Lightning Strikes!

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A pair of undergraduate students in an honors seminar proposed the following discrete model for the formulation of lightning. Place randomly generated numbers (*levels*) in each cell of an $m \times n$ grid, creating a *configuration*. Choose a starting cell along the top row, examine the neighboring cells, and (i) draw an edge to any neighbor whose level is less than or equal to our current level (such a cell has become *visited*), (ii) list the visited cells in a queue, and (iii) start the process over at the beginning of the queue, proceeding until the queue is empty.

The pictures in Figure 1 were computer generated from this model, with a 50×50 grid, the cell values chosen uniformly from the set $\{0, 1, 2\}$, and the center cell in the top row as the starting point. Each picture corresponds to a different initial distribution of the integers in the cells.



Figure 1: Several simulations on a 50×50 grid; one lightning strike.

We are interested the fate of the resulting path, and would especially like to be able to compute the probability that some portion of the path reaches the bottom of the grid. We think of this case as *success*, or more colloquially, a *lightning strike*. Besides being fun to think about, it turns out that in its proper generality, the question is highly non-trivial. There are tons of related open questions, most of which are accessible to undergraduates.

Early results obtained in collaboration with Lauren Sobral, who was an undergraduate at the time.

> on Wednesday February 5th 2020 at 4:00pm room 4113 of the Student Commons Building free pizza afterwards