Ball Aerospace Satellite Coverage Optimization

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Abstract

Ball Aerospace is preparing to launch satellites into orbit that are equipped with CIRiS (Compact Infrared Radiometer in Space) technology used to track evapotranspiration from the earth's surface. It is crucial that Ball Aerospace can achieve full earth coverage in a twenty-four-hour period, and that this be done with the least amount of satellites possible. Given the initial results, we have reason to believe that seven satellites will not sufficiently achieve full earth coverage in a twenty-four hour period. This, of course, is assuming that all coverage must be attained during the day.
[1]

Introduction

Ball Aerospace is in the process of sending satellites into space. The satellites have the ability to provide coverage of the earth, and are being used to track evapotranspiration levels. This could provide numerous benefits for various applications including the agriculture industry, giving them information to maximize their irrigation patterns. It could also provide valuable information regarding the carbon cycle which could lead to a more efficient allocation of water resources globally. For our project, we will undertake the optimization of satellite coverage by focusing on the implementation of a satellite scheduling

[1]**Cyril Waymel:** Roles consisted of coming up with the approaches and thought processes of how to implement the maximization function; specifically finding the time spacing between the satellites that would minimize overlap and maximize coverage. Secondly, to implement the coding to distinguish between satellites so that we can reference them individually and inspect their ground tracking intervals, and area covered elements in order to provide visual graphics. Thirdly, to come up with visualizations/graphs to support the group's claims. **Simon:** Helped to create the day and night functions and represent them graphically. Also came up with the idea of assigning a separate time-space for each of the satellites. **Ben:** Worked on rough draft. Before this searched workbook for areas where code could be combined into fewer cells / made tidier. Interpreted code used for plotting and provided suggestions for how different commands for coverage could be written. **Erik:** Helped with the implementation of distinguishing satellite identities in the reformatted version of the code as well as streamlining the code for potential functions to be made out of the cells that were merged. Also helped to write the rough draft with the help of Ben, Cy, and Simon.

function which will allow us to accurately predict the best spacing of satellites to eliminate overlapping coverage while maximizing total coverage. This will lead to validation, or possibly correction of Ball Aerospace's current hypothesis about the least amount of satellites necessary, allowing them to optimize their resources and finances.

We will use the previous work, completed by Bielich, Matesi, and Le, as the baseline for our approach. As mentioned in the resulting research paper, *A Study into Satellite Coverage and Orbital Equations*, Ball Aerospace is sending low-cost satellites with CIRiS (Compact Infrared Radiometer in Space) cameras into space in order to capture images of the earth's surface. These satellites will be following a sun-synchronous, near polar orbit (almost crossing the poles). The major difference between our work and that of the previous group is that we intend to find models that will allow Ball Aerospace to accomplish the task with at most six satellites by enhancing the existing code to fit our needs for an optimized satellite scheduling function. We hope to create an accurate simulation with functions that will identify daytime and nighttime coverage, and avoid instances of satellite overlap. This function will also hopefully be able to eliminate gaps between satellites as a part of our optimization problem. Accomplishing this will give us the information to either confirm or challenge, that Ball Aerospace's current method is optimized.

Methods

In the research paper *A Study into Satellite Coverage and Orbital Equations*, there is an extensive listing and explanation of each of the Keplerian elements involved in this ECEF (Earth centered Earth fixed) orbit. To be specific, there are six parameters that are necessary to describe the motion of one celestial body in relation to another for a two-body problem. A Brief overview of these elements is listed below:

1. Semimajor axis (a)- half the length of the line segment that runs through the center and both foci, with ends at the widest points of the perimeter.

2. Eccentricity (e)- parameter that determines the amount by which its orbit around another body deviates from a perfect circle (0 being perfectly circular and 1 being a parabolic escape orbit).

3. Inclination (i)- measurement of the satellite orbit's tilt around from the earth's equatorial plane.

4. Longitude of ascending node (Ω)- the angle between the reference direction and the upward crossing of the orbit on the reference plane.

5. Argument of Periapsis (ω)- Is the angle between the ascending node and the periapsis, and is measured in the direction of the satellite's motion.

6. True Anomaly (ν)- the position of the orbiting body along the trajectory, measured from periapsis.

While the objective of the previous group was to set up a fully functional simulation given the input parameters, our objective is to improve and enhance this model specifically in its flexibility regarding time. After much deliberation, we decided the best approach was to uniformly space the satellites rather than have variable spacing in order to avoid any gaps in the equator coverage and to provide a pattern that would be reliable for long-term simulations. The first issue with the current model is that as each satellite begins its orbit, its initial time is set to zero rather than the time elapsed since the first satellite started. This is a problem because the configuration of the satellite coverage is dependent on their start times. While the j indexing does create the spatial separation of the satellites in orbit, there was no way to reference each satellite's time parameter relative to another satellite.

Secondly, in the existing code in section 4, where the researchers were beginning to collect the longitudinal data when the satellites cross the equator, the identity of each of the satellites is lost. There is an appending function that aggregates all of the longitudes regardless of which satellite is crossing and where the crossing is occurring (whether during the day or night). Not only is the identity of the satellite lost in this cell, but in each of the subsequent cells in which the swath, intervals captured, and ground tracking is calculated. This is an evident disadvantage for us given our final goal.

The process of enhancing the existing code occurred in stages. We first focused on creating satellite-specific data. This means that for every variable that was created throughout the code we needed to be able to reference that variable for each individual satellite and even specific iterations within that satellite. For example, it was crucial for us to be able to distinguish and to know which intervals of swath coverage pertained to which satellite. We then shifted our focus on the individual time-spaces. This consisted in altering the existing code which had a single time variable that was initiated at the start time of the first satellite and placing that within a for-loop which created a time-space for each satellite relative to the first. We could then manipulate the spacing between satellites by incrementing their start times as a decimal multiplier of their orbital period.

Once we had these two goals completed, we could then begin to evaluate how different spacing increments between the satellites' start times were related to the overlap or gaps of their swath intervals and thus how to optimize the spacing.

In addition, Dr. Osterman has mentioned that it is preferable that we only count the earth coverage that occurs on the sunny side of the earth. Therefore it is necessary to formulate a function that distinguishes between day and nighttime crossings of the equator.

To create the code for day/night function, we assume the initial location is 0 longitude with the initial time 0AM. The day time is between 6AM and 6PM which means the initial day time location is between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Since the Earth rotates 2π longitude per day, then the day time location rotates $\frac{\pi}{180}$ radians for every 240 seconds. When the sunrise longitude is smaller than the sunset longitude, the day time equator longitude is:

$$\psi_{day} \in [\psi_r - t * rad, \psi_s - t * rad]$$

and when the sunrise longitude is bigger than the sunset longitude, the day time equator longitude is:

$$\psi_{day} \in \{x|x \geq \psi_r - t * rad\} \cup \{x|x \leq \psi_s - t * rad\}$$

where t is the time that satellite have traveled, rad is the longitudes that the earth rotate every second, ψ_r is the initial sunrise location and ψ_s is the initial sunset location . The longitudes outside these interval are night time location.

Results

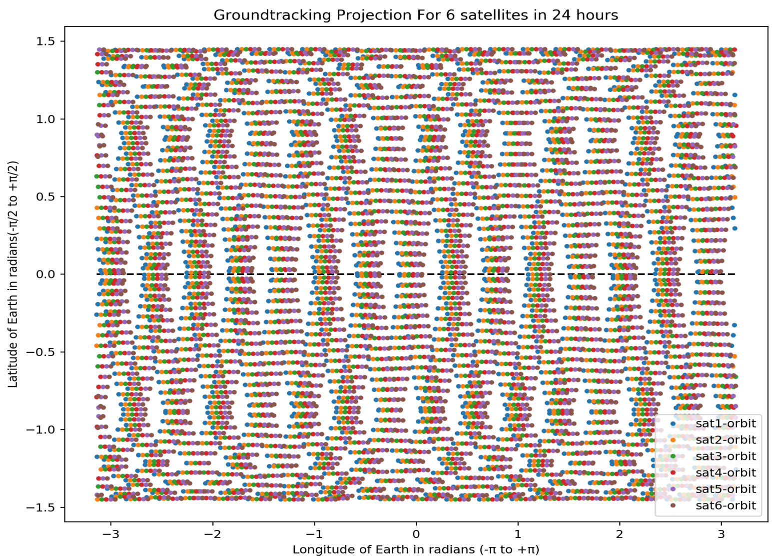
Through enhancing and manipulating the code we were able to provide the following results displayed in 2-D and 3-D plots. We have produced the results from six to ten satellites in order to show the progression of satellite daytime coverage. They are grouped by their plot types. All of the results shown are produced based on a 24 hour simulation period in addition to certain assumptions and parameters stated in the discussion portion of this report.

Figures 1(a)-1(e) show the ground tracking projection for the satellites. They display the trajectory of each satellite as they orbit around the earth; the horizontal dashed line at zero on the y-axis locates the equator, and the x-axis frames the circumference of the earth in radians.

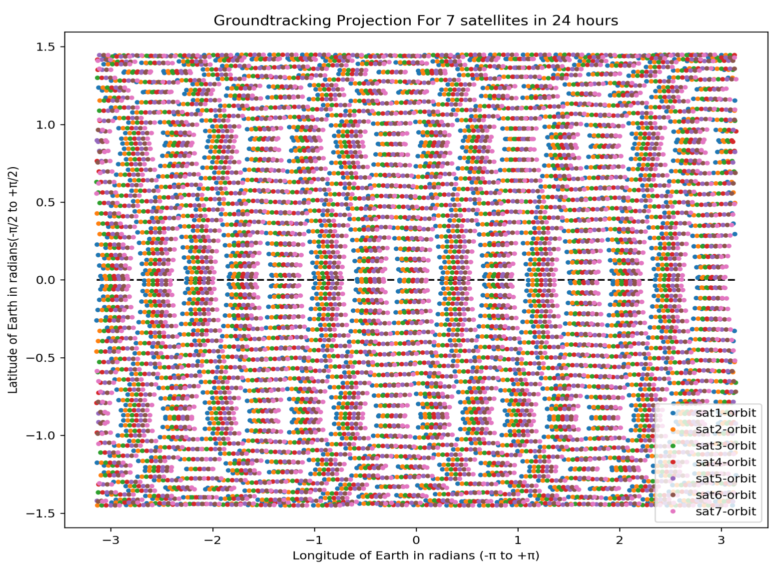
Figures 2(f)-2(j) show the 3-Dimensional trajectories of each satellite. The q1, q2, and q3 axes are displayed in meters.

Figures 3(k)-3(o) reveal the longitudinal locations for each equatorial crossing per satellite. Only the daytime crossings are plotted within the bounds of sunrise (red line) and sunset (blue line) currently established in our code as 6AM and 6PM ($\frac{\pi}{2}$ and $\frac{3\pi}{2}$ in radians) respectively. These sunset and sunrise times are defined as user inputs and can easily be modified to fit Ball Aerospace's purposes.

Figures 4(p)-4(t) display the individual coverage by satellite as well as the cumulative coverage for all satellites. The left-hand y-axis pertains to the individual satellite plots which increase in coverage every period as they cross the equator. The right-hand y-axis is scaled appropriately to the cumulative plot which sums all coverage. It is with no less than ten satellites that we achieve full earth coverage.

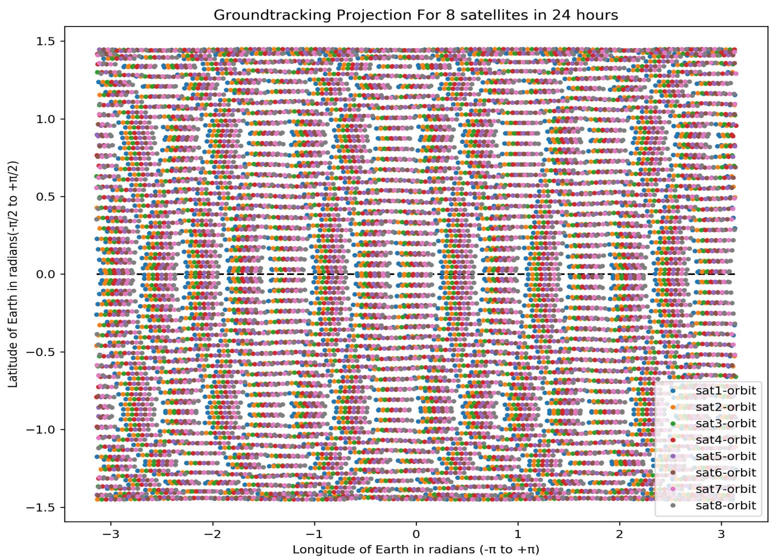


(a) 6 Satellites

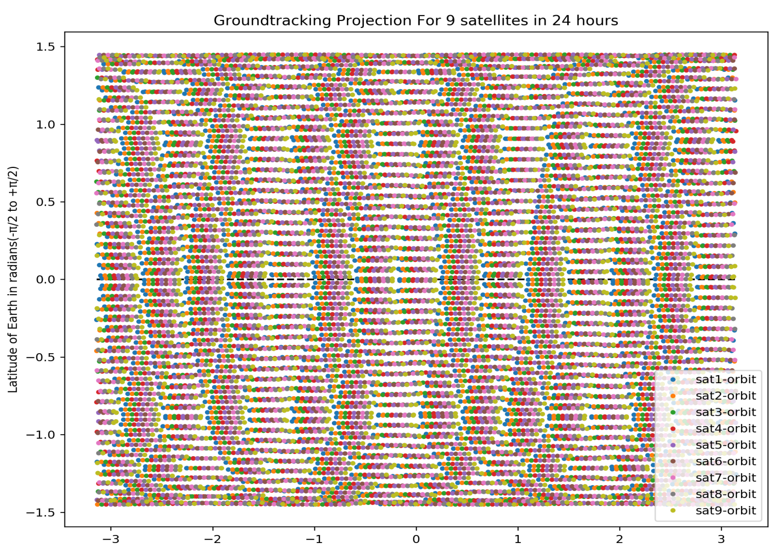


(b) 7 Satellites

Figure 1: Ground Tracking

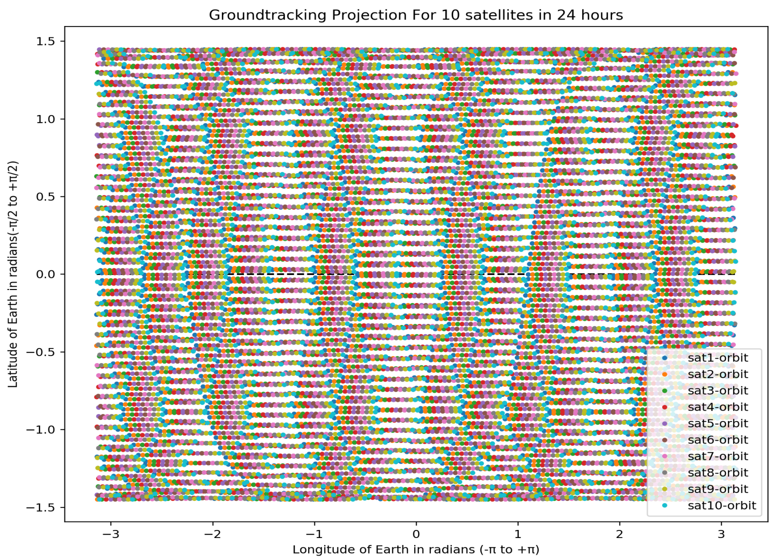


(c) 8 Satellites



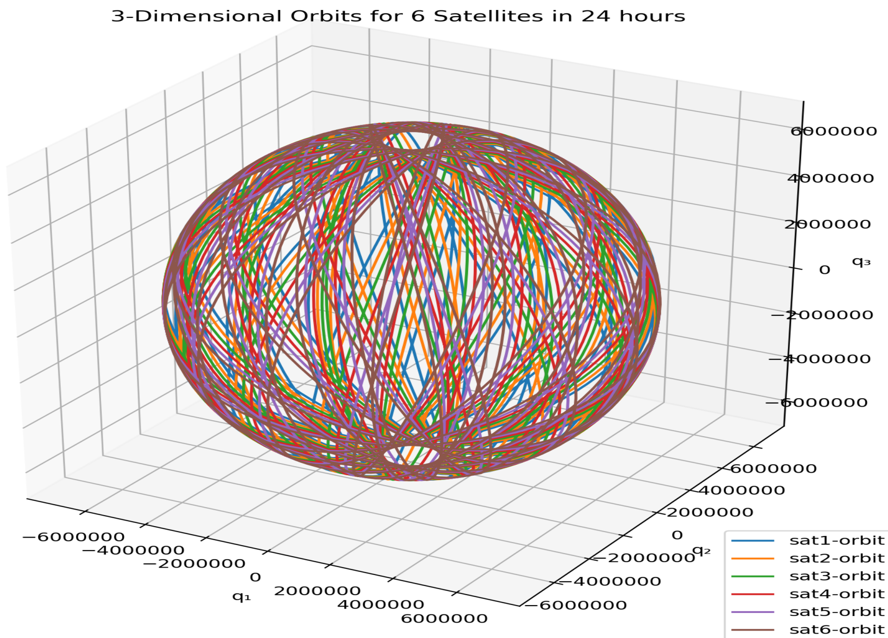
(d) 9 Satellites

Figure 1: Ground Tracking

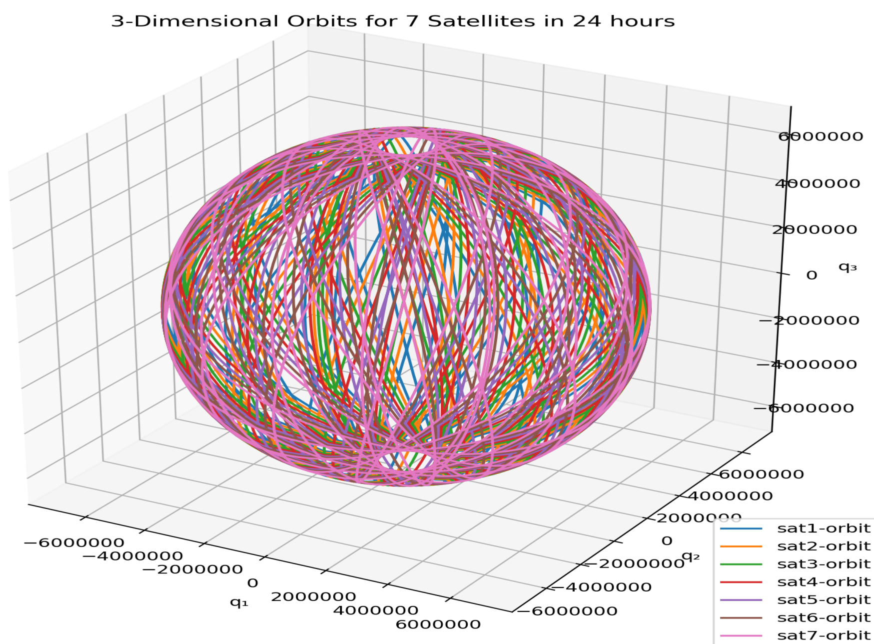


(e) 10 Satellites

Figure 1: Ground Tracking

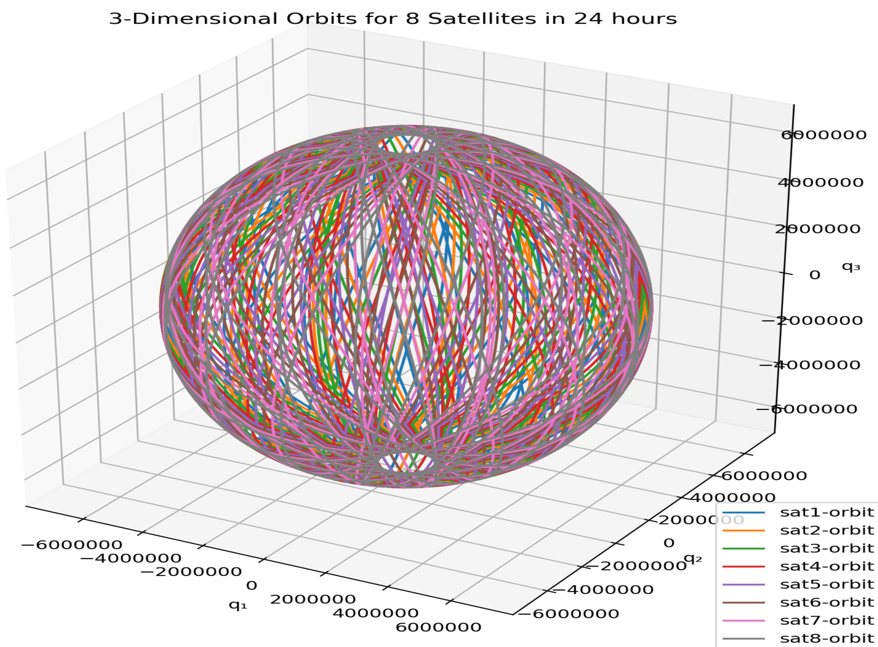


(f) 6 Satellites

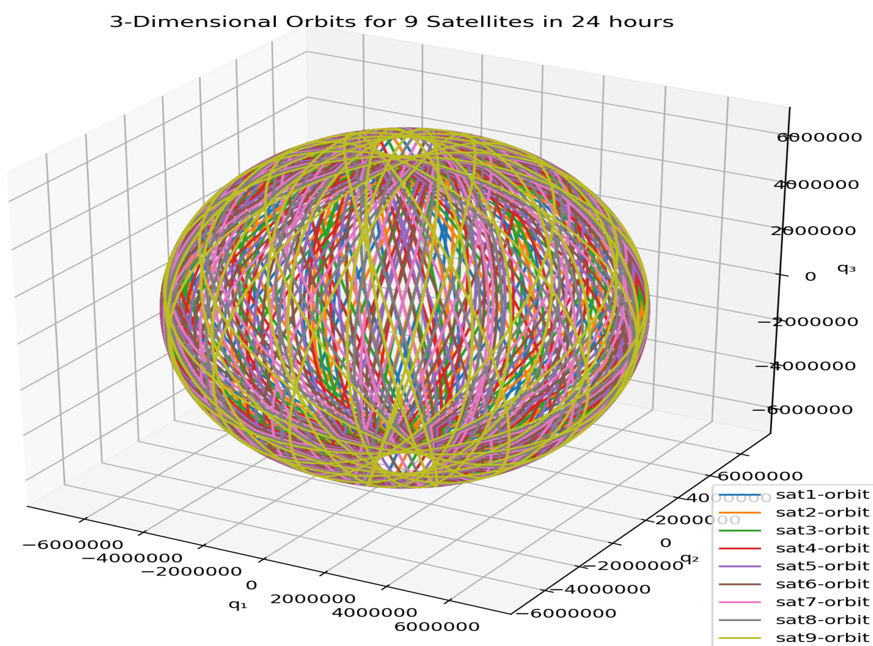


(g) 7 Satellites

Figure 2: 3-Dimensional trajectories of each satellite

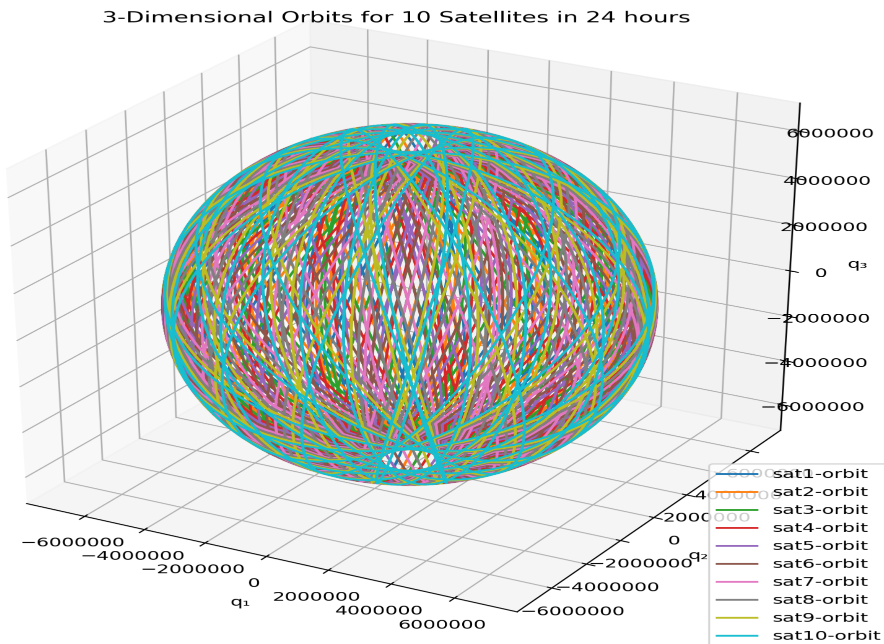


(h) 8 Satellites



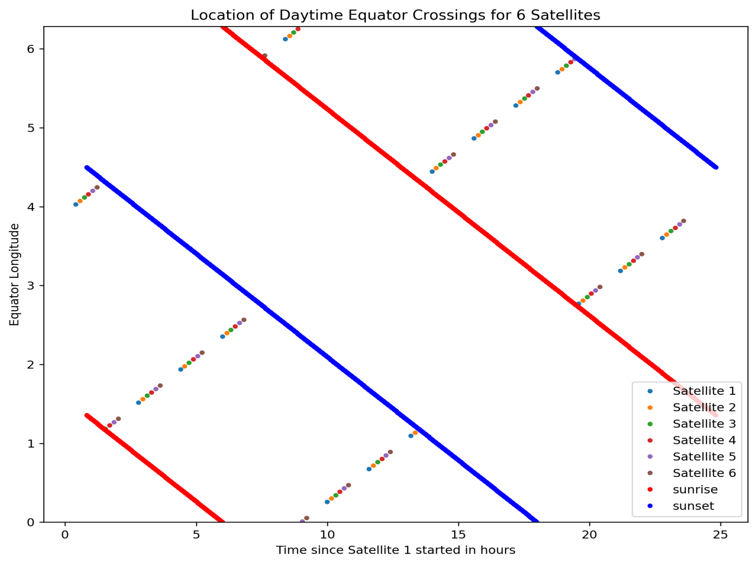
(i) 9 Satellites

Figure 2: 3-Dimensional trajectories of each satellite

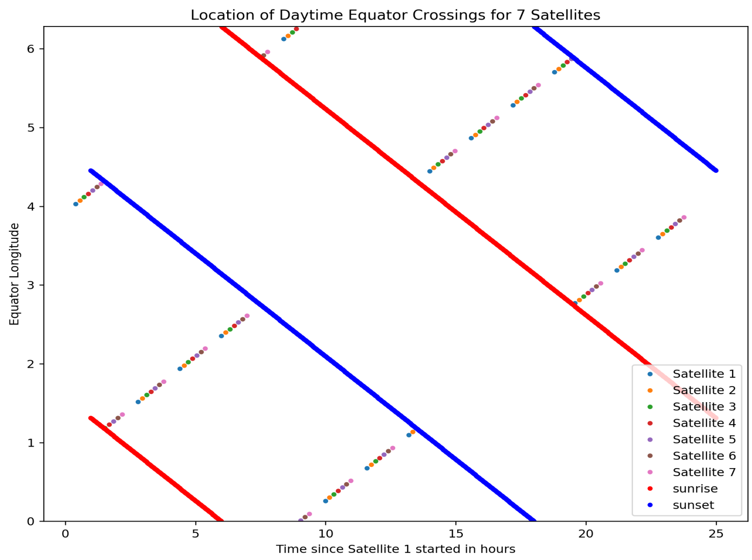


(j) 10 Satellites

Figure 2: 3-Dimensional trajectories of each satellite

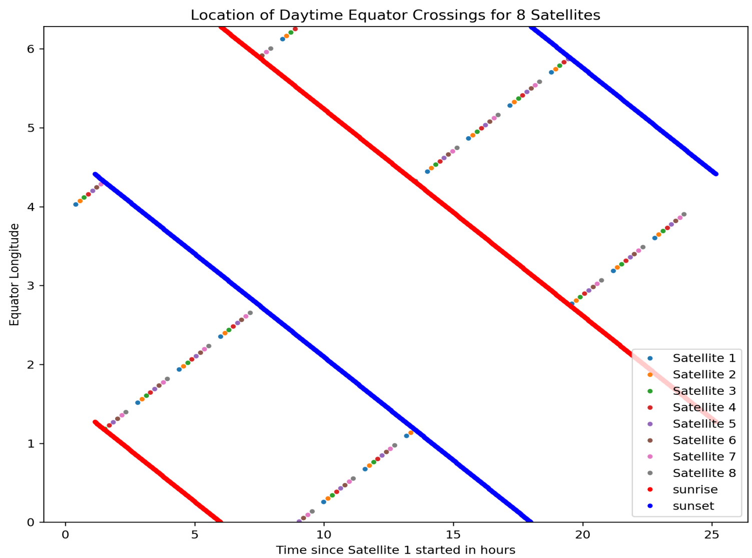


(k) 6 Satellites

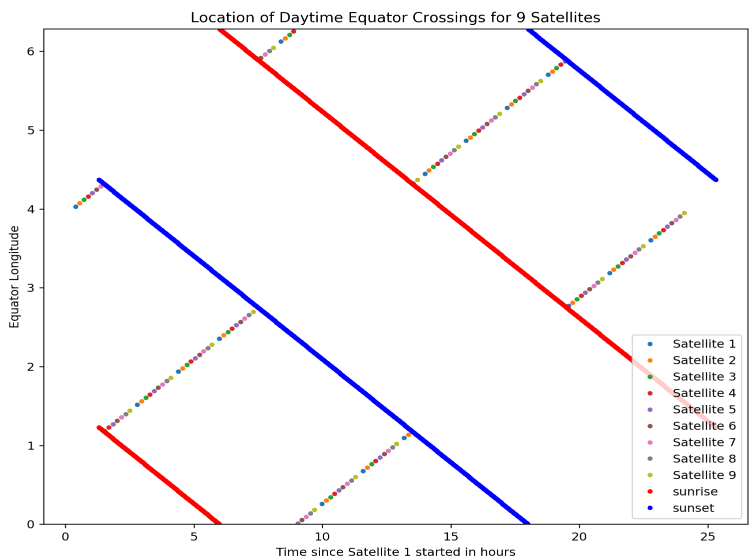


(l) 7 Satellites

Figure 3: Equatorial Crossing

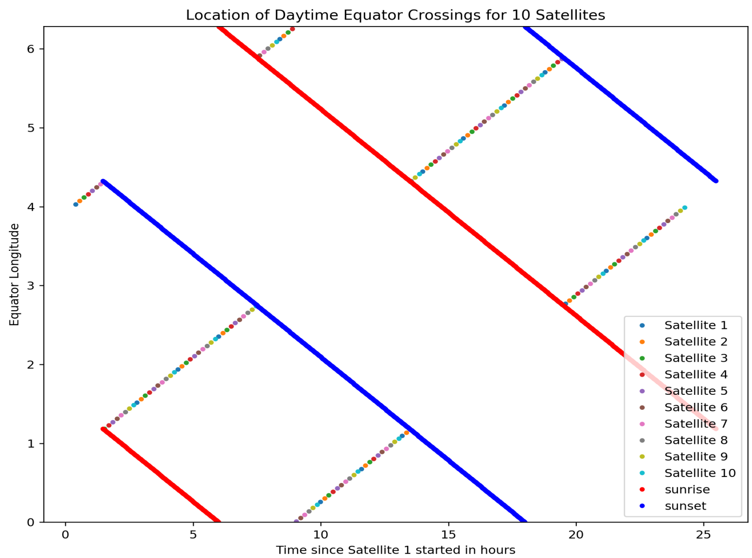


(m) 8 Satellites



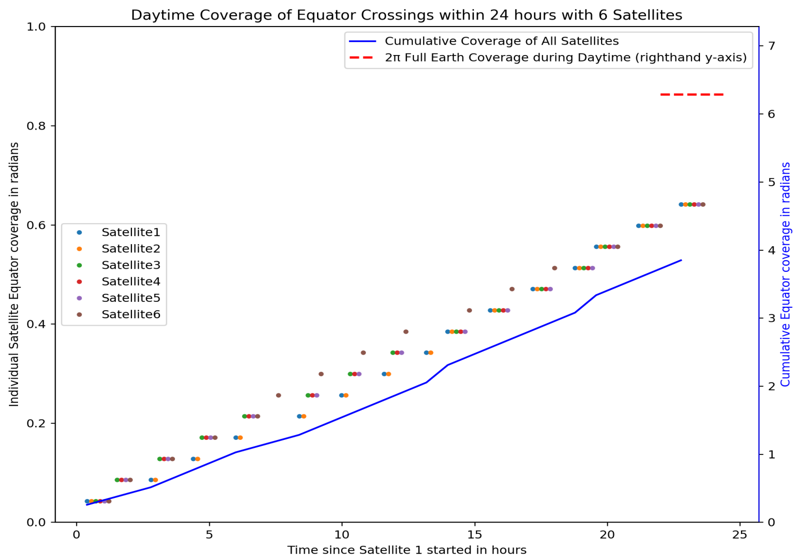
(n) 9 Satellites

Figure 3: Equatorial Crossing

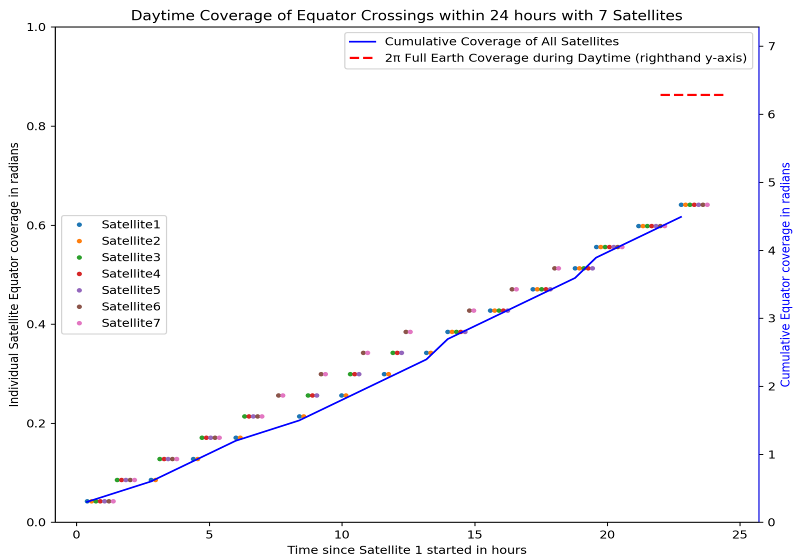


(o) 10 Satellites

Figure 3: Equatorial Crossing

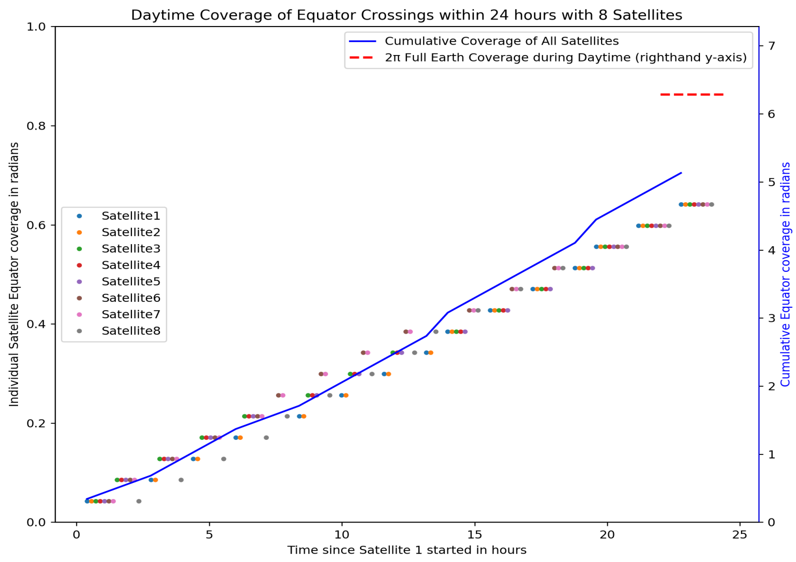


(p) 6 Satellites

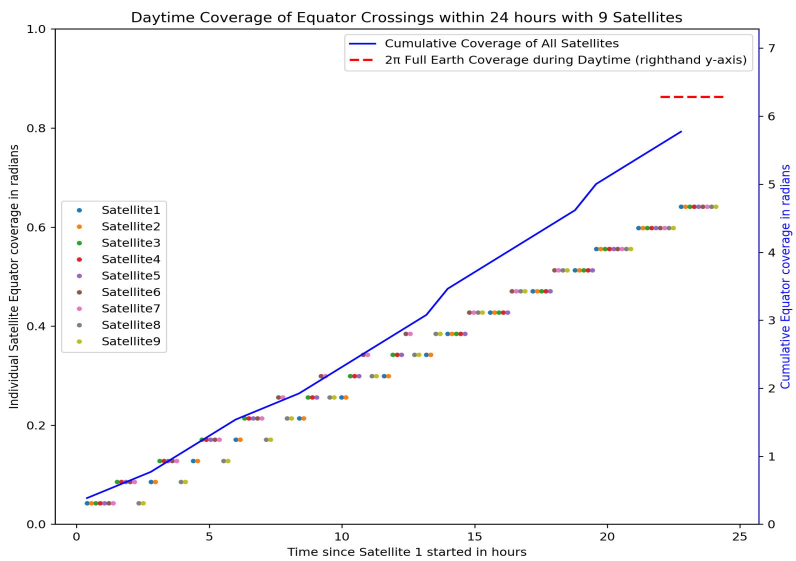


(q) 7 Satellites

Figure 4: Equatorial Crossing

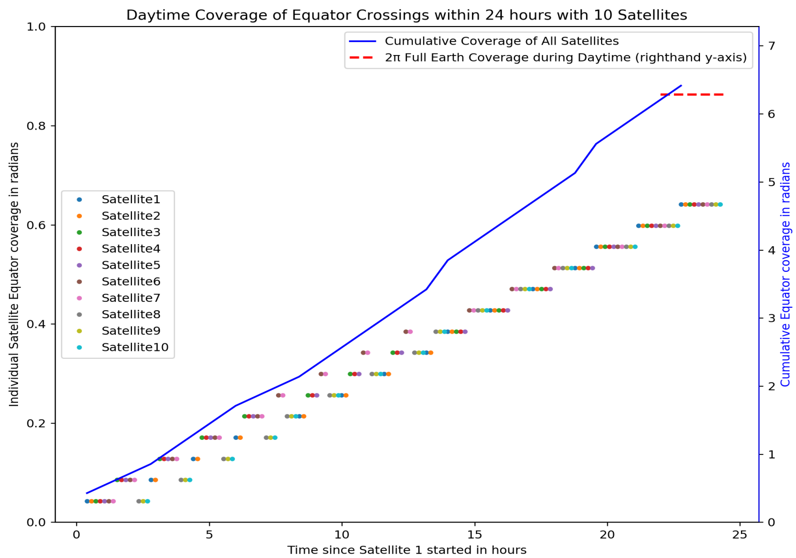


(r) 8 Satellites



(s) 9 Satellites

Figure 4: Equatorial Crossing



(t) 10 Satellites

Figure 4: Equatorial Crossing

Discussion

As mentioned above, all of these results are based off of certain assumptions and parameters which we have outlined in our notebook. First, we have simulated these orbits with the satellites at an altitude of 500 kilometers, revolving around the earth at a constant velocity. Although the swath coverage would increase if the orbits were at a higher altitude, anything further than 500 kilometers would create issues with the resolution of the satellite imaging. Second, the orbits have a minimal eccentricity of 0.01, meaning that it is a near circular orbit. Finally, due to the design of these particular satellites, they have a range of motion which allows for 30 degrees of swath coverage which is then translated to radians in the notebook.

Regarding the optimization of the spacing between satellites, this was accomplished by analyzing the endpoints of equatorial crossings for each satellite. We needed to find the difference in start times that would allow the right endpoint of the first satellite crossing to match up with the left endpoint of the second satellite crossing without any gaps or large overlaps. Since each satellite is spaced uniformly, as long as the first two matched, the pattern will continue for the full 24 hour simulation, effectively covering the entire equator. There was an attempt to create a function that would reduce the times in between launches by decimals of a trillion, however this would have overloaded the computer memory and taken too long to process. We continued through the process of trial and error, starting with an interval of time and shrinking the interval each iteration in order to get the overlap as small as possible. We were able to find a spacing between satellites, expressed as a fraction of the period, that reduces the overlap to 66 meters per crossing. This overlap is negligible and does not affect our results as it amounts to a total of 0.002% of the earth's circumference. Even with the simulation of 9 satellites we are only able to cover 91.9% of the earth's equator (shown in the table below), reducing the overlap to 0 would not produce significant changes in the results.

Because we were only interested in obtaining daytime coverage, our equator longitude graphs, which show time since first satellite launch on their x axes, have noticeable gaps in data during the time intervals which are denoted as nighttime. The graphs for cumulative coverage use the assumption that when coverage $= 2\pi$ radians, or approximately 6.28 radians, then full earth coverage has been attained.

We also discovered that we could reduce the orbital period to exactly ninety minutes to ensure that each location on the earth would be covered at the same time every day by reducing the altitude to 215.033 kilometers. This, however, is below the minimum altitude threshold to keep the satellite in orbit.

The respective coverages for six to ten satellites in both radians and percentages of coverage in a 24 hour period are as follows (Coverages are rounded to 3 significant figures):

# of satellites	Equator Covered	Percentage of the Equator Covered
6	3.848	61.3%
7	4.490	71.5%
8	5.131	81.7%
9	5.772	91.9%
10	6.414	102.1%

Conclusion

Our final notebook is a versatile tool that produces powerful visualizations with numerous implications for Ball Aerospace. The visualizations above clearly demonstrate that according to our calculations the minimum number of satellites needed to achieve full earth daytime coverage within 24 hours is 10. This is 2 more satellites than Ball Aerospace had anticipated. We believe our calculations to be accurate, however there is a possible explanation for the difference in results. We believe that some of the parameters used in Ball Aerospace's calculations might have different from our own, and possibly less restrictive, thus resulting in more coverage with fewer satellites. It would be necessary to study these inputs together in order to determine where the discrepancy lies.

If we had more time, our final notebook could be further improved by applying pre-defined optimization packages which would use all of the new variables we have created and defined this semester to achieve the desired result more efficiently. Our takeaway is a deeper understanding of orbital mechanics and the earth's rotation in a two-body problem, as well as enhanced skills in optimization and coding.

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