economic theoretical development, the model can be nonlinear, and Lagrange multipliers are applied in the usual way to determine the structure of an optimal time path that minimizes a discounted cost of abatement and permit purchase. In testing the theory, an LP version is used (the details are omitted) and compared with results from Global 2100 by Manne and Richels (1992).

KUHNER, J., AND B. HEILER. 1973. Regional Planning Models for Solid Waste Management. Chapter 16 in Deininger, 327–362.

This contains a literature review, which cites the few LP and MIP models that had been published by that time. Several LP models are described in detail, varying by static versus dynamic characteristics and by objective function (usually total cost of operation and transportation). The constraints are composed of mass balance equations, capacity limits, and waste disposal requirements. These extend naturally to MIP models with fixed charges on new capacity.

LABADIE, J. W. 1988. Program MODSIM: River Basin Network Flow Model for the Microcomputer. Department of Civil Engineering, Colorado State University, Ft. Collins, Colorado.

This gives details about stream modeling and documents the software system, MODSIM, used by the EPA. Water quality is not explicitly included, but the model can be extended to account for pollution with additional material flow equations and discharge rates. The notion of simulation is that of stepping through time with the dynamical state equations, and myopic optimization is applied at each stage to represent the behavior of the agents. Unlike conventional DP, which assumes a clairvoyance, the use of optimization is to simulate responses to the state under the assumption of rational behavior, rather than to prescribe an overall best policy.

LAKSHMANAN, T. R., AND P. NIJKAMP (EDS.). 1980. *Economic-Environmental-Energy Interactions: Modeling and Policy Analysis*. Martinus Nijhoff, Boston.

This contains a collection of papers on the entitled subject. The only one that contains a mathematical programming model is Lakshmanan and Ratick.

LAKSHMANAN, T. R., AND P. NIJKAMP (EDS.). 1983. *Systems and Models for Energy and Environmental Analysis*. Gower Publishing Co., Hampshire, England.

Many of the papers, including the editors' introduction, make interesting points about modeling the linkage among energy, environment, and the economy, but only a few give details about the model. An exception is Chapter 8: A Programming Approach as a Design for Economic Development Policy, by R. Bannink, C. Broekhof and P. Nijkamp (pp. 80-90). (Because this book is difficult to read, due to the very light font and very tight line spacing, the papers are not cited separately here.)

LAKSHMANAN, T. R., AND S. RATICK. 1980. Integrated Models for Economic-Energy-Environmental Impact Analysis. Chapter 1 in Lakshmanan and Nijkamp, 1–39.

After some general background, this gives an overview of SEAS, which is EPA's Strategic Environmental Assessment System, and an application. Of the three modules, the energy submodel is a cost-minimization allocation of energy supplies to satisfy demands in the presence of abatement policies (determined by the environmental submodel). It is not clear what the exact form of the mathematical program is, but it appears to be an LP.

LANGHAM, M. R., J. C. HEADLEY AND W. F. EDWARDS. 1972. Agricultural Pesticides: Productivity and Externalities. Chapter 5 in Kneese and Bower, 181–212.

This is a welfare economics model to determine levels of pesticides used, based on the model by Edwards, Langham and Headley (1970). The objective is to maximize producer and consumer surplus minus an "externality function" that represents the damage cost of the pesticide level. No interaction is assumed: The total damage is the sum of the damages from each pesticide. There is also a constraint that limits the use of each pesticide in each portion of the land. Piecewise approximation yields an LP model, which was applied to Dade County, Florida.

LANGLEY, J. A., B. C. ENGLISH AND E. O. HEADY. 1992. A Regional-National Recursive Model for the State of Iowa. Chapter 11 in Heady and Vocke, 251–271.

This uses LP to represent optimal Iowa production, which is linked with a national econometric model. The algorithm iterates (not recursive, as the title suggests) by using the output of one module as input to the other. Details of the models are given, and this is applied to test data.

LEBLANC, M., AND J. REILLY. 1988. Energy Policy Analysis: Alternative Modeling Approaches. Chapter 12 in Johnston et al., 244–271.

The energy model includes CO_2 emissions, which complements the one by Arthur (1988). Two approaches are discussed: parametric simulation and iterative linear programming, which obtains a partial equilibrium. Some modeling issues are raised and guidelines are offered.

LEE, B. H., AND R. A. DEININGER. 1992. Optimal Locations of Monitoring Stations in Water Distribution System. *ASCE J. Environ. Engin.* **118**(1), 4–16.

A region is defined by a network with two types of nodes: candidate monitoring station sites, and demands. An integer programming model is presented to determine where to locate sampling stations to maximize total satisfied demand. A binary variable (x) represents whether a monitoring station is located at a site, and the total number of stations is limited. The demand variables (y) are limited by the coverage provided by the stations with a linear constraint $Ax \geqslant y$, where A is a binary matrix such that $A_{ij} = 1$ if station i serves demand j (else, $A_{ij} = 0$). The model is applied to Flint, Michigan.

LEE, S. I., AND P. K. KITANIDIS. 1991. Optimal Estimation and Scheduling in Aquifer Remediation With Incomplete Information. *Water Resour. Res.* **27**(9), 2203–2217.

This begins with a review of combining groundwater simulation with optimization, citing Gorelick et al. (1984) as the first to do so. The approach taken here is adaptive, where controls respond to real-time measurements of uncertain parameters, notably transmissivities.

LEHMANN, R. 1991. Uncertainty Analysis for a Linear Programming Model for Acid Rain Abatement. *Atmos. Environ.* **25**(2), 231–240.

This describes how the RAINS model (Alcamo et al. 1990) represents uncertainty in the transfer coefficients. Unlike the nonlinear certainty equivalent of Guldmann (1986) and others, this model is linear. The derivation makes use of some simplifying assumptions, such as constant wind speed, and properties of receptors that are either very near or very far from a source. Some empirical results are given to indicate how well this model represents uncertainty.

LEIGHTON, J. P., AND C. A. SHOEMAKER. 1984. An Integer Programming Analysis of the Regionalization of Large Wastewater Treatment and Collection Systems. *Water Resour. Res.* **20**(6), 671–681.

The problem is to select a regionalization plan for wastewater treatment and collection, which was applied to Western Suffolk County in New York. A network is defined with nodes representing sewage treatment plants and population centers. The decision variables are average flows and levels of treatment, for which there are capacity limits. Binary variables are defined to allow capacity expansion (including new plant construction) and to represent pipe routes.

LEV, B., J. A. BLOOM, A. S. GLEIT, F. H. MURPHY AND C. SHOEMAKER (EDS.). 1987. *Strategic Planning in Energy and Natural Resources*. North-Holland, New York.

The papers that contain mathematical programming models for environmental control are Ahlfeld and Mulvey; Clark and Adams; Kolstad; and Turnquist.

LIEBMAN, J. C. 1968. A Branch-and-Bound Algorithm for Minimizing the Costs of Waste Treatment, Subject to Equity Constraints. Number 10 in IBM, 193–202.

The early LP models, which find overall min-cost solutions, can contain inequities as to who pays for the pollution of a stream. This paper addresses the conflict between equity and economy by formulating categories of treatment plants (e.g., putting all paper mills in one category). The model also allows pollution to flow upstream, as well as downstream, so the earlier DP approach (Liebman and Lynn 1966) does not apply. Instead, MIP is used, as in Liebman and Marks (1968).

LIEBMAN, J. C. 1975. Models in Solid Waste Management. Chapter 5 in Gass and Sisson, 139–164.

This is a review of operations research models for solid waste collection and disposal. Those using mathematical programming are siting (MIP), capacity expansion (MIP, DP), and routing (MIP).

LIEBMAN, J. C., AND W. R. LYNN. 1966. The Optimal Allocation of Stream-Dissolved Oxygen. *Water Resour. Res.* **2**(3), 581–591.

This was the first DP model for water quality control, based on the first author's thesis (1965). Similar to the early LP models, it minimizes the cost of providing waste treatment to satisfy DO concentration levels.

LIEBMAN, J. C., AND D. H. MARKS. 1968. A Balas Algorithm for Zoned Uniform Treatment. *ASCE J. Sanit. Engin.* **94**(4), 585–593.

This is an integer programming model that extends the early LP models for water quality control by considering a finite number of treatment levels in each zone (defined by reaches). This is reformulated as a 0-1 integer program to which Balas' implicit enumeration algorithm was applied (this was a new technique at that time, so its performance to solve large-scale problems was not well understood).

LINDHOLM, O. G. 1978. Modelling of Sewerage Systems. Chapter 10 in James, 227–246.

This describes an NLP model of a sewerage system design built for the Norwegian government. Two objectives are min cost and min leakage of waste water from the system.

LIU, B., AND E. S. YU. 1977. *Air Pollution Damage Functions and Regional Damage Estimates*. Technomic, Westport, Conn.

This is a detailed study of how to estimate damage caused by air pollution. A well formed damage function can serve as an objective in an air quality model, such as minimizing damage as a function of variables whose levels can be controlled. This study proposes a primitive theoretical framework for optimal policies, using Lagrange multipliers in the usual way to deduce properties of optimality. Its main contribution, however, is the detailed derivation of damage functions.

LOAICIGA, H. A. 1989. An Optimization Approach for Groundwater Quality Monitoring Network Design. *Water Resour. Res.* **25**(8), 1771–1782.

This is a combinatorial optimization model (classified as MIP) to determine whether or not an observation is made at a sampling site. The objective is to minimize the variance of the error, which is a quadratic function. In addition to a budget constraint, there are linear equations that require the estimators to be unbiased. The basic model is extended to be dynamic.

LOEHMAN, E., D. PINGRY AND A. WHINSTON. 1974. Cost Allocation for a Regional Pollution Treatment System. In Conner and Loehman, 223–250.

This extends the LP models by Graves, Hatfield and Whinston (1969, 1972) with a nonlinear cost function. Variations are analyzed to enable market forces to replace quality constraints, such as taxes and an incremental cost allocation scheme (to achieve better equity among polluters).

LOHANI, B. N., AND K. B. HEE. 1983. A CCDP Model for Water Quality Management in the Hsintien River in Taiwan. *Int. J. Water Resour. Dev.* **1**(2), 91–114.

This is a chance-constrained model to minimize operation costs, using DP as the solution method. The cost is a nonlinear function of the level of treatment at each reach. State equations represent BOD and DO concentrations over the (ordered) reaches. By using a binary decision rule, the certainty equivalent simply constrains the ith stage, whose decision variable is the level of treatment at reach i.

LOHANI, B. N., AND N. C. THANH. 1978. Stochastic Programming Model for Water Quality Management in a River. *J. Water Pollut. Control Fed.* **50,** 2175–2182.

This extends the Liebman and Lynn (1966) DP model to address a question of equity: How much should each polluter pay? Total treatment cost is minimized subject to a chance constraint that represents risk of violating a DO standard. Using a linear decision rule, the certainty equivalent becomes simple inequalities on the levels of treatment. Then two policies are analyzed in connection with the equity question. The first policy is to require a percentage of BOD removal from each plant; the second is to require all plants to achieve the same level of risk for DO violation.

LOHANI, B. N., AND N. C. THANH. 1979. Probabilistic Water Quality Control Policies. *ASCE J. Environ. Engin.* **105**(4), 713–725.

This extends the early water quality control models by considering stochastic stream flows with a chance constraint. The model was reported by the authors in 1978, and this paper applies it to the Hsintien River in Taiwan.

LOUCKS, D. P. 1976. Surface-Water Quality Management Models. Chapter 6 in Biswas, 219–252.

This reviews the early LP models and some extensions to illustrate their applications for a variety of scenarios.

LOUCKS, D. P., AND H. D. JACOBY. 1972. Flow Regulation for Water Quality Management. Chapter 9 in Dorfman et al., 363–431.

The nature of the flow regulation pertains to rates of discharge into a stream at locations associated with plants that need to eliminate waste. The levels of pollutants are measured by DO concentration at specified points (not the same as plant locations). The primary objective is to determine a least-cost solution for water delivery when there might be a drought. Water quality is included as requirements according to the class of water use (there are four classes that differ by DO level requirements). In the complete model, a Pareto optimum is obtained by maximizing a weighted sum of net benefits. The net benefit of each participant is a piecewise linear utility function of its discharge.

LOUCKS, D. P., C. S. REVELLE AND W. R. LYNN. 1967. Linear Programming Models for Water Pollution Control. *Mgmt. Sci.* **14**(4), B166–B181.

Drawing on earlier works by Thomann and Sobel (1964), Sobel (1965), Deininger (1965), Kerri (1966) and Liebman and Lynn (1966), this presents an extensive development of the entitled models. The problem is to determine a least-cost control of effluents into a stream at designated locations. Each discharge affects the downstream DO levels, and reductions in the waste discharges have associated costs. The fundamental LP model is to minimize cost subject to flow constraints and bounds on DO concentrations and on the levels of the controls.

LOUCKS, D. P., J. R. STEDINGER AND D. A. HAITH. 1981. *Water Resource Systems Planning and Analysis*. Prentice-Hall, Englewood Cliffs, N. J.

This is a comprehensive, introductory text on the entitled topic. Early chapters set the foundation, including presentations of LP, NLP, and DP models. Part IV, which deals specifically with water quality management, presents simulation models first (Chapter 9). This describes the standard stream modeling with dynamical equations. The last chapter (10) presents a least-cost NLP model with exogenous quality standards. The variables are pollution reduction levels that can be controlled by operating plants whose sites can be part of the decision.

LYNN, W. R., J. A. LOGAN AND A. CHARNES. 1962. Systems Analysis for Planning Wastewater Treatment Plants. *J. Water Pollut. Control Fed.* **34**(6), 565–581.

This was the first LP formulation to minimize the cost of sewage treatment. A generalized network is formulated and flow balance equations are from first principles: input = output.

MALER, K-G. 1971. A Method of Estimating Social Benefits From Pollution Control. In Bohm and Kneese, 106–118.

This presents the primal and (Lagrangian) dual NLPs, where each consumer's utility is a function of the levels of private and public goods. For example, a private good could be the catch of fish and the associated public good is the dissolved oxygen, which affects all fish quality. The primal NLP maximizes each consumer's utility subject to a budget constraint that depends upon prices, and its dual minimizes total cost subject to a utility requirement. Assuming differentiability and quasiconcavity, Lagrange multipliers are applied to derive properties of the equilibria.

MALER, K-G. 1974. *Environmental Economics: A Theoretical Inquiry*. Johns Hopkins Press, Baltimore, Maryland.

This presents a materials-balance, general equilibrium framework for modeling environmental quality and its relations with the economy. The balance equations account for flows of goods, services, labor, and capital. These are logical constraints in the equivalent mathematical program whose solution is a general equilibrium. Other constraints include limits on variables that are in the flow accounts, or on controls that are related to the economic variables. Environmental quality is included in the welfare function, which is a present value utility in the aggregate. The point of this development is to present alternative paradigms that suggest a framework for environmental modeling based on standard economic theory. As such, it is an abstract treatment that also suggests a generic method of analysis.

MANNE, A. S., AND R. G. RICHELS. 1992. *Buying Greenhouse Insurance: The Economic Costs of Carbon Dioxide Emission Limits*. MIT Press, Cambridge, Mass.

Motivated by analyzing policies that would affect the next century, the ETA-MACRO modeling framework is extended to define their Global 2100 model. Part II gives a detailed description of the model structure, supplemented by appendices on the underlying assumptions of its parts: macroeconomic relations, electricity generation, nonelectric energy supplies, international oil trade, and carbon emissions. The model is a partial equilibrium, with oil prices as exogenous, which is equivalent to a nonlinear program whose objective function is a logarithmic utility function of consumption. Most of the constraints are linear; some are linearizations of nonlinear functions. Carbon emissions are determined by processes, notably electricity generation, for which emission rates are presumed known.

MANNE, A. S., AND T. F. RUTHERFORD. 1993. International Trade in Oil, Gas and Carbon Emission Rights: An Intertemporal General Equilibrium Model. Technical Report, Economics Department, University of Colorado, Boulder.

This extends the Global 2100 model (Manne and Richels 1992) to obtain a general equilibrium; in particular, oil import and export limits are removed and oil prices are imputed, rather than exogenous. Then three issues are addressed with the model: 1) impacts of carbon emission limits on future oil prices, 2) leakage, and 3) quantification of gains from trade in carbon emission rights. (The term "leakage" pertains to an increase in one region's CO_2, relative to global reduction. This can occur from relocation or from the use of substitute fuels if oil prices increase.) In addition, this paper demonstrates the computational effectiveness of using sequential joint maximization to obtain an intertemporal general equilibrium.

MANNE, A. S., AND C-O. WENE. 1992. MARKAL-MACRO: A Linked Model for Energy-Economy Analysis. Informal Report BNL-47161. Brookhaven National Laboratory, Upton, N.Y.

This reports on some practical problems with linking MARKAL and ETA-MACRO to form MARKAL-MACRO (see Abilock and Fishbone 1979, Hamilton et al. 1992). MARKAL is a linear program that was written in OMNI, and ETA-MACRO is a nonlinear program that was written in GAMS. The language differences presented some implementation problems with the first effort to link them. (The present version of MARKAL-MACRO is written entirely in GAMS and solved as an NLP.)

MARKS, D. H. 1972. Water Quality Management. Chapter 17 in Drake et al., 356–375.

This is an insightful review of the early LP and DP models.

MARKS, D. H. 1975. Models in Water Resources. Chapter 4 in Gass and Sisson, 103–137.

This gives a succinct account of the problem and how mathematical programming models apply to some aspects: least-cost treatment, equitable cost distribution, siting, tax schemes, and pipeline design. Generic models are given, which contain earlier models as special cases. Quality requirements could be absolute or relative to what a source currently discharges.

MARRYOTT, R. A., D. E. DOUGHERTY AND R. L. STOLLAR. 1993. Optimal Groundwater Management 2. Application of Simulated Annealing to a Field-Scale Contamination Site. *Water Resour. Res.* **29**(4), 847–860.

The models are the nonlinear programs described in Gorelick (1983), Ahlfeld et al. (1988). The design problem is to select well locations and pumping rates to minimize cost or contamination (or a linear combination). As in the earlier models, a complication is the use of simulation to solve the transport equations to determine the contamination for a particular design. This paper presents some experiments with a simulated annealing approach. (Part 1, published in 1991, presented the method of simulated annealing apart from this application.)

MATALAS, N. C. 1968. Optimum Gaging Station Location. Number 5 in IBM, 85–94.

It is assumed that the quality of information obtained from a gaging station is inversely proportional to the variance of the estimate. The model maximizes information by minimizing the sum of variances, subject to a budget constraint. Each variance is a function of all station locations due to correlation effects. The first-order Lagrangian conditions are applied to obtain the key solution property that determines whether a station should be discontinued.

MATHUR, V. K. 1976. Spatial Economic Theory of Pollution Control. *J. Environ. Econ. and Mgmt.* **3**(1), 16–28.

This uses NLP analysis of several welfare economic models in connection with taxing SO emissions. One model seeks minimum cost; another seeks maximum profit. In each case, bounds on levels of taxation are derived that achieve desired levels of emissions. The appendix (B) also describes two ways to introduce land prices into the model.

MATHUR, V. K., AND H. YAMADA. 1972. An Economic Theory of Pollution Control. *Papers Region. Sci. Assoc.* **28,** 223–235.

This postulates a welfare economic model with convexity structure to apply Lagrangian duality. The model is Max $W(x, y, R)$ subject to $T(x, y, z) = 0$ and $R = F(v(y) - a(z), R^0) = 0$. The variables are x = production level of pollution-free good, y = production level of pollution-generating good, and z = production level of pollution-preventive (or abatement) good. All functions are assumed to be twice continuously differentiable, and the welfare function (W) is concave. The first equation represents limited production with free substitutability among the three types of goods, where T is concave. The second equation defines the level of pollution (R), where v is strictly increasing and convex, a is strictly increasing and concave, and R^0 is the initial stock. F is assumed to be strictly increasing and convex in its first argument. The model is extended to two pollution-generating goods (y_1, y_2).

MCNAMARA, J. R. 1976. An Optimization Model for Regional Water Quality Management. *Water Resour. Res.* **12**(2), 125–134.

This extends the early LP models by considering a variety of pollution abatement techniques in a geometric program. The use of DP to allow nonlinearities is rejected on computational grounds, due to the size of the state space for the extension (though the author's 1971 Ph.D. thesis did show how geometric programming can be used to solve each stage of a DP formulated with a manageable state space). Following the early models, quality is measured by levels of BOD concentration, and the stream is partitioned into reaches in the usual way. Nonlinearities enter the constraints and costs, as described in Ecker and McNamara (1971) and Ecker (1975). This model incorporates flow regulations, and it is illustrated with an application to the Upper Hudson River in New York.

MEISTER, A. D., AND E. O. HEADY. 1992. Assessment of Water and Land Resources for U.S. Agriculture Within an Interregional Competition Framework. Chapter 3 in Heady and Vocke, 79–108.

This is the same LP as by Nicol and Heady (1992), except that the producing regions are aggregated to about half the number. Many of the later chapters refer to this one as the LP they used in their studies. A conclusion of this study is that constraints on land and water use would redistribute farm income in favor of areas with modest rainfall and level land.

MEYER, P. D., AND E. D. BRILL, JR. 1988. A Method for Locating Wells in a Groundwater Monitoring Network Under Conditions of Uncertainty. *Water Resour. Res.* **24**(8), 1277–1282.

This uses simulation and maximal location covering iteratively. Given well sites, the simulation determines sets of wells that detects contamination at each of several plumes. The maximal location covering problem has the form: Maximize $\sum_i c_i y_i$: $\sum_{j \in S^i} x_j \geq y_i$, $\sum_j x_j = N$, $x_j, y_i \in \{0, 1\}$, where N is the number of wells to be located, $x_j = 1$ means a well is located at site j, and $y_i = 1$ means plume i is detected by some well (in its associated set, S_i). The sets, $\{S_i\}$ were determined by the previous simulation.

MHAISALKAR, V. A., J. K. BASSIN, R. PARAMASIVAM AND P. KHANNA. 1993. Dynamic Programming Optimization of Water-Treatment-Plant Design. *ASCE J. Environ. Engin.* **119**(6), 1158–1175.

This describes the DP model and a case study to minimize the cost of wastewater treatment by a sequence of process units: rapid-mix, slow-mix, sedimentation, and rapid sand filter. Each type of unit has design variables, such as detention time in the mix units and filtration rates in the filters. The optimal sequencing of the processes is solved by DP.

MIDDLETON, A. C., AND A. W. LAWRENCE. 1974. Cost Optimization of Activated Sludge Systems. *Biotech. and Bioengin.* **16,** 807–826.

This is an NLP model to determine unit processes and their operational characteristics. Constraints include physical relationships and bounds. The cost function is the sum of annual operation and maintenance costs plus discounted, unitized capital costs.

MIDDLETON, A. C., AND A. W. LAWRENCE. 1976. Least Cost Design of Activated Sludge Systems. *J. Water Pollut. Control Fed.* **48**(5), 889–905.

This is the same NLP as the authors published in 1974.

MILLER, C., D. M. VIOLETTE AND J. LENT. 1982. Economic Approaches to Controlling Stationary Source Air Emissions, A Quantitative Assessment of Control Strategies for Nitrogen Oxide. Chapter 11 in Tolley et al., 257–294.

This paper formulates an integer program to choose a (discrete) level of control for each source that minimizes total control cost. The emissions from a source under a particular level of control is modeled by EPA's Real-time Air-quality Model (RAM).

MILLER, W. L., AND D. M. BYERS. 1973. Development and Display of Multiple-Objective Project Impacts. *Water Resour. Res.* **9**(1), 11–20.

This presents a public investment MIP model, where binary variables are used to represent which of several structural designs is to be used. Parametric programming is used to generate a frontier function for maximum benefit relative to social objectives. Tradeoffs are seen from displays, such as between net dollar benefit and some level of sediment (e.g., of phosphorus or nitrogen).

MORRISON, M. B., AND E. S. RUBIN. 1985. A Linear Programming Model for Acid Rain Policy Analysis. *J. Air Pollut. Control Assoc.* **35**(11), 1137–1148.

This LP, called OMEGA (Optimization Model for Emission Generating Alternatives), minimizes total cost to produce and deliver coal to satisfy energy demand, subject to specified reductions in SO_2 emissions. The cost is the total levelized cost of delivered coal plus the cost of pollution control equipment. Extensive sensitivity analyses (95 cases) are used to explore the effects of assumptions, such as demands, costs, and retrofitting (versus coal switching).

MULLER, F. 1973. An Operational Mathematical Programming Model for the Planning of Economic Activities in Relation to the Environment. *Socio-Econ. Plan. Sci.* **7,** 123–138.

This is a welfare economic model, using NLP methods. Economic relations, such as with production, labor, and employment, are represented by an input-output (linear) equation. This is augmented with emissions, estimated from a diffusion model, giving another linear subsystem. Decision variables include levels of investments to abate pollution. Two models are LPs. The NLP (model III) contains bilinear terms resulting from the product of levels of production times levels of investment.

MURPHY, F. H. 1994. Convergence Properties of NEMS. Working Paper, Department of Management, Temple University, Philadelphia.

This begins by showing how the modular structure of the National Energy Modeling System (NEMS) employs Bender's decomposition. Other decomposition approaches, such as Lan and Fuller, are considered in connection with using Gauss–Seidel iterations. The difficulties encountered with convergence in an earlier version of NEMS is overcome through deeper understanding of the iterative process when computing an economic equilibrium by fixed-point computation using LP (see text). The analysis also applies to recent air quality control models that use a similar modularity, for example, Duraiappah (1993).

NAGADEVARA, V., AND E. O. HEADY. 1992. Interregional Analysis of Soil Conservancy and Environmental Regulations in Iowa Within a National Framework. Chapter 2 in Heady and Vocke, 64–75.

This is an LP with four parts: land and water resource availability, crop and livestock production, commodity transportation network, and demands (both domestic and foreign). The objective is to minimize total cost. The authors applied this to Iowa, using different policy instruments to reduce soil erosion. They conclude that Iowa farmers will bear a significant income loss, and other farmers will realize benefits.

NAYAYAN, R., AND B. BISHOP. 1978. A Residuals Management Model for Regional Environmental Quality. *J. Environ. Syst.* **8**(2), 139–155.

This is a linear program with decision variables, x_{ijk} = level of kth residual at the ith source using the jth treatment. The model is generic in that the associated discharge rates, A_{pijk}, for the pth pollutant could be into any part of the environment. The example application, however, is for the Utah Basin, with BOD and DO concentrations the measure of water quality.

NICHOLSON, G. S., E. E. PYATT AND D. H. MOREAU. 1970. A Methodology for Selecting Among Water Quality Alternatives. *Water Resour. Bull.* **6**(1), 23–33.

This is a discrete DP model over a finite horizon that chooses discharge rates to minimize cost. The cost is also a function of the temperature, which depends upon the layer from which the water is withdrawn. The state is the total water discharge rate, which is the sum of the decision rates. The paper includes a case study of the Savannah River basin.

NICOL, K., AND E. O. HEADY. 1992. Interregional Competition Modeling of a National Soil Conservancy Program. Chapter 1 in Heady and Vocke, 27–63.

This is an LP model designed to answer the question, Does agriculture have sufficient production capacity to meet domestic and export demands and also contribute to improvement of the environment through reducing the quantity of sediment discharged into the nation's waterways? The LP has 223 producing regions (covering the continental U.S.), 51 water supply regions (in the western U.S.), and 30 market regions (which are also aggregated to form 7 reporting regions). This regional structure is chosen to correspond to the USDA data base. To measure soil damage, the model uses the classification scheme of the USDA, for which there are eight primary classes, seven of which are subdivided. This paper describes the LP in detail and its data sources. Among its conclusions is that soil erosion can be reduced significantly with only small increases in commodity prices with low exports.

NIJKAMP, P. 1977. *Theory and Application of Environmental Economics*. North–Holland, New York.

This is a succinct, lucid text on environmental economics. Chapter 4 presents some mathematical programming models, beginning with interregional input-output equations. A quadratic programming model is presented, where the objective is to minimize total square deviation from target pollutant levels. LP models can have an objective to minimize cost, maximize income, or maximize consumption. Dual prices are applied in an example of economic analysis, relative to the objective chosen. This fundamental modeling framework is extended in subsequent chapters, including one on multiple objectives, then in several chapters that extend the modeling framework to dynamic control systems.

NIJKAMP, P. 1980. *Environmental Policy Analysis: Operational Methods and Models*. John Wiley, New York.

This extends the author's previous work (1977) in several ways. There is special attention to multiple objective models, distinguishing between "hard" and "soft." Hard models presume complete knowledge of quantitative measures, such as utility and penalty (like square deviation from targets). Soft models are qualitative, based on ordinal information, like preferences. In addition, geometric programming is used to optimize a geometric mean of multiple objectives, rather than the usual arithmetic mean.

NORDHAUS, W. D. 1992. An Optimal Transition Path for Controlling Greenhouse Gases. *Sci.* **258**(20 Nov.), 1315–1319.

This gives a brief description of the DICE model, which stands for Dynamic Integrated Climate-Economy. It uses NLP to determine a dynamic, economic equilibrium that maximizes a discounted utility function of per-capita consumption and population.

NORDHAUS, W. D. 1993. *Managing the Global Commons: The Economics of Climate Change.* MIT Press, Cambridge, Mass.

This is a detailed description of the DICE model (1992) plus analysis results. The economic relations are the standard ones that follow a Ramsey growth model, using a Cobb–Douglas production output with constant returns to scale. A key submodel in DICE is composed of the climate-emissions-damage equations. The first equation relates greenhouse gas (GHG) emission (E) to level of control (μ), and production output (Q) over time (t): $E(t) = (1 - \mu(t))\sigma(t)Q(t)$, where σ is the trend parameter such that $\sigma(t)Q(t)$ is the greenhouse gas emission without controls. The time-varying control variable, μ, is the fractional reduction in GHG emissions, determined by optimization. The emissions determine the CO_2 concentration (M) for assumed constant retention (β) and transfer (δ) rates:

$M(t) - 590 = \beta E(t-1) + (1-\delta)(M(t-1) - 590)$. The accumulations of GHGs determine the increase of surface warming (F), called radiative forcing: $F(t) = 4.1 \log(M(t)/590)/\log(2) + O(t)$, where $O(t)$ is an exogenous term that represents the relatively negligible effect of other gases. Then the climate change is given by two recursive equations: $T(t) = aT(t-1) + bF(t) + cT^*(t-1)$ and $T^*(t) = a^*T^*(t-1) + b^*T(t-1)$, where T is the increase in global average temperature of the atmosphere and upper level of the oceans, and T^* is the increase of temperature in the deep oceans. The parameters (a, b, c, a^*, b^*) are exogenous, determined by thermal capacities and transfer rates. The relationship between global temperature increase and income loss (D) is the equation: $D(t) = 0.00144T(t)^2Q(t)$. Finally, the cost (TC) of reducing emissions of GHGs is: $TC(t) = 0.0686\mu(t)^{2.887}Q(t)$. (Particular parameter estimates are discussed, such as the proportionality constant and exponent.) Subject to these equations and economic relations, the nonlinear program maximizes the present value of a utility function that depends on per capita consumption and labor.

OKADA, N., AND Y. MIKAMI. 1992. A Game-Theoretic Approach to Acid Rain Abatement: Conflict Analysis of Environmental Load Allocation. *Water Resour. Bull.* **28**(1), 155–162.

The model allows coalitions among receptors. For any particular coalition, the rest of the game's solution is obtained by LP. The LP is to maximize the lowest bound on pollution quotas for average reductions of pollutants per source forming the coalition. Another LP, equivalent to Fortin and McBean (1983), is solved to obtain a fully cooperative allocation.

OLSON, K. D., E. O. HEADY, C. C. CHEN AND A. D. MEISTER. 1992. An Interregional Competition Quadratic Programming Analysis of Two Environmental Alternatives for U.S. Agriculture. Chapter 9 in Heady and Vocke, 207–233.

The quadratic programming model is a departure from the usual LP approach, and it is based on the early Ph.D. dissertations of Plessner (1965), Hall (1969), Stoecker (1974), and Chen (1975). (None of these were published and are not cited here.) The objective is to maximize profit, and the quadratic terms arise due to linear projections of demands multiplied by the dependent vector of prices. Among its conclusions is that restrictions on nitrogen or insecticide use causes only slight increases in farm commodity prices, although regional production patterns change.

ORTH, H., AND W. AHRENS. 1976. Optimization Methods for Planning Waste Water Management Systems. In Brebbia, 99–113.

This begins with an LP model of the entitled problem, then adds fixed charges for capacity expansion using MIP. DP is briefly discussed as a solution technique and applied to Abidjan at the Ivory Coast.

OSTFELD, A., AND U. SHAMIR. 1993. Optimal Operation of Multiquality Networks I: Steady-State Conditions. *ASCE J. Water Resour. Plan. and Mgmt.* **119**(6), 645–662.

This is essentially a review of earlier models, leading into their companion paper in the same issue.

OSTFELD, A., AND U. SHAMIR. 1993. Optimal Operation of Multiquality Networks II: Unsteady-State Conditions. *ASCE J. Water Resour. Plan. and Mgmt.* **119**(6), 663–684.

This extends the usual NLP model by removing the steady-state assumptions. The unsteady state can result from time-varying water quality at supply nodes and/or from changing flows in the system. Head quality constraints are put into the objective with a large penalty term to obtain feasible solutions to the rest of the model.

OTT, W. R. 1977. Development of Criteria for Siting Air Monitoring Stations. *J. Air Pollut. Control Assoc.* **27**(6), 543–547.

This briefly describes criteria, which are used in some math programming models. For example, the federal air quality standards are interpreted in the context of measuring the criterion: human population exposure.

PARVIN, M., AND G. W. GRAMMAS. 1976. Optimization Models for Environmental Pollution Control: A Synthesis. *J. Environ. Econ. and Mgmt.* **3**(2), 113–128.

This uses an input-output system of equations in a quadratic program that seeks to minimize total damage cost. Decision variables are production levels, and this model is compared with a related LP.

PEARCE, D. W. 1976. *Environmental Economics*. Longman, London, U.K.

This book is similar to the one by Maler (1974), but with more analysis of Pigovian taxes to abate pollution.

PEARCE, D., E. BARBIER AND A. MARKANDYA. 1990. *Sustainable Development: Economics and Environment in the Third World*. Edward Elgar Publishing Limited, Hants, U.K.

This defines "sustainable development" as not decreasing over time elements like per-capita real income, health, and access to resources. Environmental quality is a part of this, and an appendix to Chapter 3 (Economic appraisal and the natural environment) presents an NLP model that incorporates an environmental damage function into its total net revenue objective.

PECK, S. C., AND T. J. TEISBERG. 1992. CETA: A Model for Carbon Emissions Trajectory Assessment. *Energy J.* **13**(1), 55–77.

This gives an overview of CETA, which extends the Global 2100 model (Manne and Richels 1992) by adding a dependence of CO_2 concentration on CO_2 emissions, and of global mean temperature on CO_2 concentration. Furthermore, a damage function is included in the costs that is dependent on global temperature. The optimization determines a trajectory of controls of greenhouse gas emissions and finds the resulting optimal trajectory to be very sensitive, in the longterm, to the degree of nonlinearity of the damage function. For example, a cubic function yields far greater control of emissions than a linear function. Other conclusions are given concerning optimal control policies under different scenarios.

PECK, S. C., AND T. J. TEISBERG. 1993. Optimal Carbon Emissions Trajectories When Damages Depend on the Rate or Level of Global Warming. Technical Report, Electric Power Research Institute, Palo Alto, Calif. (*Climatic Change*, 1994, to appear).

This applies the authors' CETA model (1992), except that the damage function depends on the rate of change in global temperature, rather than on its level. As in their earlier study, the authors find that the degree of nonlinearity of the

damage function is an important determinant of the extent of optimal emissions control in the longterm. Experiments are conducted to investigate how stringent the damage function would need to be to justify a policy of stabilizing emissions.

PINGRY, D. E., AND A. B. WHINSTON. 1973. A Regional Planning Model for Water Quality Control. Chapter 4 in Deininger, 61–90.

After reviewing the basic approach, an NLP model is presented. Quality is constrained by bounds on variables that represent sectional levels of BOD, DO concentrations, and temperatures. The objective is to minimize abatement cost subject to water quality constraints and the usual BOD and DO balance equations. The nonlinearities are ratios of flow augmentations from previous sections to that of a current section.

PINTÉR, J. 1987. A Conceptual Optimization Framework for Regional Acidification Control. *Syst. Anal. Model. and Sim.* **4**(3), 213-226.

This incorporates air quality constraints into an energy production and consumption LP model. The objective is a nonlinear, separable cost function, which is linearized.

PINTÉR, J. 1991. Stochastic Modelling and Optimization for Environmental Management. *Ann. Opns. Res.* **31**, 527-544.

This gives a collection of stochastic programming models for water quality control, with extensive references to both the general methodology and specific applications. Some of the models have a high degree of linearity, but they all have at least one nonlinear constraint.

PINTÉR, J., J. W. MEEUWIG, D. J. MEEUWIG, M. FELS AND D. S. LYCON. 1993. ESIS—An Intelligent Decision Support System for Assisting Industrial Wastewater Management. *Ann. Opns. Res.* (to appear).

This describes the basis of ESIS (Environmentally Sensitive Investment System), which is designed to assist both industry and government for policy analysis. It incorporates artificial intelligence, data base technology and visualization tools with economic models and operations research techniques. The core is a nonlinear program, which is dynamic and can have integer restrictions (as for capacity expansion). State equations describe waste stream characteristics, and the governing functions need not satisfy convexity properties. The optimization problem, therefore, is potentially complex (depending upon the user's specifications). Branch and bound is used to obtain a global optimum with a Lipschitzian search method. This paper includes a numerical example to illustrate how ESIS works.

PINTÉR, J., AND L. SOMLYODY. 1986. Optimization of Regional Water-Quality-Monitoring Strategies. *Integrated Design of Hydrological Networks* (Proceedings of the Budapest Symposium), IAHS Publ. No. 158, 259-268.

This is an MIP model of how many samples to take from each of several monitoring stations and find routes to collect them. The objective is to minimize total cost, subject to a specified level of statistical accuracy. The accuracy constraints are nonlinear (possibly nonconvex).

PLOURDE, C. G. 1972. A Model of Waste Accumulation and Disposal. *Canadian J. Econ.* **5**(1), 119-125.

This is a welfare economics model that uses NLP to determine optimal waste control, such as garbage. The Adynamics assume waste accumulates, except for that which is disposed by either biodecomposition or recycling. An arbitrary concave utility function is used to determine properties of a steady-state solution.

QUERNER, I. 1993. *An Economic Analysis of Severe Industrial Hazards*. Phusica-Verlag, Heidelberg, Germany.

This is mostly a risk analysis approach to the entitled problem. One chapter (IV) uses NLP analysis for cost minimization. The Lagrange conditions are applied in a standard way to infer properties of an optimal policy (no algorithms are presented).

RAUSSER, G. C., R. E. JUST AND D. ZILBERMAN. 1980. Prospects and Limitations of Operations Research Applications in Agriculture and Agricultural Policy. In Yaron and Tapiero, 17-40.

This is an introduction into farmers' decisions and their connection with agricultural policy making. After a careful description of objectives and constraints, NLP is used to determine optimal production plans and land transactions for a given technology. Then a competitive equilibrium (NLP) model considers the dynamics of the farm industry, notably technological change and land markets. Raising the issue of uncertainty, a framework is suggested that uses both optimization and simulation. Although not dealing specifically with environmental control, the modeling framework offers a foundation for integrating quality control into an economic equilibrium.

REMSON, I., AND S. M. GORELICK. 1980. Management Models Incorporating Groundwater Variables. In Yaron and Tapiero, 333-356.

The authors show how to incorporate previous works on groundwater quality control (particularly their own in Aguado and Remson (1974) and Gorelick et al. (1979) into agricultural models that include irrigation plans. Much of the emphasis is on LP because most farm management optimization models use LP (particularly those that seek to optimize their land use).

REMSON, K. A., E. AGUADO AND I. REMSON. 1974. Tests of a Groundwater Optimization Technique. *Ground Water* **12**(5), 273-276.

This tests an LP model to obtain a min-cost pumping strategy. The concern was whether the solution would behave sufficiently close to the actual performance to validate the LP approach as a viable management technique. Numerical groundwater models were used, and the LP solutions passed the test.

REVELLE, C. S., D. P. LOUCKS AND W. R. LYNN. 1967. A Management Model for Water Quality Control. *J. Water Pollut. Control Fed.* **39**(7), 1164-1183.

This is an early application of LP to water quality that introduces the use of inventory equations from hydraulic principles. Other differences from the earlier models pertain to the derivation of the flow equations in such a way that the model's scope is enhanced.

REVELLE, C., J. COHON AND D. SHOBRYS. 1991. Simultaneous Siting and Routing in Disposal of Hazardous Wastes. *Trans. Sci.* **25**(2), 138-145.

The problem is to locate storage facilities for spent fuel rods from nuclear reactors. The model is a standard representation of routing and siting, using binary variables, but it has two objectives: minimize transportation (ton-miles) and minimize perceived risk (people-tons). A frontier function is generated by parametric programming.

REVELLE, C. S., D. P. LOUCKS AND W. R. LYNN. 1968. Linear Programming Applied to Water Quality Management. *Water Resour. Res.* **4**(1), 1–9.

This is an early application of LP to water quality, similar to what the authors published in 1967. The difference is the simplification of the flow equations, which greatly reduces the dimensionality of the LP.

RINALDI, S., AND R. SONCINI-SESSA. 1978. Optimal Allocation of Artificial Instream Aeration. *ASCE J. Environ. Engin.* **104**(1), 147–160.

This is a DP model to locate aerators that reduce DO concentrations in a stream. The problem is simplified by proving (under assumption of constant flow rate) that it is optimal to locate the aerators where the DO level is equal to the standard.

RINALDI, S., R. SONCINI-SESSA, H. STEHFEST AND H. TAMURA. 1979. *Modeling and Control of River Quality*. McGraw-Hill, New York.

This is a text, which begins with flow models apart from optimization. After a chapter that introduces mathematical programming generically, some of the standard control problems are described. This begins with a nonlinear program to design a wastewater treatment facility. Steady-state control models are presented first with LP, then with NLP, and finally with DP. Proceeding to unsteady-state control, which includes feedback mechanisms, Lagrangian duality is introduced and applied to a quadratic programming formulation of an emergency control problem. Other problems, treated later in the text, introduce other mathematical programming techniques, such as dealing with multiple objectives.

ROSSMAN, L. A. 1980. Synthesis of Waste Treatment Systems by Implicit Enumeration. *J. Water Pollut. Control Fed.* **52**(1), 148–160.

This begins with a table that lists six features of the design problem and a representative collection of 10 optimization models that had been published. None of the models contains all six features, and this paper presents such a model as a mixed integer NLP.

ROWE, M. D., AND D. HILL (EDS.). 1989. Estimating the National Costs of Controlling Emissions From the Energy System. Technical Report BNL-52253, Brookhaven National Laboratory, Upton, N. Y.

This is a comprehensive report of a study that had 28 contributors (listed in the report). It begins with an overview of the entitled problem and the MARKAL model. This is an LP that represents the energy processes and polluting emissions, notably CO_2, SO_2 and nitrogen oxides (NO_x). The report has a complete chapter on the MARKAL model, followed by 10 chapters on its applications in different countries. The scenario descriptions suggest how MARKAL is used for policy analysis (also see Abilock and Fishbone 1979).

RUSSELL, C. S. 1971. Models for Investigation of Industrial Response to Residuals Management Actions. In Bohm and Kneese, 141–163.

This contains an interindustry LP for the generic residuals management problem (see text). To overcome some of the difficulties described, the LP is linked (conceptually) with environmental impact models, which contain receptor damage functions. The LP determines discharges, which are inputs to the environmental model. This determines steady-state residuals, which are inputs to the damage functions.

RUSSELL, C. S. 1973. *Residuals Management in Industry: A Case Study of Petroleum Refining*. Johns Hopkins University Press, Baltimore, Maryland.

This book describes residuals management (see text), as it developed from considerations of pollution control. It is an outgrowth of the author's works plus others at Resources for the Future. Chapter II describes the LP model.

RUSSELL, C. S. 1973. Application of Microeconomic Models to Regional Environmental Quality Management. *Am. Econ. Rev.* **63**(2), 236–243.

The author gives a brief tutorial and describes work that was being done at Resources for the Future. The generic LP model for water quality control is presented as the "didactic model" in the context of residuals management, and the author suggests this model structure also applies to air quality control. (For that reason, we classify this paper as integrated.)

RUSSELL, C. S., AND W. O. SPOFFORD, JR. 1972. A Quantitative Framework for Residuals Management Decisions. Chapter 4 in Kneese and Bower, 115–179.

This is essentially what is in Russell (1971) with more details about residuals management.

RUSSELL, C. S., AND W. J. VAUGHAN. 1974. A Linear Programming Model of Residuals Management for Integrated Iron and Steel Production. *J. Environ. Econ. and Mgmt.* **1**, 17–42.

This applies the LP described by Russell (1971, 1973) to consider how waste discharges from iron and steel production into a stream or into the air are affected by effluent taxes. Among their conclusions, they show that continuous casting results in less water pollution, and an increase in the price of scrap iron results in less scrap at steel mills, which increases water pollution.

RUSZCZYÁNSKI, A. 1993. Water Quality Management: Problem Formulations and Solution Methods. Working Paper WP-93-36, IIASA, Laxenburg, Austria.

This is a succinct review of all types of mathematical programming models for water quality control.

SAYGIDEGER, O., AND E. O. HEADY. 1992. A Study of the Tradeoffs Between Soil Erosion Control and the Cost of Producing the Nation's Agricultural Output. Chapter 5 in Heady and Vocke, 126–136.

This uses the generic LP model (see text), but with goals represented by two objectives: minimize soil erosion and cost. This is transformed to a weighted sum in the usual manner, and a parametric study of the weights gives a tradeoff function for different policies.

SCHLOTTMANN, A. M. 1977. *Environmental Regulation and the Allocation of Coal: A Regional Analysis*. Praeger, New York.

This contains LP models for coal allocation, subject to deterministic supply and demand constraints. In one model, sulfur emissions are explicitly constrained; in another it is taxed. Reclamation costs of strip mining are included in the models.

SEINFELD, J. H. 1972. Optimal Location of Pollutant Monitoring Stations in an Airshed. *Atmos. Environ.* **6**, 847–858.

This uses NLP within an iterative scheme to solve the entitled problem. A system of partial differential equations describes the concentration of each of several contaminants

over time in an airshed. The error is defined as the difference between this model's solution and observed values, where the observations are taken from specified locations. The objective is to minimize the total square error, which depends upon the locations of the monitoring stations (and what each station can monitor, for example, the equipment might be to monitor only CO, but not SO_2). The reformulation of the problem results in seeking to maximize the determinant of a covariance matrix. Overall, the NLP is not convex, and a gradient descent method is used until it converges.

SEINFELD, J. H., AND C. P. KYAN. 1971. Determination of Optimal Air Pollution Control Strategies. *Socio-Econ. Plan. Sci.* **5**, 173–190 (reprinted in Daetz and Pantell, 165–182).

This is an interesting use of problem decomposition into three subproblems. The first subproblem is an LP model similar to Kohn (1978): Given a set of mass emissions, determine a least-cost control to achieve required reductions. The second subproblem is a dynamic nonlinear program that seeks the values of the time-varying mass emissions that minimize total cost. The major complication in this subproblem is the use of an airshed simulation model to evaluate atmospheric concentrations of pollutants. The third subproblem is the airshed pollutant concentrations, which is a transport and diffusion model that includes reaction kinetics and mass emissions as functions of time and location. The decomposition of the optimization portion into the first two subproblems separates the linear portion, which has high dimension, from the nonlinear portion, which has low dimension.

SEITZ, W. D., C. R. TAYLOR, R. G. F. SPITZE, C. OSTEEN AND M. C. NELSON. 1979. Economic Aspects of Soil Erosion. *Land Econ.* **55**(1), 28–42.

This uses an LP model whose objective is to maximize the total producer and consumer surplus in the corn and soybean market. The activities include land allocations to crops having different characteristics for soil erosion. The basic model is short term, but these authors also applied it to analyze long-term effects.

SHAFER, J. M., AND M. D. VARLJEN. 1993. Coupled Simulation-Optimization Approach to Wellhead Protection Area Delineation to Minimize Contamination of Public Ground-Water Supplies. In Hon, 567–570.

The model is an NLP that was developed by others (for example, Ahlfeld and Mulvey 1987). The contribution of this paper is the use of a penalty function to solve the NLP.

SHAFIKE, N. G., L. DUCKSTEIN AND T. MADDOCK III. 1992. Multicriterion Analysis of Groundwater Contamination Management, *Water Resour. Bull.* **28**(1), 33–43.

This LP model is like those in Willis (1979), Gorelick (1982), and Ahlfeld et al. (1988), except the multiple objectives are not combined with exogenous weights into a single-objective function. Instead, the technique of compromise programming is used with the L_p metric.

SHIH, C. S. 1970. System Optimization for River Basin Water Quality Management. *J. Water Pollut. Control Fed.* **42**(10), 1762–1804.

This DP model minimizes total water supply and waste treatment costs, similar to ReVelle et al. (1967), net of benefits. Wastes are either conservative or nonconservative. This means they are not changed (except by dilution or evaporation), or they are biodegradable, respectively. Each reach defines a stage of the DP, and the usual flow balance equations on BOD and DO concentrations comprise the state transitions.

SHIH, C. S., AND J. A. DEFILIPPI. 1970. System Optimization of Waste Treatment Plant Process Design. *ASCE J. Sanit. Engin.* **96**(2), 409–421.

This uses DP for the entitled problem, where total annual cost is minimized.

SHIH, C. S., AND P. KRISHNAN. 1969. Dynamic Optimization for Industrial Waste Treatment Design. *J. Water Pollut. Control Fed.* **41**(Oct), 1787–1802.

This is a DP model for industrial wastewater treatment design. A treatment process is chosen at each stage whose output feeds into the next stage. The state variables are the BOD levels of the influent and the effluent (the final level is constrained).

SIEBERT, H. 1992. *Economics of the Environment*. Springer–Verlag, Berlin, Germany.

This is a significant revision of the author's earlier (1981) book on this subject. It gives a detailed development of environmental economics from the approach of optimal resource allocation. As with other integrated modeling approaches, the notion of pollution is abstracted, but the author gives particular attention to air pollution. Using NLP techniques, notably Lagrangian duality principles, a variety of social issues is addressed, including property rights and different policy instruments for using market economics to control pollution. Most of the analysis is deterministic, but the last chapter considers "risk and environmental allocation."

SINGPURWALLA, N. D. 1975. Models in Air Pollution. Chapter 3 in Gass and Sisson, 61–102.

This only briefly includes an LP model, which is to minimize the total cost of fuel used by sources, subject to each source's energy requirements and a total air quality limit at each of several receptors. Emission rates and meteorological parameters are known for each fuel, and those that emit less pollutants either cost more or produce less energy (or both).

SMEERS, Y. 1981. On the Economics of Time Varying River Quality Control Systems. In Dubois, 463–503.

This begins with a brief review of formulating a mathematical program from the Streeter–Phelps equations, then introduces the "load curve": the probability that the level does not exceed the acceptable load. This probability is a function of the acceptable load and could refer to any of several steady-state values, including the BOD or DO concentration. The extended framework is intended to capture uncertain and time varying properties, like flows, to find a minimum-cost mix of treatment plants. There are several new concepts, such as series versus parallel treatment, and the paper seems to aim for mathematical simplicity as a priority. For that reason, no complete mathematical program is presented, other than those simple enough to be solved analytically.

SMEERS, Y., AND D. TYTECA. 1984. A Geometric Programming Model for the Optimal Design of Wastewater Treatment Plants. *Opns. Res.* **32**(2), 314–342.

The entitled model extends the geometric programming model by Ecker and McNamara (1971) and considers the validity of the cost function in greater detail.

Sobel, M. J. 1965. Water Quality Improvement Programming Problems. *Water Resour. Res.* **1**(4), 477–487.

Following Thomann and Sobel (1964), this was an early LP formulation of water quality control. The decision variables are the levels of decrease (x_j) of discharge into a stream at location j that currently depletes oxygen downstream. These are typically treatment plants whose discharges are called "effluent." In the constraints, $Ax \geqslant b$, Ax is the DO deficit reduction at each segment of the stream, and b represents target reductions in the discharges. Bounds of the form $x \leqslant U$ represent the maximum reductions, where U is the vector of present discharge rates. The objective is a linear cost function, cx, where c_j is the cost of a unit reduction at location j. Alternative objectives are considered, such as maximizing a benefit/cost ratio. Using a standard reformulation technique, the model is a linear program. Then uncertainty about the DO concentrations is modeled with a quadratic program, which accounts for the variance of DO improvement in each segment.

Stavins, R. N. 1990. Alternative Renewable Resource Strategies: A Simulation of Optimal Use. *J. Environ. Econ. and Mgmt.* **19,** 143–159.

This uses an optimal control model to address the question, Has wetland depletion and conversion to agricultural cropland been excessive? The 'simulation' is the idea of how economic agents behave in a dynamic market that ascribes a land value for its allocation to cropland. The methodology is Lagrangian (or Hamiltonian), which puts it in the realm of NLP analysis.

Sugiyama, H. 1989. Improving Water Quality by Optimal Aeration Control via Dynamic Programming. In Esogbue, 261–275.

The state variables are the levels of BOD and DO over time. The decision variables are the aeration rates, and the objective is the sum square BOD and DO deficits plus an energy cost, which is presumed to be a quadratic function of the decision variables.

Tang, C., E. D. Brill, Jr. and J. T. Pfeffer. 1987. Optimization Techniques for Secondary Wastewater Treatment System. *ASCE J. Environ. Engin.* **113**(5), 935–951.

This paper exploits the structure for computational efficiency, using a decomposition strategy with GRG, and compares the results with Ecker's (1975) geometric programming model.

Tang, C. C., E. D. Brill, Jr. and J. T. Pfeffer. 1987. Comprehensive Model of an Activated Sludge Wastewater Treatment System. *ASCE J. Environ. Engin.* **113**(5), 952–969.

This presents the model given by the authors in the same year (*op. cit.*) with a focus on its use for analysis of design options.

Tarassov, V., H. J. Perlis and B. Davidson. 1969. Optimization of a Class of River Aeration Problems by Use of Multivariable Distributed Parameters, Control Theory. *Water Resour. Res.* **5**(3), 563–573.

This is a control theory approach to the problem originally modeled by LP (Deininger, 1965) and DP (Liebman and Lynn, 1966). The advantage is computational exploitation of the quasilinear partial differential equations that describe the mass transport. The authors state this was "the first application of multivariable optimal control theory to water pollution problems."

Taylor, C. R., and K. K. Frohberg. 1977. The Welfare Effects of Erosion Controls, Banning Pesticides and Limiting Fertilizer Application in the Corn Belt. *Am. J. Agric. Econ.* **59**(Feb), 25–36.

This is an LP model of production and marketing of several products in the Corn Belt (like corn, soybeans, and wheat). The objective is producers' and consumers' surplus minus cost, which gives a partial equilibrium solution. The analysis first solves the LP without any pollution control. Then impacts of the following controls are measured: bans on herbicides, bans on insecticides, nitrogen restriction, soil erosion limits (per acre), and soil erosion taxes.

Taylor, C. R., K. K. Frohberg and W. D. Seitz. 1978. Potential Erosion and Fertilizer Controls in the Corn Belt: An Economic Analysis. *J. Soil and Water Conserv.* **33**(4), 173–176.

This applies the LP model described in Taylor and Frohberg (1977) and Seitz et al. (1979) to the entitled problem. With price-sensitive demands, this study concludes that control costs are passed through to consumers with little impact on the farmer.

Teller, A. 1968. The Use of Linear Programming to Estimate the Cost of Some Alternative Air Pollution Abatement Policies. Number 20 in IBM, 345–353.

This appears to have been the first application of mathematical programming for air quality control. The decision variables are the tons of each of two types of fuels used at different sources. Each fuel emits pollutants at a constant, known rate. Total cost is minimized, subject to constraints that limit the total amount of each pollutant emitted and others that require energy demands at each source. The resulting model is a linear program (also see Chilton et al. 1972, Gass, 1972).

Thomann, R. V. 1972. *Systems Analysis and Water Quality Management*. McGraw-Hill, New York.

This is a careful text, based partly on the author's seminal work, that is divided into three parts: the problem setting, the physical environment, and the socio-economic environment. In Chapter 11 (the third part), the linear programs are given in detail.

Thomann, R. V., and M. J. Sobel. 1964. Estuarine Water Quality Management and Forecasting. *ASCE J. Sanit. Engin.* **90**(5), 9–36.

This uses Thomann's extension of the Streeter–Phelps equations, focusing on the application potential of the LP model. (Also see Sobel 1965.)

Thomas, V. 1982. Welfare Cost of Pollution Control. Chapter 9 in Tolley et al. 217–235.

The welfare cost is defined as the losses in producer and consumer surplus due to emission standards to control pollution. This is evaluated with an NLP model that represents cost minimization, putting emission control constraints into the objective with a Lagrange multiplier. Setting the multipliers equal to zero corresponds to the uncontrolled minimum cost, and each multiplier value corresponds to a control limit with the usual duality relation. It is the value of the multipliers that represent the marginal cost of pollution control for any particular limit. Although this is presented in the context of air quality, the model can apply to any pollution control in the form of emission or discharge as long as the production function can be determined.

THOMAS, V. 1982. Welfare Analysis of Pollution Control With Spatial Alternatives. Chapter 10 in Tolley et al., 237–255.

This continues the analysis introduced by the author in the same book, with attention to welfare benefit.

TIETENBERG, T. H. 1974. On Taxation and the Control of Externalities: Comment. *Am. Econ. Rev.* **64**(3), 462–466.

This extends a theorem due to Baumol and Oates (1975) concerning the use of taxation to reduce polluting discharges into a stream by cost minimizing companies. This paper shows that the same NLP approach applies to reducing air pollution.

TIETENBERG, T. H. 1974. Derived Decision Rules for Pollution Control in a General Equilibrium Space Economy. *J. Environ. Econ. and Mgmt.* **1,** 3–16.

This defines "zones" to which different levels of taxes can be applied to control polluting emissions, which could be into a stream or into the air. An activity analysis approach is used to develop an NLP model that represents Pareto optimal allocation of resources. Lagrangian analysis is applied to derive solution properties with concave utility functions. A main result is that the taxed polluters reach a decentralized economic equilibrium, and the cost minimizing tax is a linear combination of pollutant emission rates in the zones.

TIHANSKY, D. P. 1973. Economic Models of Industrial Pollution Control in Regional Planning. *Environ. and Plan.* **5**(3), 339–356.

This is an integrated LP model that represents damage from air and water pollution. Constraints include standard economic relations, such as an input-output model of production. Each firm has alternative waste removal methods, where this means reduction of the pollutant to the air or water. Each alternative has an associated cost and emission (or discharge) rate, which can vary over time. Environmental quality standards are represented as simple bounds on waste emission. A second model is presented that accounts for uncertainties in the emission rates by chance constraints. Without independence, this model is an NLP, which the author approximates with piecewise linear functions.

TOLLEY, G. S., P. E. GRAVES AND A. S. COHEN (EDS.). 1982. *Environmental Policy, Volume II: Air Quality*. Ballinger Publishing Company, Cambridge, Mass.

This is the second of a five-volume, comprehensive presentation of problems, methods and ideas for understanding and improving environmental policies (volumes I, IV and V do not contain mathematical programming models). The particular chapters in Volume II that contain mathematical programming models for air quality control are Miller et al. and Thomas.

TOLLEY, G. S., D. YARON AND G. C. BLOMQUIST (EDS.). 1983. *Environmental Policy, Volume III: Water Quality*. Ballinger Publishing Company, Cambridge, Mass.

The chapters that contain mathematical programming models for water quality control are Fishelson and Yaron.

TRAIN, R. E., AND J. CARROLL. 1972. Environmental Management and Mathematics. *SIAM Newsletter* **5**(4), 2–3.

This is a succinct, informal presentation as the title suggests, but almost all of the substance (and references) is on air quality. Little is said about land and water, and there is only a hint of the emergence of environmental economics.

TRIJONIS, J. C. 1974. Economic Air Pollution Control Model for Los Angeles County in 1975. *Environ. Sci. and Tech.* **8**(9), 811–826.

Kohn's (1978) LP is solved parametrically to give the min cost as a function of reaching specified emission levels (exactly), say $G(E)$, where E_i = the level of the ith pollutant. Another model is used to determine the expected number of days the quality standard is violated, say $F(E)$. The NLP is to minimize $G(E)$ subject to $F(E) \leqslant P^0 \equiv$ max allowable number of days the standard can be violated. In the application to Los Angeles, two pollutants are considered: NO_2 and reactive hydrocarbons.

TRUJILLO-VENTURA, A., AND H. ELLIS. 1991. Nonlinear Optimization of Air Pollution Monitoring Networks: Algorithmic Considerations and Computational Results. *Engin. Optim.* **19,** 287–308.

(Note that the second author has also published under the name, J. H. Ellis.) This is a companion to the authors' model (1992), which focuses on computational results. They ran several algorithms in three computing environments. Their experiments indicate that a Hooke–Jeeves algorithm was best.

TRUJILLO-VENTURA, A., AND J. H. ELLIS. 1992. Multiobjective Air Pollution Monitoring Network Design. *Atmos. Environ.* **25A**(2), 469–479.

This is a nonconvex NLP to determine the number and location of air quality monitoring stations with four objectives: min-spatial interpolation error, max probability of violation detection, max-domain coverage, and min cost. The solution procedure has four main steps in an iteration: simulate to get first and second moments of the air quality indices, interpolate to get expected values of the air quality indices, integrate the air quality indices to get spatial coverage, and assemble the objectives. The model was applied to design an air quality monitoring network in an area surrounding Tarrogona, Spain.

TUNG, Y. K. 1986. Groundwater Management by Chance-Constrained Model. *ASCE J. Water Resour. Plan. and Mgmt.* **112**(1), 1–19.

This begins with an LP formulation of the entitled problem when aquifer properties are known with certainty. Then withdrawal rates are presumed to be distributed normally, so the equivalent chance constraint has the nonlinear standard deviation term added to the (linear) mean value. Taylor's theorem is applied to replace this with a linear approximation, so LP is used iteratively.

TURNQUIST, M. A. 1987. Routes, Schedules and Risks in Transporting Hazardous Materials. In Lev et al., 289–302.

The problem is to find a route in a given network from a specified source to a specified destination. The material transported is hazardous, such as radioactive. Each arc has multiple criteria, such as cost, population exposed, and probability of an accident. There are two complications with the criteria values: they vary with the time of day, and they are uncertain. The routing problem, therefore, is a multiple-objective stochastic program, and an "undominated solution" is sought. Without the uncertainty, an optimal solution is defined as a Pareto efficient route. The model deals with uncertainty by simulation. Given a route, the simulation is

used to evaluate its efficiency. Routes are found by finding the entire set of efficient ones, using a labeling procedure to solve the deterministic, multiple-objective mathematical program upon replacing each arc value with the result of the last simulation. (This is classified as MIP.)

TYTECA, D. 1981. Nonlinear Programming Model of Wastewater Treatment Plant. *ASCE J. Environ. Engin.* **107**(4), 747–766.

This is a min-cost NLP model for the design of a treatment plant. The author states that earlier models were either very accurate on the operational characteristics, but rough on optimization, even just enumerating the possible designs, or they used sophisticated optimization techniques, but used oversimplified relationships.

TYTECA, D. 1981. Sensitivity Analysis of the Optimal Design of a Municipal Wastewater Treatment plant. In Dubois, 743–766.

This mostly reviews the geometric programming model described by Ecker (1975).

TYTECA, D., AND Y. SMEERS. 1981. Nonlinear Programming Design of Wastewater Treatment Plant. *ASCE J. Environ. Engin.* **107**(4), 767–779.

Described as a "companion" to Tyteca (1981), the authors present a geometric programming model for the entitled design problem.

TYTECA, D., Y. SMEERS AND E. J. NYNS. 1977. Mathematical Modeling and Economic Optimization of Wastewater Treatment Plants. *CRC Crit. Rev. Environ. Control* **8**(Dec), 1–89.

This is a survey that synthesizes all of the works through 1976 that pertain to optimal design and/or operation of wastewater treatment plants. It also provides a succinct description of the fundamental equations and models used for wastewater treatment, apart from optimization. The primary objective considered was cost minimization (discounting used in dynamic models). The authors point out that another objective is efficiency maximization. Efficiency of a process is typically measured by the ratio, $BOD_{out}:BOD_{in}$.

VANSTEENKISTE, G. C. (ED.). 1978. Modeling, Identification and Control in Environmental Systems. *Proceedings of the IFIP Working Conference on Modeling and Simulation of Land, Air and Water Resource Systems*, North-Holland, Amsterdam, The Netherlands.

This collection of papers on the entitled subject had only one paper that used mathematical programming for environmental control: Hazeghi et al.

WADE, J. C., AND E. O. HEADY. 1977. Controlling Nonpoint Sediment Sources eith Cropland Management: A National Economic Assessment. *Am. J. Agric. Econ.* **59**(1), 13–24.

This is an LP concerned with adjustments in crop production to achieve sediment quality goals. The LP structure follows the paradigm given in the text, and the data source is USDA. The model was applied to evaluate five sediment control policies, none of which stood out as best.

WADE, J. C., AND E. O. HEADY. 1992. An Interregional Model for Evaluating the Control of Sediment from Agriculture. Chapter 6 in Heady and Vocke, 139–161.

This applies the LP by Meister and Heady (1992) to the entitled problem, with goals set for the year 2000. A conclusion is that although the cost to agriculture (and hence to society) can be high for extreme levels of environmental control, a reasonable level can be achieved at a relatively small cost.

WAGNER, B. J., AND S. M. GORELICK. 1987. Optimal Groundwater Quality Management Under Parameter Uncertainty. *Water Resour. Res.* **23**(7), 1162–1174.

This is a chance-constrained nonlinear program to determine an optimal pumping policy for the control of groundwater contamination (also see Willis and Yeh 1987, Ahlfeld and Heidari 1994). Using parameter estimation, the chance constraint, $P\{z \geqslant Z\} \geqslant \alpha$, is reformulated to the form: $\mu + \theta\sigma \leqslant Z$, where θ depends upon α.

WATANABE, T., AND H. ELLIS. 1993. Robustness in Stochastic Programming Models. *Appl. Math. Model.* **17**, 547–554.

(Note that the second author has also published under the name J. H. Ellis.) This considers a two-stage recourse model for acid rain control, where the decision variables are SO_2 reductions, and the uncertainty is in the transfer coefficients. Simulation is used to measure the robustness of a policy, and the model is extended to include a robustness function in the objective aimed at minimizing sensitivity to selected parameters.

WATANABE, T., AND H. ELLIS. 1993. Stochastic Programming Models for Air Quality Management. *Comput. and Opns. Res.* **20**(6), 651–663.

(Note that the second author has also published under the name J. H. Ellis.) Five stochastic programming models are presented for identifying cost-effective acid rain control strategies. Four are chance constrained, stemming from the early work of Ellis et al. (1985–1986); one is a recourse model, described in the authors' 1993 companion paper. The first two models differ in row independence assumption, and the next two differ in the objective function. All five models were implemented for 32 source regions, and the results were compared. More interesting, each model's result was entered as a starting point for each of the other four. The two-stage recourse model did very well and was computationally less expensive to run.

WIESNER, M. R., C. R. O'MELIA AND J. L. COHEN. 1987. Optimal Water Treatment Plant Design. *ASCE J. Environ. Engin.* **113**(3), 567–584.

This is a pair of NLP models for the entitled problem. The first assumes contact filtration; the second, direct filtration (chemical addition is followed by flocculation prior to filtration).

WELSCH, H. 1993. An Equilibrium Framework for Global Pollution Problems. *J. Environ. Econ. and Mgmt.* **25**(1) (Part 2), S64–S79.

A world economy is postulated with production and damage functions of the level of pollutant emission that are twice continuously differentiable and strictly increasing from zero. In addition, the production function is strictly concave, and the damage function is strictly convex. As the pollution level approaches zero, the production function's rate diverges and the damage function's rate approaches zero. Each country seeks to maximize its income, defined as the difference between production and damage. A solution is a Nash equilibrium point, which can be found by nonlinear (convex) programming. The model is extended to include cooperative abatement policies. With further assumptions about the underlying functions, theorems are presented about the structure of a cooperative abatement policy that is optimal in a global sense.

WERCZBERGER, E. 1974. A Mixed-Integer Programming Model for the Integration of Air-Quality Policy into Land-Use Planning. *Papers Region. Sci. Assoc.* **33**, 141–154.

This extends a land-use LP model to include linear air quality constraints. The continuous decision variables are x_{ikra} = the level of ith activity in shelter k of area r using technology a. A binary variable is introduced: $y_{ir} = 0$ if the ith activity is located in area r (at any shelter, using any technology). The following data are presumed: b_i = the desired maximum pollutant concentration in any area using the ith activity; q_r = the background pollution in area r (external to the activity levels); r_{ia} = the average emission rate of activity j, using technology a; p_{sr} = the rate of contribution to the pollution in area r resulting from pollution in area s. Then the air quality constraint is: $\sum_{j,k,s,a} (r_{ja}p_{sr})x_{jksa} - My_{ir} \leq b_i - q_r$ (for all i, r), where M is sufficiently large to render the constraint redundant when $y_{ir} = 1$. (Note that $y_{ir} = 0$ forces the x variables to satisfy the air quality limit, $b_i - q_r$.) The author adds two additional constraints that relate the x and y variables.

WHIFFEN, G. J., AND C. A. SHOEMAKER. 1993. Nonlinear Weighted Feedback Control of Groundwater Remediation Under Uncertainty. *Water Resour. Res.* **29**(9), 3277–3289.

The model is the same as in Culver and Shoemaker (1992), except the pollution concentration is uncertain. The feedback mechanism obtained by differential dynamic programming is known to have favorable properties of optimality when the errors are negligible. This paper considers the effect of two types of errors: bias, such as misestimating the average hydraulic conductivity, and scatter, which is the error in the node values for the mesh.

WILLIS, R. 1976. Optimal Groundwater Quality Management: Well Injection of Waste Waters. *Water Resour. Res.* **12**(1), 47–53.

This is a planning and design model that considers secondary wastewater treatment in conjunction with reservoir supply, using linear mass transport equations. The cost is a nonlinear function of flow variables, and there are binary variables to represent the selection of process units. (This is classified as MIP.)

WILLIS, R. 1979. A Planning Model for the Management of Groundwater Quality. *Water Resour. Res.* **15**(6), 1305–1312.

The model's objective is to manage conjunctively the water supply and quality resource of a groundwater basin. There can be multiple objectives, which are combined into one objective equal to a weighted sum of the objectives. Each objective depends upon the injection rates and mass concentrations, constrained to satisfy standard flow equations. In addition, there are linear constraints for water target requirements and load disposal. The groundwater quality requirements are simple bounds on mass concentration levels. Assuming linear objectives and flows, the model is an LP. A hypothetical example is used to demonstrate the application, using parametric programming to derive a linear relationship between maximum injection concentrations and quality standards for the aquifer system.

WILLIS, R., AND W. W-G. YEH. 1987. *Groundwater Systems Planning and Management*. Prentice-Hall, Englewood Cliffs, N. J.

This is an introductory text to the entire subject, beginning with an overview. Chapters 2 and 3 present the groundwater flow equations and the mass transport problem, respectively. Chapter 5 gives a brief introduction to all types of mathematical programming (LP, NLP, DP), including stochastic programming. Chapter 7 presents specific mathematical programming models for groundwater quality management.

WOLOZIN, H. (ED.). 1966. *The Economics of Air Pollution*. W. W. Norton, New York.

This collection of papers is an interesting first effort to investigate the application of welfare economics to air pollution control. No mathematical model is presented, but a background is set that led to the emergence of environmental economics (see Kneese and Bower 1968). There is a staff report (pp. 192–272), by the U.S. Senate Public Works Committee, that answers such questions as these: What are pollutants? What are impacting factors? What can be done?

YAKOWITZ, S. 1982. Dynamic Programming Applications in Water Resources. *Water Resour. Res.* **18**(4), 673–696.

This is an extensive survey of the entitled subject. Although it has 145 references, only a small number of these pertain to water quality; the majority pertain to reservoir management, irrigation, and general background on hydrology and DP methodology. The water quality problems included in this survey are succinctly described and serve as an excellent introduction to the literature as of its publication date.

YARON, D. 1983. Chance-Constrained Modeling of Water Quality Control With Seasonality. Chapter 6 in Tolley et al., 97–114.

Previous LP and NLP models assumed that the water quality is a deterministic outcome of water treatment. This extends those models by considering the water quality to be a random variable with a known distribution. Then a chance constraint is defined, given the desired quality requirement and a probability bound. The constraint has the form: $P\{Q_i(u) \geq q\} \geq \alpha$, where $Q_i(u)$ is the water quality in the ith season for control level u; q is the desired quality requirement; and α is a specified level of acceptable probability of compliance.

YARON, D., AND C. S. TAPIERO (EDS.). 1980. *Operations Research in Agriculture and Water Resources*. North-Holland, Amsterdam.

This collection of papers deals broadly with the entitled applications with only some attention to environmental quality control. Those cited here, either because of their direct relevance, or as good background, are Horner and Dudek; Rausser, Just and Zilberman; and Remson and Gorelick.

YEH, W. W-G. 1992. Systems Analysis in Ground-water Planning and Management. *ASCE J. Water Resour. Plan. and Mgmt.* **118**(3), 224–237.

This review includes a brief description of LP, MIP, and NLP models of the entitled problem.