ALCUBIERRE WARP DRIVE:

Hyper-fast travel within general relativity

*"The most exciting phrase to hear in science, the one that **heralds new discoveries**, is not '**Eureka!**' but 'That's funny...''*

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ABSTRACT

Light speed has come to be known as the cosmic speed limit on natural phenomena in their local space-time. But this is the business of special relativity. General relativity, on the other hand, provides a broader picture within which superluminal speeds, properly defined are not only possible but seem to have been a reality since the inception of the universe. According to cosmology, parts of the universe may be receding from us at FTL (faster than light) speeds, which would put them completely beyond the cosmic horizon and, therefore, completely isolated from us. Objects in those isolated regions of the universe e.g., galaxies may be nearly at rest with respect to their local environment as well as the cosmic microwave background radiation, but their respective environments are receding from one another at speeds exceeding that of light.

Alcubierre has proposed a means by which we may be able to mimic the inherent expansion mechanism of the universe. This proposed metric describes the curvature of space-time surrounding a region of flat space housing a starship that is at rest with respect to its local environment. In effect the starship would be "propelled" in much the same way as a person standing on a conveyer belt inside an airport terminal would without having to take a single step (putting aside how they arrived on that belt). The advantages to this kind of scenario are great as we shall see later, because motion that is due to the expansion and contraction of space-time itself is inherently immune to the "penalties" imposed by special relativity on objects traveling in space. Alcubierre has shown that a starship equipped with such a drive would be in free fall even when it accelerates relative to observers outside the effects of the warp apparatus; the environment of such a vessel would be travelling along a geodesic and any measured acceleration of the vessel would be due to the environment itself accelerating. Of course, there would be significant tidal forces at the edges of the vessel's local environment. And those tidal forces would be proportional to the amount of curvature, which would in turn be proportional to how well-defined those edges would be. We keep in mind that space-time is referred to as a four-dimensional mathematical structure that, in fact, vibrates, contracts, expands and can be warped by mass and energy. Thus, the idea behind warp drive is a purely local expansion of the space-time "behind" the ship and an opposite contraction of the space-time in the direction in which the ship is meant to travel (in front of the ship), which is reminiscent of the science fiction space-

time distortion associated with “warp drive”. In fact, it was the word *warp* in the title “warp drive” from science fiction that induced this investigation of the possibility of FTL travel by warping space-time.

INTRODUCTION

The postulates of relativity theory tell us that movement through space is no longer independent of movement through time. We, therefore, conclude that space and time are no longer independent but are intertwined in such a way that movement through time, the rate at which “clocks” tick, depends on the manner of movement through space, that is it depends on velocity. Thus, space and time are no longer separate entities that could be treated independently; space in itself is no longer absolute, and neither is time. To quote Minkowski, “Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality” (Minkowski, 1908). And to quote his former student Albert Einstein, “The distinction between past, present and future is only a stubbornly persistent illusion”.

It is with this kind of understanding that we embark on this brief journey to examine a warp drive concept proposed by theoretical physicist Miguel Alcubierre (Alcubbiere,1994) while working at the Physics and Astronomy Department of Cardiff University in Wales, Great Britain.

What is most attractive about the idea of FTL (Faster than Light) speed is that it would allow a starship to travel at arbitrarily high speeds, free of the limitations imposed by special relativity on motions in local space-time, which brings us immediately to the following clarification regarding variation of mass with velocity:

$$m(u) = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}, \tag{1-1}$$

where m_0 is the *rest mass* as measured by observers at rest with respect to the mass, u is the absolute speed and c is the speed of light. Please note that $c \approx 3\times 10^8$m/s but it is often treated as $c = 1$. Clearly, as u approaches c, the observed mass m grows without bound. Thus, the mass of the starship, however small to begin with, would be an unbounded function of its speed. It is this limitation that makes it impractical to aim for speeds comparable to the speed of light in a local space-time.

Of course, light (electromagnetic radiation) has no rest mass, which is why it can, and indeed must, travel at speed c; also, c is dictated by Maxwell’s equations.

DEFINITION OF FORCE AND RELATIVISTIC KINETIC ENERGY

In this section we present reasons for avoiding challenging the speed of light in its own domain, local space-time, and why resorting to a radically different approach, manipulating space-time itself, is a much more practical alternative, despite the as-yet unsolved obstacles. Consider the following special-relativistic equation for the force

$$F = m\frac{du}{dt} + u\frac{dm}{dt}, \tag{2-1}$$

where m is relativistic mass and is given by equation (1-1)

Equation (2-1) implies that F is a function of several quantities which are:

a. the mass m of the starship,
b. acceleration $\frac{du}{dt}$,
c. speed u and
d. $\dot{m} = \frac{dm}{dt}$.

All of these are increasing in magnitude, without bound. Another issue associated with any attempt to travel at a relativistic speed is time dilation, which would "delay" all clocks in the frame of the travelling astronauts allowing them, in reality and for all intents and purposes, to travel into the future that is, slower than do people relatively at rest.[1] See equation 2-2 (Weinberg)

$$dt' = dt\sqrt{1 - v^2} \tag{2-2}$$

So what would it take to travel at the speed of light?

Tackling kinetic energy in the context of special relativity provides the following insight; that the total energy E of a particle, its momentum $p = m(v)v$ and its rest-mass energy $E_0 = m_0c^2 \leq E$ must satisfy

$$E^2 - {E_0}^2 = p^2c^2 = E_0^2\frac{v^2}{c^2-v^2}. \tag{2-3}$$

Clearly, if an object can move at speed $v = c$ with $0 < E = E_0/\sqrt{1 - v^2/c^2} < \infty$, then the following equivalent statements must hold:

$$(E_0 = 0 \text{ and } m_0 = 0) \quad \Leftrightarrow \quad (v = c).^2 \tag{2-4}$$

The above discussion demonstrates that in special relativity, c is an unattainable upper bound on the speed $v = \sqrt{1 - E_0^2/E^2}\, c < c$ of massive objects moving through space-time, and that electromagnetic radiation, being massless, must indeed move at the speed of light; and even if,

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[1] Time dilation can also be the result of a gravitational difference between two locations, a fact that is accounted for in satellite design since satellites operate in an environment where gravity is decreased, so they travel into the future faster.

somehow, we could convert a mass into light it could not possibly travel faster than *c* and a trip to Alpha Centaury would still take well over four years in one direction.

The galaxies were (and are) not traveling faster than *c* nor as fast as *c* in their local space-time, instead the space-time is expanding. In a similar fashion, it is the space-time itself that is meant to "propel" the spaceship; in other words, the space-time gets warped and the starship is riding on it.

Issues with Warp Drive Implementation

In order to be able to travel to Vega (26 light years away) in about two months, our starship would have to travel at 26×12/2 ≈ two hundred times the speed of light on average, which will be seen below to require an amount of energy on an astronomical scale, literally. On the other hand, the Alcubierre drive presents a solution that directly tackles the nature of space-time itself and is seen as a solution, in principle, to the barriers presented by special relativity. The type of energy required by the warp drive physics is negative energy or exotic matter which is still unidentified. This may be a problem for quantum mechanics, but, however we attempt to classify the nature of the problem the mathematics suggests that "exotic" matter may exist. Side effects of the Alcubierre drive could devastate an entire planet or even an entire star system. The Alcubierre drive would position the ship at the center or of a large, flattened sphere that contains exotic matter capable of warping space-time. To see how this mechanism could have devastating effects, consider that space is not truly completely empty but contains high-energy particles. Such particles would be collected and trapped by the warp bubble encompassing the ship (hopefully without interfering with the drive or the bubble) but upon arrival at its destination the ship would have somehow to disengage the warp bubble causing the high-energy particles to shoot into the void, devastating anything in their path. It is predicted that so-called Hawking's radiation would fry the starship. Hawking's radiation is in essence black-hole radiation and is a result of black-hole evaporation, in which, as the name suggests, black holes evaporate and ultimately disappear. However, black-hole radiation has bearing on this discussion because a black hole seems to be a better-understood example of the warping of space-time. As Hawking's radiation temperature is inversely proportional to the amount of energy embodied in the warp (Hawking, S. W, 1974-03-01), we can expect lethal doses of radiation to be emanating from the warp, and such doses would have short wavelength according to the following equation $T \approx \frac{10^{23} K \cdot kg \cdot c^2}{E}$ (Lopresto, 2003)

The space-time metric in General Relativity

We can start by considering *n*-dimensional Cartesian coordinates

$$x^1, x^2, \ldots, x^n \tag{3-1}$$

(where the superscripts are not exponents). Now suppose we want a measure of the incremental distance *ds* between two points in 3-dimensional space (of course, we can choose *n* to be higher than 3). Then we have

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2, \tag{3-2}$$

which is nothing more than the Pythagorean Theorem and which can be condensed into the following form,

$$ds^2 = \delta_{\mu\nu}\, dx^\mu dx^\nu, \tag{3-4}$$

where $\delta_{\mu\nu}$ is the Kronecker delta and we note that once repeated indices on 2 levels imply summation. Outside the context of General Relativity, one would normally use the summation symbol $\sum$. Since μ and ν are summed over 1 to *n*, we say that the quantity on the right is fully contracted and is therefore a scalar (having no indices left), which is to be expected since the quantity on the left must be invariant to coordinate choices.

This same procedure and notation can be generalized to Minkowski space and beyond, as we discuss next. The following defines the Minkowski metric $\eta_{\mu\nu}$:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu. \tag{3-5}$$

Here μ and ν range from 0 to 3 as in 4-dimensional space and $\eta_{\mu\nu}$ is a 4×4 matrix with diagonal entries -1, 1, 1, 1. Thus, with $x^0 = ct$, the expanded form of equation (3-5) is

$$ds^2 = -(cdt)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2. \tag{3-6}$$

Equation (3-6) suggests that space-time has a causal structure; that is when ds2 > 0 the space-time interval is space-like and communication cannot happen. When ds2 < 0 the interval is time-like (preventing communication) and when ds2 = 0 the interval is light-like (communication at speed c). *ds* is known as the proper-distance increment that is agreed upon by all observers. This interval is usually written in terms of the proper-time increment $d\tau$ and the speed of light *c*. That is,

$$ds^2 = -c^2 d\tau^2 \tag{3-7}$$

but in natural coordinates we set $c = 1$ for simplicity, which yields

$$d\tau^2 = (dt)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2. \tag{3-8}$$

The Minkowski metric is applicable only to inertial frames of reference i.e., those moving at constant velocities relative to each other in flat space-time; whereas when we consider warp drive, then we are in the realm of warped space-time where masses, in following their respective geodesics, are no longer moving at constant velocities relative to each other unless the region they move through is so small that tidal forces cannot be detected. This brings us to what are known as *locally inertial frames of reference.*

We know from General Relativity that energy and matter warp space-time, and we assume that a warped space-time manifold is smooth (differentiable) unless we are working with black holes. However, regardless of how warped a smooth manifold is, if we zoom in enough we can still find a region that can be treated as flat space and bring the Minkowski metric to bear. The greater the gravitational field, the smaller the appropriate region if we are to avoid tidal forces. What this means is that a freely falling observer in such a region will not be able to detect the existence of a gravitational field and the physics of such a region can be reduced to that of inertial reference frames. This insightful discovery is what is known as *the principle of equivalence*.

Since the Minkowski metric offers such limited usefulness, we turn our attention to the general metric of General Relativity. This metric is a rank-2 tensor i.e., a bilinear mapping from vector pairs to scalars. It describes different features of a smooth manifold related to how we measure spatiotemporal displacement under changes in coordinates. In fact, the metric tensor is a demonstration of how the spatial and temporal components of space-time are interdependent. That is, its derivatives measure and describe geodesics; the metric is the essential subject matter of relativity. It is, in fact, a generalization of the Newtonian potential. We know that when dealing with weak gravitational fields and the matter causing the space-time curvature not moving at relativistic speeds, then the metric tensor can be safely replaced by the Newtonian potential allowing us to work with Newton's gravity theory. Otherwise, the metric embodies information about the curvature of the space-time manifold and about how coordinate basis vectors change. The Einstein field equations (EFE) enforce how the metric changes in response to changes in energy-momentum distribution. In fact, the metric tensor is a demonstration of how the spatial and temporal components of space-time are interdependent.

The proper time τ in a local inertial coordinate system can be derived from (Weinberg)

$$d\tau^2 = -\eta_{\alpha\beta} d\xi^\alpha d\xi^\beta, \tag{3-9}$$

where ξ^α (α = 0, 1, 2, 3) are the coordinates of the inertial i.e., freely falling coordinate system. $\eta_{\alpha\beta}$ is the Minkowski metric, but not a tensor under general transformations. If we use a different coordinate system $x^\mu = x^\mu(\xi)$ that might be curvilinear or anything else that we please then the coordinates ξ^α become functions of the 4 x^μ, leading by the differential chain rule to

$$d\tau^2 = -\eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} dx^\mu \frac{\partial \xi^\beta}{\partial x^\nu} dx^\nu, \tag{3-10}$$

or

$$d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu, \tag{3-11}$$

where $g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu}$ (3-12) is the general metric tensor. As a tensor, its components and the basis vectors change in such a way that it remains invariant for all observers in all reference frames. This tensor is a construction consisting of time and the usual three-dimensional space we are familiar with, and can be explicitly calculated according to the formula

$$g_{\mu\nu} = -\frac{\partial\xi^0}{\partial x^\mu}\frac{\partial\xi^0}{\partial x^\nu} + \frac{\partial\xi^1}{\partial x^\mu}\frac{\partial\xi^1}{\partial x^\nu} + \frac{\partial\xi^2}{\partial x^\mu}\frac{\partial\xi^2}{\partial x^\nu} + \frac{\partial\xi^3}{\partial x^\mu}\frac{\partial\xi^3}{\partial x^\nu}. \tag{3-14}$$

THE ACLUBIERRE METRIC

We start with the EFE

$$R_{\mu\nu} - Rg_{\mu\nu}/2 + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{4-1}$$

Each term in (4-1) is a rank-2, 4-dimensional tensor, and we will proceed to define each term according to the order in which it appears in (4-1).The first two terms are collectively known as the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/2, \tag{4-2}$$

where $R_{\mu\nu}$ is the Ricci tensor, which tracks spatiotemporal 4-volume changes in the neighborhood of geodesic lines, which becomes handy as the geodesic lines diverge or converge. This is why the Ricci tensor represents gravity in the EFE; it tells us how volume changes due to the curvature of space-time we are in, but it does not tell us how the shape of a region changes (Eigenchris.”Ricci Tensor Geometric Meaning.*Youtube*.YouTube. October 14 2019.Web). The Ricci tensor is obtained from the Riemann tensor by contracting the latter (Parker & Christensen 1994)

$$R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu}. \tag{4-3a}$$

and is also given more explicitly by (4-3b) in terms of the affine connection as (Weinberg)

$$R_{\mu\nu} = \partial_\nu \Gamma^{\lambda}{}_{\lambda\mu} - \partial_\lambda \Gamma^{\lambda}{}_{\mu\nu} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\lambda} - \Gamma^{\lambda}{}_{\mu\nu}\Gamma^{\sigma}{}_{\lambda\sigma} \tag{4-3b}$$

where the affine connection is discussed later on and is given by (4-7). This brings us to the Riemann tensor, a rank-4 tensor that can be expressed entirely in terms of (derivatives of) the Affine connection and in which all of the curvature of General Relativity is encoded. The Christoffel symbols are a means by which changes in the metric tensor are registered.

R is the Ricci scalar that compares the surface area of a disc in curved space to the surface of a disc with the same perimeter in flat space (Treude, 2011. 130). We can imagine a disc in a plane with a given perimeter and a circle with the same perimeter on a sphere. Clearly the surface area of the circle on a sphere will be greater than the area of the disc in a plane because the curvature allows more area within the same perimeter (Eigenchris,"Geometric Meaning Ricci Tensor/Scalar.*YouTube*,YouTube.October 25 2019.Web), it can be obtained by contracting the Ricci tensor $R_{\mu\nu}$ (Weinberg) using the inverse of the metric tensor (the contravariant metric tensor) $g^{\mu\nu}$as in (4-4)

$$g^{\mu\nu}R_{\mu\nu} = R. \tag{4-4}$$

Λ is the cosmological constant which Einstein introduced because the prevalent perception at the time was that the universe was not changing despite the fact that Newtonian mechanics suggested that the galaxies and other forms of matter should have been pulling on each other, which should have caused the universe to contract. Thus, the cosmological constant is a term that acknowledges and paves the way for the mechanism that balances out the attractive effects of the galaxies and allows the universe to remain fixed. The derivation of the EFE suggests that the cosmological constant can also be viewed as the constant of integration (Stanford."Cosmology Lecture by Leonard Susskind".*Youtube*.Youtube. Jan 28 2013.Web) component of the stress-energy tensor that is relevant to the Alcubierre drive. The remaining diagonal components, that is T_{11}, T_{22}, T_{33} are the normal stress components while the remaining six entries are sheer stress (Misner, Thorne, & Wheeler, 1973)

The following is a road map of the dependencies of the objects that constitute the EFE.

$$g_{\mu\nu} \to \Gamma^{\alpha}_{\beta\mu} \to R^{\alpha}{}_{\beta\mu\delta} \to \{R_{\alpha\beta} \to R\} \to \{G_{\mu\nu} = 8\pi T_{\mu\nu}\}.$$

As we can see, the metric tensor $g_{\mu\nu}$ is the ingredient of the affine connection $\Gamma^{\alpha}_{\beta\mu}$ which is the ingredient of the Riemann tensor, while the Riemann tensor yields the Ricci tensor which in turn yields the Ricci scalar. Both the Ricci tensor and the Ricci scalar are explicit terms in the EFE which when solved (if possible) in response to a given stress-energy tensor $T_{\mu\nu}$, yields $g_{\mu\nu}$. That is, solving the EFE would yield $g_{\mu\nu}$ that describes the geometry of the space-time dictated by $T_{\mu\nu}$.

Suppose now that we want to consider a particle in free fall (the only forces acting upon it are gravitational forces) then according to the principle of equivalence discussed earlier, the particle is in a freely falling coordinate system ξ^{α} following a straight-line path through space-time that is given by (Weinberg)

$$\frac{d^2\xi^{a}}{d\tau^2} = 0, \tag{4-4}$$

where τ is the proper time as given by (3-9). And suppose we want to relate this coordinate system to another coordinate system x^{μ}, then (4-4) becomes (Weinberg)

$$\frac{d}{d\tau}\left(\frac{\partial\xi^{\alpha}}{\partial x^{\mu}}\frac{dx^{\mu}}{d\tau}\right) = 0. \tag{4-5}$$

By multiplying (4-5) by $\frac{\partial x^{\lambda}}{\partial \xi^{\alpha}}$ and using the differentiation product rule

$$\frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial x^{\lambda}}{\partial \xi^{\alpha}} = \delta_{\mu}^{\lambda},$$

it can be shown that (4-5) yields (Weinberg)

$$\frac{d^2 x^{\lambda}}{d\tau^2} + \Gamma_{\mu\nu}^{\lambda} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} = 0 \qquad \text{(4-6a)}$$

that is the equation of motion; this in fact is the geodesic equation where $\Gamma_{\mu\nu}^{\lambda}$ is the affine connection defined by (Weinberg)

$$\Gamma_{\mu\nu}^{\lambda} = \frac{\partial x^{\lambda}}{\partial \zeta^{\alpha}} \frac{\partial^2 \zeta^{\alpha}}{\partial x^{\mu} \partial x^{\nu}}. \qquad \text{(4-7a)}$$

Equation (4-6a) represents four coupled equations that must be solved for simultaneously, and the solution to (4-6a) is $x^{\lambda}(\tau) = x^0(\tau), x^1(\tau), x^2(\tau), x^3(\tau)$ which is the path of the particle in an arbitrary coordinate system.
And while at it let us write the affine connection explicitly

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\alpha} (\partial_{\mu} g_{\alpha\nu} + \partial_{\nu} g_{\alpha\mu} - \partial_{\alpha} g_{\mu\nu}), \qquad \text{(4-7b)}$$

And please note that (4-7b) is derived using the covariant derivative and is premised on the metric compatibility given by $D_{\alpha} g_{\mu\nu} = 0$ where D_{α} is the covariant derivative. This is where the Riemann tensor comes to the rescue and is given next in two versions (Weinberg)

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu} \Gamma_{\nu\sigma}^{\rho} - \partial_{\nu} \Gamma_{\mu\sigma}^{\rho} + \Gamma_{\mu\lambda}^{\rho} \Gamma_{\nu\sigma}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{\mu\sigma}^{\lambda} \qquad \text{(4-8a)}$$

$$R_{\lambda\mu\nu\kappa} := g_{\lambda\sigma} R^{\sigma}{}_{\mu\nu\kappa}$$

The Riemann tensor is proportional to the second covariant derivative (and we emphasize the word second) of the separation vector between geodesics, which means that the Riemann tensor keeps track of the acceleration in the deviation of geodesics thus revealing the presence of curvature and its extent.

Alcubierre introduces a simple metric motivated by the expansion of the universe; this metric is meant to show how to use an expansion of space-time to move a spaceship away from a location (defined by a planet or a star for instance) and how to use a contraction of space-time to approach a destination at an arbitrary speed where the enormous FTL travel is due to the expansion of space-time. This is what is known as the Alcubierre Metric which, in the context of General Relativity, allows a warp bubble to form in a previously flat region of space-time and travel at superluminal speeds, effectively. We note here that the Alcubierre metric is a solution to EFE because (a) it describes a pseudo-Riemannian manifold with a signature (–, +,+,+) that renders it a Lorentz manifold, (b) it is consistent with the restrictions of GR and with the relativistic effects and (c) the outcome of this metric is a stress-energy tensor that meets the boundary conditions ---deferring to below a consideration if it is physically permissible. In other words, one would guess a trial metric with enough unknown parameters and then calculate the Christoffel symbols (the connections) from this trial metric. The Christoffel symbols are used to calculate the components of the Riemann curvature tensor which can be reduced to the Ricci tensor which can still be reduced the curvature scalar. Now we have all the ingredients for the Einstein tensor, which allows us to write the EFE for that particular trial metric and finding the unknown parameters by matching boundary conditions. The Acubierre metric is also asymptotically Minkowski as required.

Alcubierre used Einstein's field equations to calculate the stress energy tensor when in a traditional approach to system of partial differential equations, one would start with a known stress energy tensor source $T_{\mu\nu}$, and try to solve the equations; but this direction would be exceedingly difficult for a general $T_{\mu\nu}$. However, following the designer approach, we can start with the kind of geometry that we want for our purposes; in other words we start with a metric that describes the geometry we want, which would allow us to calculate the Ricci tensor $R_{\mu\nu}$ and Ricci scalar R leading us to the right-hand side of the equations, the stress energy tensor. The designer approach is far easier than the alternative, but it usually produces nonphysical or nonsensical results.

The Alcubierre metric would create a bubble, the interior of which is the inertial reference frame for any object inhabiting it. A great advantage to this design is that since the starship is not moving within this bubble, nor is it required to, but is catching a ride as it is being carried along as the local space-time itself moves, conventional relativistic effects such as time dilation would not apply. It is in this sense that the rules of space-time and the laws of relativity would not be violated. As it turns out, even though Alcubierre implemented the designer method by choosing the required geometry, he modeled the idea on the early expansion of the universe and the emerging stress-energy tensor is also consistent with what cosmology tells us about the mechanism responsible for the accelerating expansion of the universe; that is negative energy or the dark energy that accounts for 69% (Ade, Aghanim, Alves, et al of the total, 2013, Carroll, 2007) energy. In this sense, the Alcubiere metric is indeed a solution to the EFE for some $T_{\mu\nu}$ that must be examined next.

This Alcubierre metric describes a foliation of space-like hypersurfaces and is given generically by:

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j, \qquad (4\text{-}12)$$

where: α is the lapse function that gives the interval of proper time between nearby hypersurfaces; β^i is the shift 3-vector that lies in the direction of motion of the starship; and γ_{ij} is a positive-definite 3-dimensional metric tensor on each of the hyperspaces that form the foliation, thus γ_{ij} is a Cartesian metric.

Then let $\alpha = 1$, and it should be noted that $\alpha^2 - \beta_i\beta^i \geq 0$, $\beta^x = -v_s(t)f(r_s(t))$, $\beta^y = \beta^z = 0$, and $\gamma_{ij} = \delta_{ij}$, where

$$v_s = \frac{dx_s}{dt}, \tag{4-13}$$

and $x_s(t)$ is an arbitrary function of time that describes the trajectory of the starship. We note that this trajectory is a geodesic since world-lines of particles subject to gravity only are geodesics. Also,

$$r_s(t) = [(x - x_s)^2 + y^2 + z^2)]^{1/2}, \tag{4-14}$$

and

$$f(r) = \frac{\tanh(\sigma(r + R)) - \tanh(\sigma(r - R))}{2\tanh(\sigma R)}, \tag{4-15}$$

is a spherically symmetric form function with σ and R are arbitrary parameters and $r = r_s$. We note the following.

- The lower limit on R is greater than the size of the spaceship since R is the "radius" of the warp bubble.
- $f(r)$ is a top-hat function in the limit as $\sigma \to \infty$.
- Also, $f(r)$ is spherically symmetric because $r = \sqrt{x^2 + y^2 + z^2}$ and $r \to \infty$ for $\sigma > 0$ implies that $f(r) \to 0$, which implies that ds^2 is asymptotically Minkowski. So, by construction, $f(r) \to 0$ outside the warp bubble.
- σ is inversely proportional to the thickness of the wall of the warp bubble. The larger is this parameter the thinner is the wall of the bubble, and (as we will see) the higher is the energy density.
- $\lim_{\sigma \to \infty} f(r) = 1$ for $-R \leq r \leq R$, $\lim_{\sigma \to \infty} f(r) = 0$ otherwise. Thus, $f(r)$ has the value $f = 1$ inside the warp bubble and $f = 0$ outside the bubble.

Notice that

$$\gamma_{ij}dx^i dx^j = dx^2 + dy^2 + dz^2 \quad (\text{since } \gamma_{ij} = \delta_{ij}), \tag{4-16a}$$

$$2\beta_i dx^i dt = 2\beta_x dx dt \quad (\text{since } \beta^y = \beta^z = 0), \tag{4-16b}$$

and so

$$-(\alpha^2 - \beta_i\beta^i)dt^2 = -[1 - (-v_s(t)f(r_s))^2]dt^2 = -dt^2 + v_s{}^2 f(r_s)^2 dt^2 \tag{4-16c}$$

and when we substitute (4-16) into (4-12) we get

$$ds^2 = -dt^2 + v_s{}^2 f(r_s)^2 dt^2 - 2v_s f(r_s) dx dt + dx^2 + dy^2 + dz^2, \qquad (4\text{-}17)$$

which is factored into the Alcubierre metric tensor given by

$$ds^2 = -dt^2 + (dx - v_s f(r_s) dt)^2 + dy^2 + dz^2. \qquad (4\text{-}18)$$

Now, naturally, we should be interested in how much the space-time is being stretched or compressed and, more importantly, what is the value as measured by the Eulerian observers that are in free-fall. That is, observers whose four-velocity is orthogonal to the spatial hypersurfaces and who are following geodesic paths. For this purpose we resort to the extrinsic-curvature tensor as it is well known from differential geometry that the trace of the extrinsic curvature tensor provides a measure of the extent to which the space-time is warped. Alcubierre explains that the 3-geometry of the hypersurfaces is flat and that the information about the curvature of space-time is contained in the extrinsic-curvature tensor which is the rate of change of the unit normal to the hypersurfaces. This tensor is given by

$$K_{ij} = \frac{1}{2\alpha}\left(D_i\beta_j + D_j\beta_i - \frac{\partial g_{ij}}{\partial t}\right) \qquad (4\text{-}19a)$$

which is the extrinsic curvature tensor given in terms of the shift function and the 3-metric Keeping in mind that $\gamma_{ij} = \delta_{ij} = g_{ij}$ implies $\frac{\partial g_{ij}}{\partial t} = 0$, which yields

$$K_i^i = \partial_i \beta^i, \qquad (4\text{-}19b)$$

where the summation is over all spatial coordinates, but since $\beta^y = \beta^z = 0$ we are left with

$$\beta^x = -v_s(t) f(r_s(t)),$$

which implies

$$\partial_x \beta^x = \partial_x(-v_s(t) f(r_s(t))).$$

Now we carry out the derivative with respect to x and get the following result that is known as the expansion of the volume elements which is the Alcubierre warp bubble:

$$\theta = -\alpha K_i^i$$

with $\alpha = 1$, and finally we have, with $v_s = \frac{dx_s(t)}{dt}$

$$\theta = -v_s \frac{x - x_s}{r_s} \frac{df}{dr_s}. \qquad (4\text{-}20a)$$

And assuming that the starting point $x = 0$, then 4-20a becomes

$$\theta = v_s \frac{x_s}{r_s} \frac{df}{dr_s} \tag{4-20b}$$

In what follows we present a derivation of the extrinsic curvature tensor, also known as the second fundamental form, by Misner, Thorne and Wheeler (MTW) as it is outlined in their book *Gravitation* (1973).

Let $\boldsymbol{n}$ be a normal 4-vector standing at the point p on the hypersurface Σ, transport it parallel to itself to the point $p + dp$ where we have another vector that is normal to the surface Σ. We keep in mind that $d\boldsymbol{n}$ which is the change in the normal vector $\boldsymbol{n}$ is nothing more than a change in the direction of $\boldsymbol{n}$; the magnitude of $\boldsymbol{n}$ is kept a constant as we are interested only in the manner in which the direction of $\boldsymbol{n}$ changes as a measure of the curvature of the surface Σ as observed in the higher dimensional ambient space. The limiting concept of the vector-valued displacement $d\boldsymbol{n}$ can be regarded is lying in Σ and is therefore perpendicular to $\boldsymbol{n}$. We also know that dp lies in Σ; thus, depending linearly on $d\boldsymbol{p}$, $d\boldsymbol{n}$ can be represented in the form

$$d\boldsymbol{n} = -\boldsymbol{K}(d\boldsymbol{p}). \tag{eq 21.59 MTW}$$

Here, the linear operator $\boldsymbol{K}$ is the extrinsic curvature tensor and is defined to be positive if the tips of the two unit normal vectors located at the points p and $p + dp$ are closer together than their tails are; it is reminiscent of a concave surface in 3D. Next, we replace $d\boldsymbol{p}$ with a special tangent vector, the basis vector $\boldsymbol{e}_i$, $d\boldsymbol{p} = \boldsymbol{e}_i dx^i$, where $\boldsymbol{e}_i \cdot \boldsymbol{n} = 0$, which yields

$$\nabla_i \boldsymbol{n} = -\boldsymbol{K}(\boldsymbol{e}_i) = -K_i^j \boldsymbol{e}_j \tag{eq 21.60 MTW}$$

where K_i^j are the components of the linear operator $\boldsymbol{K}$. The scalar product of (21.60) with the basis vector $\boldsymbol{e}_m$ reveals the symmetry of the extrinsic curvature tensor. After we carry out the covariant derivative of $\boldsymbol{e}_i \cdot \boldsymbol{n} = 0$ that yields

$$\boldsymbol{n} \cdot \nabla_i \boldsymbol{e}_m + \boldsymbol{e}_m \cdot \nabla \boldsymbol{n} = 0 \text{ or equivalently } -\boldsymbol{e}_m \cdot \nabla \boldsymbol{n} = \boldsymbol{n} \cdot \nabla_i \boldsymbol{e}_m$$

we get from 21.60 MTW

$$K_{im} = K_i^j g_{jm} = K_i^j (\boldsymbol{e}_j \cdot \boldsymbol{e}_m) = -\boldsymbol{e}_m \cdot \nabla \boldsymbol{n} = \boldsymbol{n} \cdot \nabla_i \boldsymbol{e}_m$$

which, with the covariant derivative of the basis vector $\boldsymbol{e}_m$ given by $\nabla_i \boldsymbol{e}_m = \Gamma_{mi}^{\mu} \boldsymbol{e}_{\mu}$, becomes

$$K_{im} = \boldsymbol{n} \cdot \boldsymbol{e}_0 \Gamma_{mi}^0 = \boldsymbol{n} \cdot \boldsymbol{e}_0 \Gamma_{im}^0 = \boldsymbol{n} \cdot \boldsymbol{\nabla}_m \boldsymbol{e}_i = K_{mi}. \tag{eq 21.61 MTW}$$

Thus $K_{im} = K_{mi}$. This knowledge of K_{ij} reveals information regarding the metric that is intrinsic to the embedded space by revealing how the vectors $\boldsymbol{n}, \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ change under parallel transport. Thus, one arrives, following Israel (1966) at the equations of Gauss and Weingarten:

$$\nabla_i e_j = K_{ij} \frac{\boldsymbol{n}}{\boldsymbol{n} \cdot \boldsymbol{n}} + \Gamma_{ji}^h e_h \tag{eq 21.62 MTW}$$

$$\boldsymbol{n} \cdot \boldsymbol{n} = \epsilon(\boldsymbol{n}) = \begin{cases} 1, & \boldsymbol{n} \ is \ pacelike \\ -1, & \boldsymbol{n} \ is \ timelike \end{cases} \text{(Israel, 1966)}$$

The covariant derivative of a vector $\boldsymbol{A}$ in the direction of the i-th coordinate in Σ has for its h-th covariant component

$$A_{h^{|}i} = e_h \cdot \nabla_i \boldsymbol{A} = \frac{\partial A_h}{\partial x^i} - A^m \Gamma_{mhi}. \qquad \text{(eq 21.57 MTW)}$$

Here, "the notation of the vertical stroke notation distinguishes this covariant derivative from the 21.62 MTW tells us how each basis vector in Σ changes, and since we also know the covariant derivative of a vector $\boldsymbol{A}$ in Σ as given in 21.54 MTW, we can rewrite 21.54 MTW as 21.63 MTW by carrying out the covariant derivative of $\boldsymbol{A}$ as follows

$$\nabla_i \boldsymbol{A} = \nabla_i (A^j e_i) = e_j \nabla_i A^j + A^j \nabla_i e_j.$$

From 21.62 MTW we have

$\nabla_i \boldsymbol{A} = e_j \nabla_i A^j + A^j (k_{ij} \frac{\boldsymbol{n}}{\boldsymbol{n} \cdot \boldsymbol{n}} + \Gamma_{ji}^h e_h)$ which, after rearranging terms, becomes

$\nabla_i \boldsymbol{A} = e_j \nabla_i A^j + A^j \Gamma_{ji}^h e_h + A^j k_{ij} \frac{\boldsymbol{n}}{\boldsymbol{n} \cdot \boldsymbol{n}}$ in which the first two terms are collectively (as in the context of 21.57 MTW) $\boldsymbol{A}^j_{|i}$. Here $\boldsymbol{A}^j_{|i}$ is the covariant derivative in Σ and not in the 4-geometry.

And the last equation becomes the following 21.63 MTW:

$$\nabla_i \boldsymbol{A} = \frac{\partial A^j}{\partial x^i} \boldsymbol{e}_j + K_{ij} A^j \frac{\boldsymbol{n}}{\boldsymbol{n} \cdot \boldsymbol{n}} \qquad \text{(eq 21.63 MTW)}$$

Equations 21.62 and 21.63 reveal that the parallel transport we are talking about here is with respect to the ambient or embedding space, the 4-geometry. And when we compare the covariant derivative of $\boldsymbol{n}$ as an intrinsic operation

$$(d\boldsymbol{n})_i = \left[\frac{\partial n_i}{\partial x^k} - \Gamma_{ik}^{\sigma} n_{\sigma}\right] dx^k = N \Gamma_{ik}^0 dx^k \qquad \text{(eq 21.65 MTW)}$$

with the change in $\boldsymbol{n}$ as measured by the extrinsic curvature tensor

$$(d\boldsymbol{n})_i = -K_{ik} dx^k \qquad \text{(eq 21.66 MTW)}$$

$$K_{ik} = -N \Gamma_{ik}^0$$

where $\boldsymbol{n} = (n_0, n_1, n_2, n_3) = (-N, 0, 0, 0)$ implying $\frac{\partial n_i}{\partial x^k} = 0$ and thus we have

$K_{ik} = -N \Gamma_{ik}^0 = \frac{-N}{2} \left[g^{00} \Gamma_{0ik} + g^{0p} \Gamma_{pik}\right]$ where we carried out the raising operation

$g^{00} \Gamma_{0ik} = \Gamma_{ik}^0$ and $g^{0p} \Gamma_{pik} = \Gamma_{ik}^0$ hence the $\frac{1}{2}$ in front of the previous equation.

With the help of 21.44 MTW below

$$\begin{matrix} g^{00} & g^{0m} \\ g^{k0} & g^{km} \end{matrix} = \begin{matrix} \frac{-1}{N^2} & \frac{N^m}{N^2} \\ \frac{N^k}{N^2} & g^{km} - \frac{N^k N^m}{N^2} \end{matrix}$$

$$K_{ik} = \frac{-N}{2}\left[g^{00}\Gamma_{0ik} + g^{0p}\Gamma_{pik}\right] = \frac{1}{2N}\left[\Gamma_{0ik} + N^p\Gamma_{pik}\right]$$

$$\Gamma^0_{ik} = g^{00}\Gamma_{0ik} + g^{0j}\Gamma_{jik} \qquad\qquad \Gamma^{\alpha}_{\beta\gamma} = g^{\mu\nu}\Gamma_{\mu\beta\gamma} \text{ eq 8.24c MTW}$$

$$K_{ij} = \frac{1}{2\alpha}\left(\partial_i\beta_j + \partial_j\beta_i - 2\Gamma^k_{ij}\beta_k - \frac{\partial\delta_{ij}}{\partial t}\right)$$

Then we recognize that $D_jV_i = \frac{\partial V_i}{\partial x^j} - V^k\Gamma_{kij}$ is the covariant derivative of V_i, which justifies writing the last expression for K_{ik} as follows

$$D_jV_i = \frac{\partial V_i}{\partial x^j} - V^k\Gamma_{kij} \qquad \text{(Covariant derivative in embedded manifold)}$$

$$\gamma_{ij} = \delta_{ij}$$

$$k_{ij} = \frac{1}{2\alpha}\left[D_i\beta_j + D_j\beta_i - \frac{\partial\delta_{ij}}{\partial t}\right]$$

which implies

$$K_{ik} = \frac{1}{2N}\left[\frac{\partial N_i}{\partial x^k} + \frac{\partial N_k}{\partial x^i} - \frac{\partial g_{ik}}{\partial t}\right] \qquad \text{(eq 21.67 MTW)}$$

The above was the calculation by *Misner Thorne and Wheeler*.

We note the following revelations from equation (4-20):

- θ is directly proportional to the speed v_s, thus the faster the spaceship travels, the greater is the stretching function (something to be expected). Also, θ represents the amount of energy embedded in the warp. As an example, if we shrink the mass of the sun down 32km in radius it becomes a black hole and as a result, the space-time around its limb will be warped so much that light traveling near the sun's surface can be deflected 180°. In other words, θ measures how much energy is invested by showing us the extent to which space-time is bent.
- Also, as mentioned earlier, the top-hat function $f(r_s(t))$ is zero outside the warp bubble and is 1 inside it for large σ, which means that $\frac{df(r)}{dr_s}$ vanishes inside as well as outside the warp bubble and has enormous magnitude near the bubble boundary.
- Also, the absolute increase of σ means a faster approach of the shape function to the top-hat condition.

Figure depicts θ as a function of $\left(x - x_s, \sqrt{y^2 + z^2}\right)$ for different σ. Since θ is given in terms of the trace of the extrinsic curvature tensor, we recall that the entries of the extrinsic tensor are the projections of the second partial derivatives of the position vector **r** onto the normal line **n** to the hypersurface Σ (Guggenheimer*,* 1977, Chapter 10). The position vector has its origin at the observer's location. The trace of $\boldsymbol{K}$ is the mean average curvature as measured in the ambient embedding space The position vector r is referenced to the location of the observer that is in free fall. The trace of K is the mean average curvature as measure in the ambient embedding space. Also, θ is a measure of the energy density that constitutes the shell, so to speak, of the warp bubble.

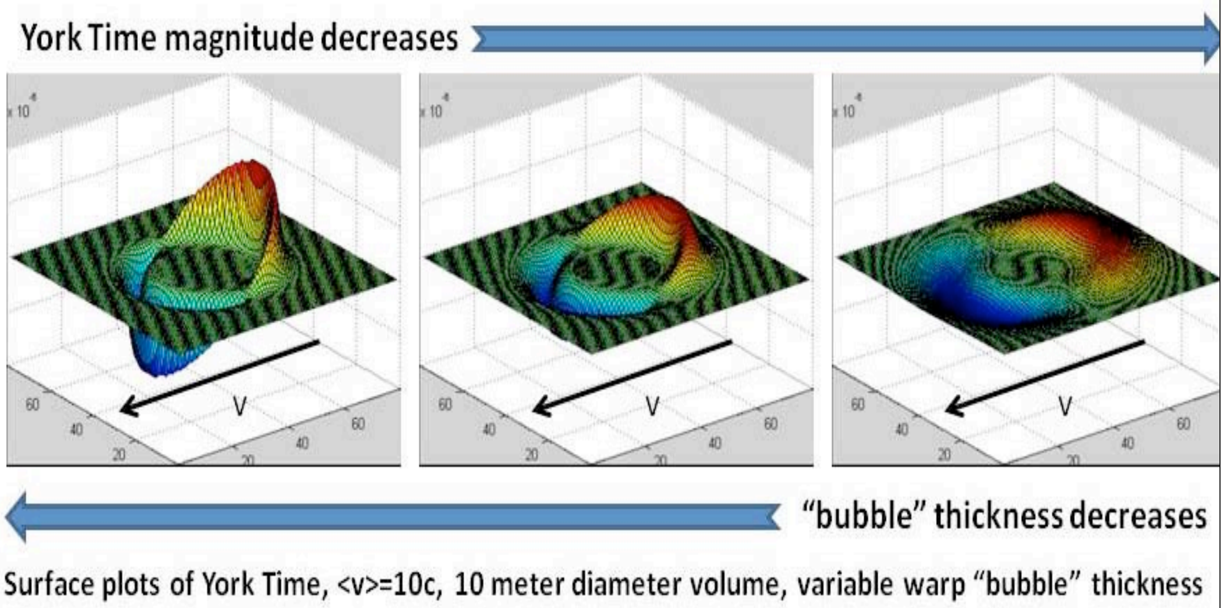

Figure 2: Warp Field Mechanics 101. Dr. White

The vanishing of θ is good news, since we don't want the crew, the ship nor anyone or anything outside the warp bubble to be warped in any way. And let us see what happens when we substitute the following quantities, $dy = dz = 0$ and $x = x_s(t)$ and $v_s = \frac{dx_s}{dt}$ in (4-12), keeping in mind that inside the warp bubble $f(r_s) = 1$.

$$-d\tau^2 = ds^2 = -dt^2 + (dx_s - \frac{dx_s}{dt}dt)^2 + 0 + 0 = -dt^2,$$

or simply

$$d\tau^2 = dt^2. \quad (4\text{-}21)$$

Equation (4-21) confirms that, as mentioned earlier, the proper time inside and outside the bubble will be the same. Thus there are no time dilation and no twin-paradox issues to be concerned with (again, neglecting the question of entering and exiting the bubble).

Figure 3 demonstrates the manner in which space-time is expanded behind the spaceship and compressed in front of it. The net effect is similar to surfing a wave because the ship is not travelling in space but rather it is riding a space-time wave and is traveling at the speed of the warp wave.

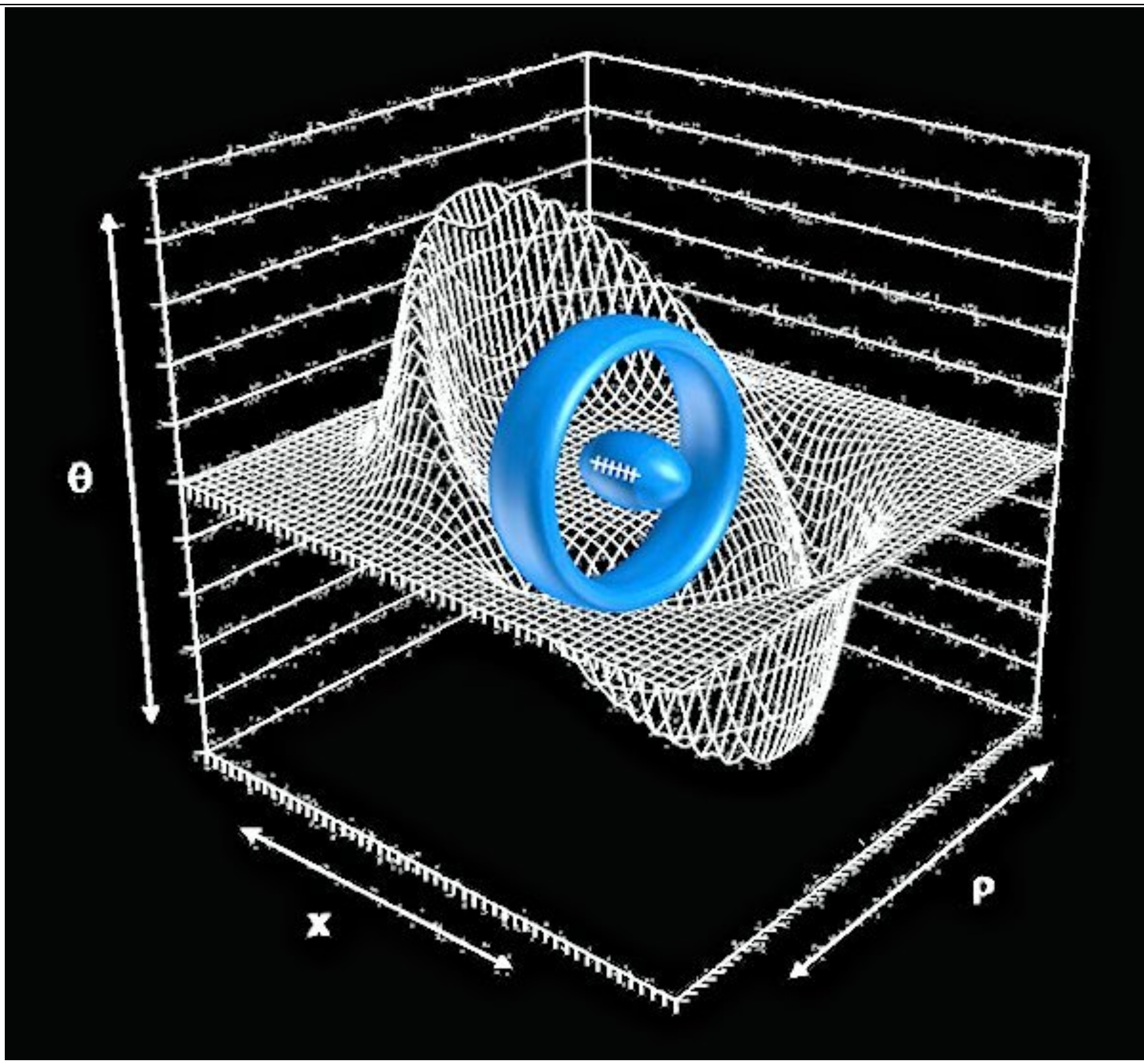

Figure 3: Image: Harold G. "Sonny" White

When Alcubierre's metric is modeled, there are a number of fascinating phenomena that are worth noting. As indicated in Figure 3, the space-time is flat everywhere except within a certain region with a defined radius, which renders the distortion highly localized. In addition, the time-dilation issue spoken of earlier is not an issue, because the time within the flat region inside the distortion is the same as that of an outside observer. However the region itself moves along a time-like curve, resembling geodesics. Also, the coordinate acceleration is a function of time but the proper acceleration along the path of the center of the distortion is zero.

Now we come to the weak energy conditions that Alcubierre declares are violated by his metric.

The **weak energy condition** (WEC) is stated as follows:

$$T_{\mu\nu}V^{\mu}V^{\nu} \geq 0 \tag{4-22}$$

in which V^{μ} is the 4-velocity of the Eulerian observers and $T_{\mu\nu}$ is the stress–energy tensor. The interpretation of the WEC is that the local energy density should be non-negative (Curiel, 2014). The calculations will be simplified using an orthonormal reference frame. We use the EFE to get the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R = 8\pi T_{\mu\nu}, \tag{4-23}$$

which gives us the following more explicit expression for the WEC,

$$T_{\mu\nu}V^{\mu}V^{\nu} = \frac{1}{8\pi}G_{tt}, \tag{4-24}$$

where V^{μ} is required to be a timelike 4-vector (Visser, Matt, Barceló, & Carlos ,2000) in this case G_{tt} is given with a generic form function f by

$$G_{tt} = -\frac{v_s^2}{4}\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right], \tag{4-25a}$$

and when we insert the form function (4.15) as the argument of (4-25a) we get

$$G_{tt} = -\frac{v_s^2}{4}\frac{y^2+z^2}{r_s^2}\left(\frac{df}{dr_s}\right)^2.$$ (Francisco, Lobo, & Matt, 2004)

(4-25b)

From (4.24) and (4.25b) we have:

$$T_{\mu\nu}V^{\mu}V^{\nu} = T^{00} = -\frac{v_s^2}{32\pi}\frac{y^2+z^2}{r_s^2}\left(\frac{df}{dr_s}\right)^2 \leq 0, \tag{4-27}$$

(4-27) suggests that the energy density of the warp field is toroidal and is symmetric about the *x*-axis; also, on the *x*-axis where $y = z = 0$, the energy density is exactly zero. It is noteworthy that the amount of energy and the kind of energy required for the establishment of a warp bubble does not depend on the mass of the starship. Also, (4-27) states that the energy is negative; the Eulerian observers measure negative energy because the warp bubble is an expansion of space-time or negative curvature and not the kind of positive curvature that is caused by a planet or star. It is to be noted that, aside from $x_s(t)$, the Alcubierre metric is time-independent because it does not describe the evolution of the warp bubble or the manner in which the exotic matter is deployed around the ship; it actually presents this arrangement of exotic matter as something that has always existed. Alcubierre mentions in a long interview (Event Horizon."Can We Travel Faster Than Light? With Dr. Miguel Alcubierre".*Youtube*.youtube June 6 2019.Web) that there is no known way to assemble or disassemble the warp bubble, which would be required for controlling when the ship travels and when it should stop.

Harold White (White, 2011) and his NASA team have proposed (White, 2011, 5) an updated concept of the Alcubbiere drive that is far more energy efficient and requires positive energy . on a far less than an astronomical scale; the amount of energy wend down from being the mass of the universe to the mass of Jupiter. Figure 4 is a plot of the energy density requirements as a function of $\left(x - x_s, \sqrt{y^2 + z^2}\right)$ for different values of the thickness σ of the wall of the warp bubble.

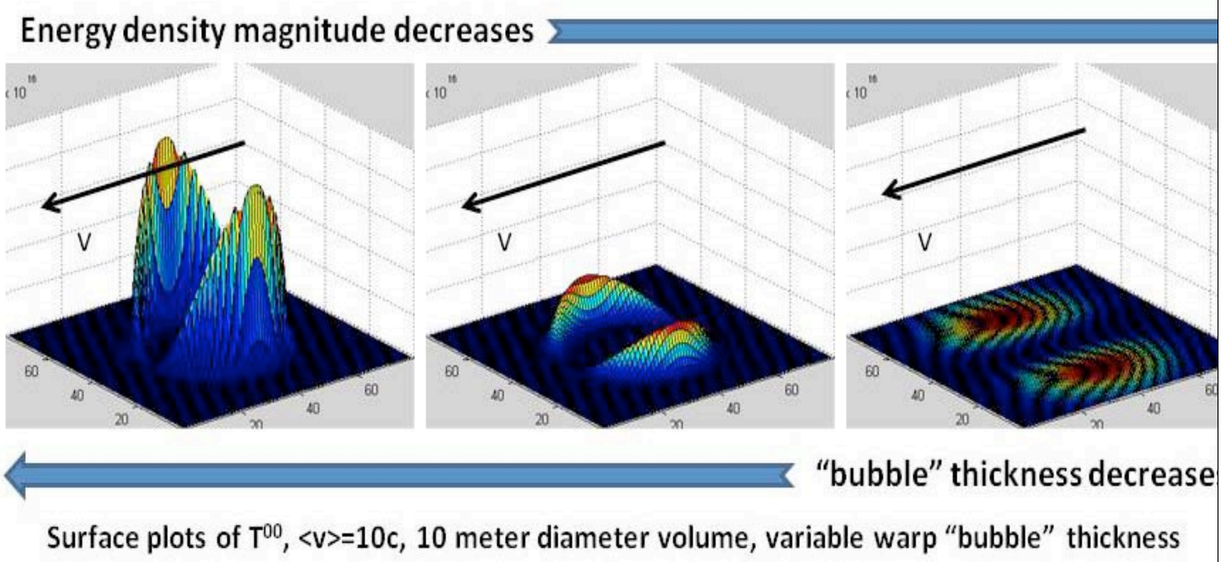

Figure 4: Warp Field Mechanics 101. Dr. White

Please note also that *G* and *c* in EFE have been suppressed by choosing *G* = *c* = 1 as is the convention. Also, there is a directional symmetry issue resulting from the symmetry of the energy density about the *x*-axis. That is, the choice of positive *x*-direction would seem arbitrary and the starship would not "know" whether –*x* or +*x* was the direction of choice, but (4-18) implies the +*x* direction. To reconcile (4-27) with (4-18) we note that $v_s = \frac{dx_s(t)}{dt}$ is a change in displacement over time in a specific direction of x_s. In other words, v_s has a direction.

Equation (4-27) is an apparent direct "violation" of the WEC. To address this point, let us recall that the left-hand-side of the EFE is a universal law of the space-time geometry while the right-hand-side, the stress-energy tensor, is not universal but depends on the particular local matter and energy distribution and is evaluated for validity upon "the variety of energy conditions in use in the relativity community driven largely by reverse engineering based on the technical requirements of how much you have to assume to easily prove the result you want" (Capozziello, Lobo, & Mimoso, 2018). The point is that the non-negativity condition on the stress-energy tensor as an input hypothesis has precluded the emergence of "weird" physics. Theoretical physics has established that there might be "negative energy" out there in the universe. For example, "on 10 April 2017, physicist Peter Engels and his team created negative effective mass by reducing the temperature of rubidium atoms to near absolute zero" (Cselyuszka, Sečujski, Crnojević-Bengin, 2015)

We note that the energy conditions represent criteria designed to rule out unphysical or "weird" solutions of the EFE; roughly speaking, "they crudely describe properties common to all (or almost all) states of matter and all non-gravitational fields that are well-established in physics while being sufficiently strong to rule out many unphysical "solutions" of the Einstein field equation" (Curiel, 2014).

CONCLUSION

Now that we have outlined the above energy condition let us remind ourselves that those conditions have not been shown to be universally true (Farnes, 2018, Visser, Barceló, 2000), which brings me to the question: If the Alcubierre metric were not a violation of any energy conditions, what would we conclude about its implementation and what would be the implications regarding the status of negative energy? The Casimir effect is supposed to be a demonstration that negative energy does exist. It can be argued that there is not really a chunk of negative energy residing between two conductive plates when they are brought very close to each other; the proximity of the plates eliminates certain harmonics or particles whose wavelengths are not compatible with the harmonics of the plate separation. Therefore I do not believe that the Casimir effect proves the existence of negative energy.

Also, the positive energy theorem asserts in its "standard form, broadly speaking, that the gravitational energy of an isolated system is nonnegative, and can be zero only when the system has no gravitating objects" (Schoen, Yau, 1981).

The point is, the manner in which we talk about energy should not be left at the mercy of rhetoric. The mathematics does not deny that space-time could be warped and manipulated to produce superluminal speeds. The mathematics simply says that in order to achieve superluminal speeds using the Alcubierre metric, we would need negative energy. Cosmology tells us that the universe is expanding at an accelerating rate, which (Wiltshire, David, 2007) seems to be an observational fact, and that dark energy is what seems to be responsible (Ishak, Richardson, Garred, Whittington, Nwankwo; Sussman, 2008)for the expansion of the universe. Then, based on the positive energy theorem we may conclude that the negative energy requirement is not a physical possibility for closed systems and that whatever is causing the universe to expand is still a mystery.

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