Master Syllabus for MATH 1130 (Precalculus)
Fall 2010

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Preface

- This topic syllabus is aligned with *Precalculus Essentials*, 3e, Blitzer (2010).
- This course is NOT a condensation of College Algebra [MATH 1110] and College Trigonometry [MATH 1120]. Instead, we focus on *specific* topics and skills which will help students be more successful in Calculus I & II.

(#1) Intermediate Algebra

Students should review these topics on their own. Most precalculus texts have a "Preliminary" review chapter.

(a) Evaluating and simplifying algebraic expressions; Order of operations.

(b) Set builder notation; Intersection and union of sets.

(c) Real number line; Absolute value is defined as distance; Properties of absolute value; Inequality symbols.

(d) Properties and simplification rules for real exponents; Scientific notation.

(e) Properties and simplification rules for radicals and rational exponents.

(f) Rationalizing denominators; Algebraic conjugate.

(g) Understanding the vocabulary of polynomials; Polynomial operations (FOIL, etc.).

(h) Factoring polynomials (greatest common factor, factoring by groups, repeated factorizations, etc.).

(i) Manipulating rational expressions; Simplifying complex rational expressions (try a difference quotient!).

(j) Understanding why certain algebraic expressions are more "attractive" than others, in terms of calculus.

(k) Solving general equations (linear, rational, quadratic, radical).

(l) Common formulas for area, perimeter, and volume.

(m) Interval notation; Solving basic inequalities.

(#2) Graphs of Equations

(a) 2D points & ordered pairs (absolute displacement relative to the origin; directed distance from the coordinate axes).

(b) Graphs of equations in two variables; Solution points.

(c) Identifying intercepts.

(d) Interpreting information given by graphs.

(#3) Basics of Functions & Their Graphs

(a) Graphs of relations; Domain and range of a relation (and later, functions).

(b) Algebraically determining whether an equation represents a function.

(c) Finding intercepts.

(d) General function definition;

(e) Function notation is a pattern of substitution for algebraic functions.

(f) Graphs of functions; Vertical line test

(#4) More Functions & Graphs

(a) Intervals of increasing and decreasing functions; Relative extrema points;

(b) Even and odd symmetry.

(c) Piecewise functions.

(d) Difference quotient. (Blitzer chooses a strange place to introduce this concept.)

(#5) Linear Functions & Slope

(a) Slope of a line; Slope-intercept form.

(b) Horizontal lines; Vertical lines.

(c) Point-slope form. Text gives this:

$$y - y_1 = m\,(x - x_1)\,.$$

I prefer to put y_1 on the other side, so that y is explicitly a function of x.

$$y = m\,(x - x_1) + y_1.$$

(#6) More on Slope

(a) Parallel & perpendicular lines.

(b) Slope as a *rate of change*; Average rate of change (Difference quotient should have been introduced here!).

(c) Position function; Average velocity of an object.

(#7) Tranformations of Functions

(a) Graphs of common functions (constant, identity, absolute value, quadratic, square root, cubic, cube root).

(b) Reciprocal functions (memorize these graphs and basic algebraic forms).

(c) Step & Piecewise-defined functions; Absolute value function.

(d) Vertical & horizontal shifts (memorize processes).
We note that translation also works on relation graphs (circle).

(e) Reflection through a coordinate axis (memorize processes).

(f) Vertical & horizontal stretch/shrink (memorize processes).

(#8) Composite Functions

(a) Combinations of functions (sum, difference, product, quotient, etc.).

(b) Power functions $[f(x) = ax^p,\ a \neq 0,\ p \neq 0]$.

(c) $(f \circ g)(x) = f(g(x))$.
f is the Outer function and g is the Inner function.

(d) Finding the domain of a composite function (lower priority).

(e) Decomposing functions for calculus.
Example: $f(g(x)) = \left(x^2 - 3x - 5\right)^7 \Rightarrow f(x) = x^7$, etc.

(#9) Inverse Functions

(a) How do we "undo" a function? $g(f(x)) = x$.
g kills f.

(b) Notation: $f^{-1}(x)$. This must also be a function.

(c) Finding inverse functions graphically.

(d) Horizontal Line Test; One-to-one functions.

(e) Finding inverse functions algebraically (memorize processes).

(#10) Distance & Midpoint Formulas; Circles

(a) 2D Distance formula.

(b) 2D Midpoint formula.

(c) Standard form for a circle.

(#11) Modeling with Functions

(a) Functions from verbal descriptions.

(b) Review surface area formulas.

(c) Setting up appropriate optimization problems.

(d) Apply appropriate technology to optimization problems.
[We have successfully used wolframalpha.com for many problems instead of a TI-89 graphing calculator.]

(#12) Quadratic Functions

(a) Graphs of quadratic functions; Vertex; Leading coefficient and concavity (orientation).

(b) Standard form of a quadratic function (memorize):

$$f(x) = a(x-h)^2 + k,\ a \neq 0.$$

This tells us the location of the vertex $V(h,\ k)$ and the stretch/shrink factor of the parabola. It also tells us if the parabola is concave upward or downward.

(c) Vertex formula; Axis of symmetry. $(x = -b/(2a))$.

(d) How does this relate to the Quadratic Formula?
Real solutions correspond to x-axis intercepts of quadratic functions.

(e) What happens when the discriminant $b^2 - 4ac < 0$? Review imaginary unit $i = \sqrt{-1}$.

(f) Complex conjugates; Quadratic equations with non-real roots.

(#13) Polynomial Functions (of Higher Degree) & Their Graphs

(a) Smooth, continuous graphs.

(b) This is a subset of the *power functions.*

(c) Leading Coefficient Test (memorize); End behaviors.
We can begin to use limit notation:

$$\lim_{x\to\infty} f(x) = ???$$

(d) Zeros of polynomial functions (factor if you can!); Multiplicity of zeros.

(e) Intermediate Value Theorem (obvious, but you still need to memorize the details).

(f) Strategies for graphing polynomial functions.

(#14) Polynomial Division & Synthetic Division

(a) Division when the degree of the divisor is greater than one.

(b) Synthetic division allows us to find $f(c)$ faster.
It also allows us to simplify

$$\frac{f(x)}{x-c} = q(x) + \frac{r}{x-c},$$

where $q(x)$ is the quotient and r is the remainder. Note that $(x-c)$ is a linear divisor.

(c) By the Remainder and Factor Theorems, we have

$$\begin{aligned} f(x) &= q(x)(x-c) + r \\ f(c) &= q(c)(c-c) + r = r. \end{aligned}$$

(#15) Zeros of Polynomial Functions

(a) Fundamental Theorem of Algebra (memorize all details).
(b) Rational Zero Theorem (lower priority).
(c) Descartes's Rule of Signs (lower priority); Upper & lower bounds for real zeros.
(d) How does the *multiplicity* of the zeros affect the graph of $y = f(x)$?

(#16) Rational Functions

(a) Horizontal & vertical asymptotes; End behaviors.
(b) Analyzing graphs of rational functions.
(c) Slant asymptotes (lower priority).

(#17) Nonlinear Inequalities

(a) Critical numbers; Test intervals.
(b) Solving a rational inequality.

(#18) Modeling Using Variation

(a) Direct variation with powers.
(b) Inverse variation with powers.

(#19) Exponential Functions & Their Graphs

(a) The base is b, $b > 0$ and $b \neq 1$.
(b) How does b affect the graph of $y = b^x$?
(c) Transformations.
(d) Natural base e.
(e) Applications: Compound interest; Radioactive decay; Exponential growth.

(#20) Logarithmic Functions & Their Graphs

(a) $x = b^y \Leftrightarrow y = \log_b(x)$.
(b) Transformations.
(c) $y = \ln(x) \Leftrightarrow x = e^y$.
(d) We will probably only work with bases $a = 2,\ 10,\ e$.
(e) Domains of logarithm functions.
(f) Change of base.
(g) Rewriting logarithmic expressions using the properties of logarithms.

(#21) Exponential & Logarithmic Equations

[Note that we typically leave the modeling for Calc. II.]

(a) Solving techniques.
(b) Checking for extraneous solutions.

(#22) Radian & Degree Measure (memorize everything!)

(a) Vocabulary associated with angles.
(b) Radian measure.
(c) Degree measure; Conversions.
(d) Review complementary and supplementary angles.
(e) Arc length (subtended).
(f) Angular and linear speed.
(g) Area of sector of a circle.

(#23) Right Triangle Trigonometry (memorize everything!)

(a) Yes, I prefer to do this section first...
(b) Right triangle definitions of trigonometric functions.
(c) Evaluations.
(d) Trigonometric identities.
(e) Applications.

(#24) Trigonometric Functions: The Unit Circle (memorize everything!)

(a) Wrapping function.
(b) Unit circle definition of trigonometric functions.
(c) Periodicity.
(d) Even & odd functions.

(#25) Trigonometric Functions of Any Angle

(a) Reference angles.
(b) Evaluations.
(c) Using the identities.

(#26) Graphs of Sine & Cosine Functions

(a) Amplitude; Period; Scaling.
(b) Translations; Phase shift.
(c) Mathematical modeling.

(#27) Graphs of Other Trigonometric Functions

(a) $y = a\tan(bx - c) + d$

(b) Cotangent, secant, and cosecant functions.

(c) Damping factor (I do not like this terminology. I prefer "envelope".)

(#28) Inverse Trigonometric Functions

(a) Restricting the domain.

(b) What is "arcsine"?

(c) We only need $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$.

(d) Evaluate compositions with right triangles:

$$\sin\left(\cos^{-1}(x)\right) = \sqrt{1 - x^2}.$$

(#29) (Trigonometric) Applications & Models

(a) General problem solving.

(b) Simple harmonic motion.

(#30) Using Fundamental Identities

(a) Simplifying expressions.

(b) Factoring and solving.

(c) Rewriting expressions which are "calculus-friendly".

(#31) Verifying Trigonometric Identities

(a) Conversions & simplifications.

(b) Working with each side separately.

(#32) Solving Trigonometric Equations

(a) General solving.

(b) Quadratic types.

(c) Equations involving multiple angles.

(#33) Sum & Difference Formulas

(a) Difference quotient.

(b) Reduction formulas.

(#34) Multiple Angle & Product-to-Sum Formulas

(a) $\sin(ku)$ and $\cos(ku)$

(b) $\sin^2(ku)$; Power-reducing formulas.

(c) $\sin(u/2)$

(d) Product-to-sum.

(#35) Ellipses

(a) We're not terribly concerned with the concept of *focus* or *eccentricity.*

(b) Standard forms.

(#36) Hyperbolas

(a) The associated ellipse.

(b) Asymptotes.

(#37) Polar Coordinates

(a) Locating $(r,\ \theta)$ points.

(b) Conversions: $(x,\ y) \Leftrightarrow (r,\ \theta)$.

(6.7) Graphs of Polar Equations

(a) Sketching; Comparing $r = f(\theta)$ on a rectangular graph vs. the polar graph.

(b) Some special graphs
[Circles, cardioids, and flowers.]