The Emergence of Weak, Despotic and Inclusive States*

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Abstract

Societies under similar geographic and economic conditions and subject to similar external influences nonetheless develop very different types of states. At one extreme are weak states with little capacity and ability to regulate economic or social relations. At the other are despotic states which dominate civil society. Yet there are others which are locked into an ongoing competition with civil society and it is these, not the despotic ones, that develop the greatest capacity. We develop a dynamic contest model of the potential competition between state (controlled by a ruler or a group of elites) and civil society (representing non-elite citizens), where both players can invest to increase their power. The model leads to different types of steady states depending on initial conditions. One type of steady state, corresponding to a weak state, emerges when civil society is strong relative to the state (e.g., having developed social norms limiting political hierarchy). Another type of steady state, corresponding to a despotic state, originates from initial conditions where the state is powerful and civil society is weak. A third type of steady state, which we refer to as an inclusive state, is also possible when state and civil society are more evenly matched. In this case, both parties have greater incentives to invest to keep up with the other, and this leads to the most powerful and capable type of state, while incentivizing civil society to be equally powerful as well. Our framework shows why structural factors such as geography, economic conditions or external threats have ambiguous effects on the development of a powerful state — depending on initial conditions they can shift the society into or out of the basin of attraction of the inclusive state.

Keywords: civil society, contest, political divergence, state capacity, weak states.

JEL classification: H4, H7, P16.

In Progress. Comments Welcome.

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1 Introduction

The capacity of the state to enforce laws, provide public services, and regulate and tax economic activity varies enormously across countries. The dominant paradigm in social science to explain the development of state capacity links it to the ability of a powerful group, elite or charismatic leader to dominate other powerful actors in society and build institutions such as a fiscal system or bureaucracy (e.g., Huntington, 1968). This paradigm also relates this ability to certain structural factors such as geography, ecology, natural resources and population density (Mahdavy, 1970, Diamond, 1997, Herbst, 2000, Fukuyama, 2011, 2014), the threat of war (Hintze, 1975, Brewer, 1989, Tilly, 1975, 1990, Besley and Persson, 2011, O'Brien, 2011, Gennaioli and Voth, 2015), or the nature of economic activity (Mann, 1986, 1993, Acemoglu, 2005, Spruyt, 2009, Besley and Persson, 2011). However, historically, societies with similar ecologies, geographies, initial economic structures and external threats have diverged sharply in terms of the development of their states. Moreover, in many instances in which the state has built capacity, it has not dominated a meek society; on the contrary, it has had to continuously contend and struggle with a strong, assertive (civil) society.[1]

These dynamics are illustrated by the historical evolution of the power of the state and state-society relations in Europe. European nations share not only a great deal of history and culture but also broadly similar economic conditions. And yet, the types of states and political dynamics we observe in the continent over the last several centuries are hugely varied. At one end of the spectrum, Prussia in the 19th century constructed an autocratic, militarized state under an absolutist monarchy, backed by a traditional landowning Junker class, which continued to exercise enough authority to help derail the Weimar democracy in the 1920s (Evans, 2005). Meanwhile, just to its south, the Swiss state attained its final institutionalization in 1848 not as a consequence of an absolutist monarchy, but from the bottom-up construction of independent republican cantons based on rural-urban communes. A little further south, places in the Balkans such as Montenegro never had centralized state authority at all. Prior to 1852 Montenegro was in effect a theocracy, but its ruling Bishop, the Vladika, could exercise no coercive authority over the clans which dominated the society partly via a complex web of traditions and social norms. As a consequence of this lack of state authority, blood feuds and other inter-clan conflicts were extremely common.[2]

[1] Throughout, we use "civil society" and "society"' interchangeably.

[2] Simić (1967) and Boehm (1986) emphasize the importance of clans and traditions as a constraint on state power in Montenegro (see also Djilas, 1966). For example, "Continued attempts to impose centralized government were in conflict with tribal loyalty" (Simić, 1967, p. 87), and "It was only when their central leader attempted to institutionalize forcible means of controlling feuds that the tribesmen stood firm in their right to follow their

This diversity is hard to explain based on some deep structural differences. Switzerland and Montenegro are both mountainous (which Braudel, 1966, emphasized as crucial), were both part of the Roman Empire, have been Christian for centuries, were specialized in the Middle Ages in similar economic activities such as herding, and have been involved in continuous wars against external foes. Before the founding of the Swiss Confederation in 1291, feuding was also common in that area. Scott, for example, notes: "There is general agreement amongst recent historians that the origins of the Swiss Confederation lay in the search for public order. The provisions of the *Bundesbrief* of 1291 were clearly directed against feuding in the inner cantons" (1995, p. 98; see also Bickel, 1992). The parallels between Switzerland and Prussia are even stronger. Both countries have very close cultural and ethnic roots (and historically Switzerland had been settled by Germanic tribes, particularly the Alemanni), and have shared similar religious identities before and after the Reformation. Though Prussia was not part of the Holy Roman Empire, its institutions have been heavily influenced by those of the Empire and had feudal roots similar to those of Switzerland.

In this paper, we develop a simple theory of state-society relations where the competition and conflict between state and (civil) society is the main driver of the institutional change and the emergence of state capacity (Acemoglu and Robinson, 2012). Small differences, such as those between Prussia, Switzerland and Montenegro which we further discuss below, can set off political dynamics in very different directions. In our model, though the state wishes to establish dominance over society, the ability of society to develop its own strengths (in the form of coordination, social norms and local organization) is central, because it induces the state to become even stronger in order to compete with society. Likewise, the race against the state encourages society to invest further in its capacity. When this balance between state and society is not achieved, either the state fully dominates society or society is powerful and the state remains weak. Crucially, however, when society is weak, state capacity is relatively

ancient traditions. This was because they perceived in such interference a threat to their basic political autonomy." (Boehm, 1986, p. 186).

Indeed, the first attempt at a codified law code in 1796 by Vladika Peter I reflected the fact that order in society was regulated by the institution of blood feuds and included the clauses: "A man who strikes another with his hand, foot, or chibouk, shall pay him a fine of fifty sequins. If the man struck at once kills his aggressor, he shall not be punished. Nor shall a man be punished for killing a thief caught in the act..." and "If a Montenegrin in self-defense kills a man who has insulted him ... it shall be considered that the killing was involuntary." (quoted in Durham, 1928, pp. 78-88). The Montenegrin politician and writer Milovan Djilas describes the importance of blood feuds in the 1950s thus "the men of several generations have died at the hands on Montenegrins, men of the same faith and name. My father's grandfather, my own two grandfathers, my father, and my uncle were killed, as though a dread curse lay upon them ... generation after generation, and the bloody chain was not broken. The inherited fear and hatred of feuding clans was mightier than fear and hatred of the enemy, the Turks. It seems to me that I was born with blood in my eyes. My first sight was of blood. My first words were blood and bathed in blood" (1958, pp. 3-4).

limited as well, because the state can control society easily and does not need to invest much in its own capacity (strength).[3] In addition to highlighting the vital role of the race between state and society in the development of state capacity, our theory shows that, just as in the examples discussed above, polities with similar initial conditions and subject to similar structural influences can nonetheless experience divergent state-society relations and evolution of state capacity, because they may fall into the basins of attraction of different dynamic equilibria.

This history-dependent development of state capacity and our main theoretical results are summarized in Figure 1. This figure plots the global dynamics of state-society relations. Region I illustrates the Huntingtonian path, approximating the political dynamics of Prussia. Here, the state is stronger than civil society to start with and fully dominates it; for this reason, we call this type of state a despotic one. Region III is the case in which the social norms of the society, especially in how they are able to act collectively and control political power and hierarchy, are strong and this prevents the emergence of a powerful state, paving the way to weak states as in Montenegro. Region II illustrates the happy middle ground where state and society are initially in balance, and this triggers a positive competition between the two, whereby they both become stronger over time. We refer to this path, which best resembles the Swiss experience, as one of inclusive states. This terminology is motivated by the fact that, in this case, the state is not just strong (capable) but also evenly matched with civil society, which is then able to partially check the domination of the state and the elites that control it. As the figure shows, it is in this inclusive case that the state achieves the greatest capacity. The fact that the power or capacity of the state is greater in this case than in Region I highlights that it is the competition between state and civil society that triggers greater investments by the state (or the ruler and elites controlling it) to invest further in their power.

Our theory and Figure 1 suggest that divergent political paths such as those between Prussia, Switzerland and Montenegro may lie not in large differences in structural factors, but in small differences that get amplified as a result of the competition between state and society. Such small differences indeed favored the development of a powerful state in Prussia, which emerged out of the militarized state of the Teutonic Knights in the east of the River Elbe, where feudalism was possibly the most intense in Europe (Gerschenkron, 1943, Moore, 1966, Clark, 2009). In contrast, they likely favored the weak state path in Montenegro, where the 'herdsman' culture was very strong and a legitimate political order like the Holy Roman Empire was absent for

[3] Throughout, we use power, strength and capacity interchangeably. In practice, one might wish to distinguish between the underlying, infrastructural power of the state, which then creates capacity to achieve certain goals or implement certain objectives, but in our abstract model, this distinction does not arise.

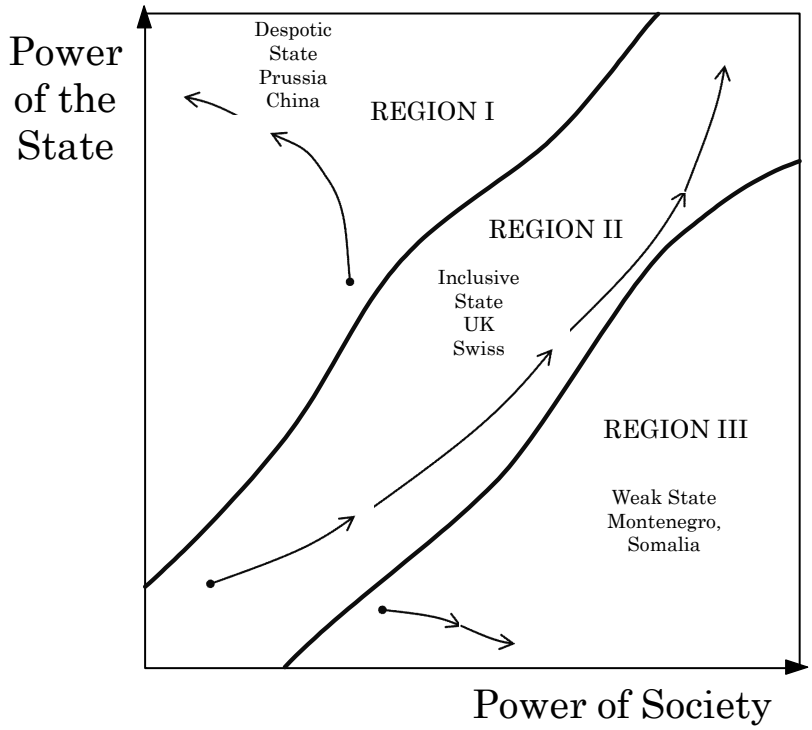

Figure 1: The emergence and dynamics of weak, despotic and inclusive states.

several centuries. In contradistinction to both of these cases, the powers of state and society were more evenly balanced in Switzerland. Differently from Prussia, Swiss peasants were 'free' (Steinberg, 2015), independent cities such as Basel, Bern and Zürich played a more important economic and political role, and the major demographic changes of the 14th century, in particular the Black Death of the 1340s, appear to have weakened the elites even further (e.g., Morerod and Favrod, 2014). Compared to Montenegro, Switzerland's history of established political order under the Holy Roman Empire and of corporations such as monasteries and cathedral chapters (Church and Head, 2013, Morerod and Favrod, 2014) may have created the small differences facilitating the emergence of a state capable of competing against civil society.

Theoretically, our setup is one of a dynamic contest between two players, the elite controlling the state and society representing non-elite citizens. At each date, the state and society both choose investments in their strength, and these strengths determine both the overall output in the economy which is distributed between the elite controlling the state and the rest of the citizens, and how this distribution takes place. We introduce some degree of economies of scale in the contest technology so that the cost of investment for either state or society becomes higher if they fall below a certain level.[4] The interplay of contest incentives and the presence of economies

[4]This appears to be a reasonable assumption both for society and the state. Many scholars have argued that

of scale underpins the emergence of three stable steady states as shown in Figure 1: when one party is significantly stronger than the other, the weaker player is discouraged from investing. But since as in contest models part of the reason why each player invests is to be stronger than the other the discouragement of the weaker party also reduces the investment incentives of the dominant player. In contrast, when the two players are evenly matched, they are both induced to invest more. These results are an application of Harris and Vickers' (1984, 1987) *discouragement effect*.[5] After illustrating the workings of our model in the simplest possible case where the players act myopically, we also show that similar results obtain when the players are forward-looking but sufficiently impatient.

Though our theory does not link the evolution of state capacity to structural factors, our general framework provides comparative static results that clarify when a society is more likely to be in the basin of attraction of different types of steady states (as already hinted by Figure 1). Specifically, changes in underlying parameters — corresponding to technologies, economic conditions and the external environment — change the basins of attraction of the three steady states. For example, starting from the basin of attraction of Region III, a need for greater coordination in the economy, resulting either because of increasing importance of public goods or national defense, can shift a society into Region II, and thus trigger the long-run development of the state. However, crucially, our framework also clarifies that the effects of structural factors are *conditional* — the same change in external threats can also shift an economy from Region II (or even Region III) to Region I, thus triggering the evolution of a despotic state and ultimately limiting the growth of state capacity. This, for example, explains why theories such as those of Tilly mentioned above, which emphasize the threat of war as a driver of state building, tend to have only limited explanatory power.

We illustrate how our approach is useful for interpreting historical dynamics using an extended historical example, the evolution of different types of states in Ancient Greece, in Section 7.[6]

Our paper is related to a number of literatures. As already discussed above, most prominent in the social science literature on state building are approaches that situate the roots of state

there are increasing returns to collective action (e.g., Marwell and Oliver, 1993, Pearson, 2000) or the acquisition of social capital (Francois, 2002). It also appears reasonable that there are fixed costs involved in the creation of fiscal systems or bureaucracies (e.g. Dharmapalaa, Slemrod and Wilson, 2011, Gauthier, 2013).

[5] See Dechenaux, Kovenock and Sheremeta (2015) for a survey of experimental evidence on discouragement effect in contests. See also Aghion, Bloom, Blundell, Griffith and Howitt (2005) and Aghion and Griffith (2008) for evidence on the discouragement effect in the context of innovation investments.

[6] Many other such examples come to mind; the post independence divergence of Botswana, the Central African Republic and Rwanda maps well into our trichotomy between inclusive, weak and despotic states, as does the historical divergence of Costa Rica, Honduras and Guatemala in Central America.

capacity in the ability of the state and groups controlling it to dominate society.[7] In addition, these approaches also emphasize the role of structural factors in triggering or preventing state building. Our theory thus sharply differs from these approaches, and has much more in common with a few works in sociology and political science emphasizing the interaction of state and society. Most importantly, Migdal (1988, 2001) emphasize that weak states are a consequence of a strong society (as in our Region III). Scott (2010) has similarly stressed the ability of people to resist the state and its interference. Putnam (1993) argued that a strong society leads to effective governance and bureaucratic effectiveness. None of these scholars note our key distinction from the previous literature — the idea that state capacity develops most strongly when state and civil society are matched in terms of their strengths and compete dynamically.[8] Acemoglu (2005) argues that the capacity of the state is highest when it is "consensually strong," but this emerges not because of competition between state and society, but as a result of a repeated game equilibrium in which citizens are expected to replace rulers who misbehave and do not provide sufficient public goods.

Our work is also related to a large literature in archaeology focusing on how societies start the process of state formation (the so-called "pristine state formation"). Most of these, for example Flannery (1999) or Flannery and Marcus (2013), emphasize a 'top-down' elite centric approach, but other work, particularly by Blanton and Fargher (2008), have placed equal weight on the role of society.

Finally, our model is an example of a dynamic contest, though most of our analysis involves myopic players. Static models of contests in economics go back at least to Tullock (1980), and have been more systematically studied in Dixit (1987), Skaperdas (1992, 1996), Cornes and Hartley (2005), and Corchon (2007). They are similar to models of (patent) races as in Loury (1979), and to all-pay auctions as studied, among others, by Baye et al. (1996), Krishna and Morgan (1997) and Siegel (2009). Our formulation uses a contest function in differences (though

[7] In addition to the works such as Huntington (1968), Tilly (1990) and Fukuyama (2011) mentioned above, this includes authors emphasizing the role of state capacity in enabling elites controlling the state to dominate society via various means, including repression (e.g., Anderson, 1974, Hechter and Brustein, 1980, Slater, 2010, Saylor, 2014).

The recent economics literature mirrors these approaches. For example, Besley and Persson (2009, 2011) focus on the incentives of the elites controlling the state to undertake investments to build state capacity and link this to the probability that they will lose power domestically and to external threats. Gennaioli and Voth (2015) develop a model of the interaction between warfare and state capacity, while Mayshar, Moav and Neeman (2011) emphasize the effect of the type of crop on state building. Other work by Acemoglu, Santos and Robinson (2013) and Acemoglu, Ticchi and Vindigni (2011) again emphasize elite incentives.

[8] Our theory is also related to a few works stressing the implications of state centralization on civil society's organization. These latter works include Tilly (1995), who illustrates these political dynamics using the British case in the 18th and 19th centuries, Acemoglu, Robinson and Torvik (2016), who develop a formal model along these lines, and Habermas (1989), who suggests that the origins of the "public sphere", which can be viewed as an important aspect of strong society, lie in the process of state formation.

we show in the Appendix that the particular formulation is not critical for the results since the *discouragement effect* arises in many standard models), introduced by Hirshleifer (1989), which is mathematically closer to all-pay auctions (e.g., Che and Gale, 2000). Dynamic contests and related racing models are more challenging and various special cases have been discussed in Fudenberg et al. (1983), Harris and Vickers (1985, 1987), Grossman and Shapiro (1987), and Konrad (2009, 2012).

The rest of the paper is organized as follows. In the next section, we introduce our main model. In Section 3, we characterize the dynamic equilibrium and steady states of this model when players are short-lived or myopic. To maximize transparency, this section uses a number of simplifying assumptions, many of which are relaxed later. Section 4 analyzes the same model with forward-looking players, and establishes that the same results when these players are sufficiently impatient. Section 5 relaxes one of the most important simplifying assumptions, allowing the investments of the state and civil society to also affect the size of the pie to be divided. In this setup, it also provides additional comparative static results on how different steady states and their basins of attraction are affected by changes in parameters. Section 7 shows how our conceptual framework is useful for interpreting the divergent paths of state capacity and state-society relations in Ancient Greece. Finally, Section 8 concludes, while the Appendix provides some generalizations and microfoundations for the setup studied in the main text.

2 Basic Model

In this section, we introduce our basic model, aimed at capturing the dynamics of conflict between state and society discussed in the Introduction. We consider the state to be controlled by a ruler or group of elites acting in a coordinated manner — motivating our convention of using elite and state interchangeably in this paper. The main decision for the elite will be how much to invest in the power of the state, which captures, among other things, the military power, the presence of state employees (i.e., what Mann, 1986, refers to as the "infrastructural power of the state") and the ability of the state to regulate and tax economic activity. On the other side, we will consider groups interacting with the state that do not have direct control over its actions. These groups could be other (e.g., local) elites or regular citizens. Throughout, we will refer to them as "civil society". Civil society, or simply society, will also invest in its power, partly as a defensive measure, to balance the power of the state. These investments correspond to efforts of the society to coordinate its activities, its local organization and social norms that are useful for limiting the power of the state (as discussed in the Introduction).

In this section we start with our general framework. We then analyze the cases in which both state and civil society are myopic (e.g., consisting of non-overlapping generations) and fully forward-looking separately. The framework we present here is reduced form. A more detailed setup in which we are more explicit about the nature of the power of state and society is outlined in the Appendix and shown to map into our reduced-form model.

2.1 Preferences and Conflict

We start with a discrete time setup, where period length is $\Delta > 0$ and will later be taken to be small, so that we work with differential rather than difference equations in characterizing the dynamics. At time t, the state variables inherited from the previous period are $(x_{t-\Delta}, s_{t-\Delta}) \in [0,1]^2$, where the first element corresponds to the strength of civil society and the second to the strength of the state controlled by the elite.

At each point in time, the elite or the state is represented by a single player, and civil society is also represented by a single player. In the next two sections, we study both the case in which these players are short-lived and are immediately replaced by another player (so that we have a non-overlapping generations model with "myopic" players), and the case in which players are long-lived and maximize their discounted sum of utilities.

At time t, players simultaneously choose their investments, $i_t^x \geq 0$ and $i_t^s \geq 0$, which determine their current strengths according to the equations:

$$x_t = x_{t-\Delta} + i_t^x \Delta - \delta\Delta, \tag{1}$$

and

$$s_t = s_{t-\Delta} + i_t^s \Delta - \delta\Delta, \tag{2}$$

where $\delta > 0$ is the depreciation of the strength of both parties between periods. Both investment and depreciation are multiplied by the period length, Δ, since they represent "flow" variables, and when period length is taken to be small, they will be suitably downscaled.[9]

The cost of investment for civil society during a period of length Δ is given as $\Delta \cdot \tilde{C}_x(i_t^x, x_{t-\Delta})$ where

$$\tilde{C}_x(i_t^x, x_{t-\Delta}) = \begin{cases} c_x(i_t^x) & \text{if } x_{t-\Delta} > \gamma_x, \\ c_x(i_t^x) + (\gamma_x - x_{t-\Delta})\, i_t^x & \text{if } x_{t-\Delta} \leq \gamma_x. \end{cases}$$

This cost function is multiplied by Δ, since it is the cost of investing an amount i_t^x during the period of length Δ (as captured by equation (1)). The presence of the term $\gamma_x > 0$, on the

[9] Assuming that depreciation is independent of the current level of the strength of the state or civil society is for convenience only. In addition, we can easily allow the two state variables to have different depreciation rates, but do not do so in order to keep the notation from becoming more cumbersome.

other hand, captures the "increasing returns" nature of conflict mentioned in the Introduction: starting from a low level of conflict capacity, it is more costly to build up this capacity. We specify this in a very simple form here, with the cost of investments increasing linearly as last period's conflict capacity falls below the threshold γ_x. This increasing returns aspect plays an important role in our analysis as we emphasize below.

The cost of investment for the state during a period of length Δ is similarly given as $\Delta \cdot \tilde{C}_s(i_t^s, s_{t-\Delta})$ where

$$\tilde{C}_s(i_t^s, s_{t-\Delta}) = \begin{cases} c_s(i_t^s) & \text{if } s_{t-\Delta} > \gamma_s, \\ c_s(i_t^s) + (\gamma_s - s_{t-\Delta})i_t^s & \text{if } s_{t-\Delta} \leq \gamma_s. \end{cases}$$

In these expressions, it will often be more convenient to eliminate investment levels and directly work with the two state variables, x_t and s_t, especially when we take Δ to be small and transition to continuous time. In preparation for this transition, let us substitute out the investment levels and observe that the cost function for civil society and state can be written as:

$$C_x(x_t, x_{t-\Delta}) = c_x\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right) + \max\{\gamma_x - x_{t-\Delta}, 0\}\frac{x_t - x_{t-\Delta}}{\Delta},$$

and

$$C_s(s_t, s_{t-\Delta}) = c_s\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right) + \max\{\gamma_s - s_{t-\Delta}, 0\}\frac{s_t - s_{t-\Delta}}{\Delta},$$

where the increasing returns to scale nature of the cost function is now captured by the max term.[10]

During the lifetime of each generation, a polity with state strength s_t and civil society strength x_t produces output/surplus given by

$$f(x_t, s_t), \tag{3}$$

where f is assumed to be nondecreasing and differentiable.[11] The dependence of the total output of the economy on the strength of the state captures the various efficiency-enhancing roles of state capacity. In addition, we allow for output to depend on the strength of civil society as well, which might be because a strong civil society prevents extractive uses of the capacity of

[10] Note that when we consider the limit $\Delta \to 0$, we obtain

$$\begin{aligned} C_x(\dot{x}_t) &= c_x(\dot{x}_t + \delta) + \max\{\gamma_x - x_{t-\Delta}, 0\}\dot{x}_t, \\ C_s(\dot{s}_t) &= c_s(\dot{s}_t + \delta) + \max\{\gamma_s - s_{t-\Delta}, 0\}\dot{s}_t. \end{aligned}$$

[11] The fact that (3) refers to output during the lifetime of each generation means that each generation will produce this quantity regardless of $\Delta > 0$. As we show more explicitly in footnote 12, this feature is important to ensure that the incentives for investment do not vanish when we consider short-lived players as in the next section and $\Delta \to 0$. (When we return to long-lived, forward-looking players, incentives for investment will not vanish and similar results apply as $\Delta \to 0$ even if (3) gives output during a period of length Δ; see footnote 15).

the state that tend to reduce the total output or surplus in the economy, or because its greater cooperation and coordination improves economic efficiency.

We next discuss how the output of society is distributed between the elite (controlling the state) and citizens. At date t, if the elite and civil society (citizens) decide to fight, then one side will win and capture all of the output of the economy (normalized to 1), and the other side receives zero. Winning probabilities are functions of relative strengths. In particular, the elite will win if

$$s_t \geq x_t + \sigma_t,$$

where σ_t is drawn from the distribution H independently of all past events. We denote the density of the distribution function H by h. The existence of the random term σ_t captures the fact that various stochastic factors impact the outcome of any conflict. Throughout, since both sides have the same assessment of the outcome of conflict, we will presume that they divide total output according to their expected shares, but whether they do so or actually engage in conflict is immaterial for our results.

This specification of the stochastic contest function implies that when the strengths of civil society and state are given, respectively, by x and s, the probability that the state will win the conflict is $H(s-x)$, and the probability that the civil society will do so is $1-H(s-x) = H(x-s)$, a property we will use frequently below.

In the Appendix, we also show that the most important qualitative features implied by this formulation of conflict between the elite (state) and society are shared by other formulation of the contest between these parties.

2.2 Assumptions

We next introduce three assumptions we will sometimes make use of. The first one is a simplifying assumption, which we impose initially and then relax subsequently:

Assumption 1 $f(x, s) = 1$ for all $x \in [0, 1]$ and $s \in [0, 1]$.

This assumption makes it transparent that the multiple steady-state equilibria and their dynamics, our main focus, are driven by the dynamic contest between the state and civil society, not because of changes in the value of the prize in this contest. It will be relaxed in Section 5.

The next two assumptions are imposed throughout.

Assumption 2 1. c_x and c_s are continuously differentiable, strictly increasing and weakly convex over $\mathbb{R}_+$, and satisfy $\lim_{x\to\infty} c_x(x) = \infty$ and $\lim_{x\to\infty} c_s(s) = \infty$.

2.

$$c_s'(\delta) \neq c_x'(\delta).$$

3.

$$\frac{|c_s''(\delta) - c_x''(\delta)|}{\min\{c_x''(\delta), c_s''(\delta)\}} < \frac{1}{\sup_z |h'(z)|}.$$

4.

$$c_s'(\delta) + \gamma_s \geq c_x'(\delta) \text{ and } c_x'(\delta) + \gamma_x > c_s'(\delta).$$

Part 1 of Assumption 2 is standard. Part 2 is imposed for simplicity and rules out the non-generic case where the marginal cost of investment at δ is exactly equal for the two parties. Part 3 is also imposed for technical convenience, and is quite weak. For example, if the gap between $c_x''(\delta)$ and $c_s''(\delta)$ is small, this condition is automatically satisfied. We will flag its role when we come to our analysis, but anticipating that discussion, it makes it much easier for us to establish the instability of some "uninteresting" steady state equilibria. Part 4 ensures that the marginal cost of each player in the increasing returns region (when $x < \gamma_x$ and $s < \gamma_s$) is greater than the marginal cost of the other player outside this region when both marginal costs are evaluated at δ — the marginal cost is evaluated at δ since, as our above transformation showed, the level of investment necessary for maintaining the steady state is δ. We will flag the role of this assumption when we come to our formal analysis.

Assumption 3 1. h exists everywhere, and is differentiable, single-peaked and symmetric around zero.

2. For each $z \in \{x, s\}$,

$$c_z'(\delta) > h(1).$$

3. For each $z \in \{x, s\}$,

$$\min\{h(0) - \gamma_z; h(\gamma_z)\} > c_z'(\delta).$$

Part 1 contains the second key assumption for our analysis — single peakedness of h around 0 (the rest of this assumption, symmetry and differentiability, is standard). This assumption not only simplifies our analysis as it ensures that $h(x-s) = h(s-x)$ and $h'(x-s) = -h'(s-x)$, but also implies that incentives for investment are strongest when x and s are close to each other. We highlight the role of this feature below as well.

Part 2 imposes that when a player has the maximum gap between itself and the other player, then it has no further incentives to invest. Part 3, on the other hand, ensures that at or near the point where conflict capacities are equal, there are sufficient incentives to increase conflict

capacity. Both of these assumptions restrict attention to the part of the parameter space of greater interest to us.

3 Equilibrium with Short-Lived Players

We now present our main results about the dynamics of the power of state and civil society, focusing on the non-overlapping generations setup, where at each point in time, each side of the conflict is represented by a single short-lived agent who will be replaced by a new agent from the same side next period. This ensures that when players take decisions today they will not internalize their impact on the future evolution of the power of either party.

3.1 Preliminaries

Suppose that the above-described society is populated by non-overlapping generations of agents — on the one side representing the elite (state) and on the other, civil society.

With these assumptions, at each time t, civil society maximizes

$$H(x_t - s_t) - \Delta \cdot C_x(x_t, x_{t-\Delta})$$

by choosing x_t (or equivalently i_t^x), taking $x_{t-\Delta}$ as given. Simultaneously, the elite maximize

$$H(s_t - x_t) - \Delta \cdot C_s(s_t, s_{t-\Delta})$$

by choosing s_t, taking $s_{t-\Delta}$ as given. A dynamic (Nash) equilibrium with short-lived players is given by a sequence $\{x^*_{k\Delta}, s^*_{k\Delta}\}_{k=0}^{\infty}$ such that $x^*_{k\Delta}$ is a best response to $s^*_{k\Delta}$ given $x^*_{(k-1)\Delta}$, and likewise $s^*_{k\Delta}$ is a best response to $x^*_{k\Delta}$ given $s^*_{(k-1)\Delta}$.

Given Assumptions 2 and 3, the investment decisions of both elites and civil society are given by their respective first-order conditions (with complementary slackness). In particular, at time t, we have:[12]

$$\begin{array}{ll} h(x_t - s_t) \leq c'_x(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } \frac{x_t - x_{t-\Delta}}{\Delta} = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t) \geq c'_x(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } x_t = 1, \\ h(x_t - s_t) = c'_x(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_x - x_t\} & \text{otherwise,} \end{array}$$

and

$$\begin{array}{ll} h(s_t - x_t) \leq c'_s(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{if } \frac{s_t - s_{t-\Delta}}{\Delta} = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t) \geq c'_s(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{if } s_t = 1, \\ h(s_t - x_t) = c'_s(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta) + \max\{0; \gamma_s - s_{t-\Delta}\} & \text{otherwise.} \end{array}$$

[12] Following up on footnote 11, we can more clearly see the role that Δ in front of the cost function plays here: without this term (or equivalently if the return was also multiplied by Δ), the marginal cost of investment would be multiplied by $1/\Delta$, and thus as $\Delta \to 0$, investments would converge to zero. This is because short-lived players that are not forward-looking do not take the impact of their instantaneous investments on future stocks (and have infinitesimal impact on the current stock).

The first line of either expression applies when the relevant player has chosen zero investment so that its state variable shrinks as fast as it can (at the rate δ), or is already at its lower bound $x_t = 0$ or $s_t = 0$. In this case, we have the additional cost of investment on the right-hand side, and also the optimality condition given by a weak inequality, since at this lower boundary, the marginal benefit could be strictly less than the marginal cost of investment. The second line, on the other hand, applies when the state variable takes its maximum value, 1, and in this case the marginal benefit could be strictly greater than the marginal cost of investment. Away from these boundaries, the third line applies, and requires that the marginal benefit equal the marginal cost. Note also that, the marginal benefit for civil society is the same as the marginal benefit for the state — since $h(s_t - x_t) = h(x_t - s_t)$. On the other hand, we also have from Assumption 3 that changes in the marginal benefits of the two players are the converses of each other — that is, $h'(s_t - x_t) = -h'(x_t - s_t)$.

Now letting $\Delta \to 0$, we obtain the following continuous-time first-order optimality (and thus equilibrium) conditions

$$\begin{array}{ll} h(x_t - s_t) \leq c'_x(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } \dot{x}_t = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t) \geq c'_x(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } x_t = 1, \\ h(x_t - s_t) = c'_x(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{otherwise,} \end{array} \tag{4}$$

and

$$\begin{array}{ll} h(s_t - x_t) \leq c'_s(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } \dot{s}_t = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t) \geq c'_s(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } s_t = 1, \\ h(s_t - x_t) = c'_s(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{otherwise.} \end{array} \tag{5}$$

In what follows, we work directly with these continuous-time first-order optimality conditions. Moreover, it is straightforward to see that in continuous time, away from the boundaries of $[0, 1]^2$ these first-order optimality conditions will hold as equality, and thus the dynamics of state and civil society strength can be represented by the following two differential equations:

$$\begin{aligned} \dot{x} &= (c'_x)^{-1}(h(x - s) - \max\{\gamma_x - x, 0\}) - \delta \\ \dot{s} &= (c'_s)^{-1}(h(s - x) - \max\{\gamma_s - s, 0\}) - \delta. \end{aligned} \tag{6}$$

The roles of the two key assumptions highlighted above, the single-peakedness of h and the increasing returns aspect of the cost function, are evident from (6). First, when x and s are close to each other, $h(x - s)$ is large, and thus both of these variables will tend to grow further. Conversely, when x and s are far apart, $h(x - s)$ is small, and investment by both parties is discouraged. This observation captures the key economic force that will lead to the emergence of different types of state-society relations and states in our setup (in the Appendix, we see that this same property holds with other formulations of the contest function). Secondly, the presence

of the max term implies that once the conflict capacity of a party falls below a critical threshold (γ_x or γ_s), there is an additional force pushing towards further reduction in this capacity.

3.2 Dynamics of the Strength of Civil Society and the State

Our main result in this section is summarized in the next proposition.

Proposition 1 *Suppose Assumptions 1, 2 and 3 hold. Then there are three (locally) asymptotically stable steady states:*

1. $x^* = s^* = 1$.
2. $x^* = 0$ *and* $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ *and* $s^* = 0$.

This proposition thus shows that there exists three relevant (asymptotically stable) steady states, one corresponding to an inclusive state, one corresponding to a despotic state and one to a weak state. The economic intuition, as already anticipated, is that when we are in the neighborhood of the steady state $x^* = s^* = 1$, $h(x - s)$ is large, encouraging both parties to move for further towards $x^* = s^* = 1$. In contrast, in the neighborhood of $x^* = 0$ or $s^* = 0$, $h(x - s)$ is small, and neither party has as strong incentives to invest.[13]

The steady states presented in Proposition 1 and their local dynamics are also depicted in Figure 2. Our analysis so far establishes the dynamics in the neighborhoods of these steady states, and we next turn to a characterization of global dynamics.

3.3 Proof of Proposition 1

We start with a series of lemmas on the steady-state equilibria of this model, and their stability properties. Before presenting these results, we remark that there can be three types of steady states: (i) those in which the party in question (say the civil society) chooses a positive level conflict capacity, and thus we will have $x_t^* = x^* \in (0, 1)$, so that the marginal cost of investment is simply $c_x'(\delta) + \max\{\gamma_x - x^*, 0\}$, which is equal to the benefit from this conflict capacity; (ii) those in which we have zero conflict capacity, in which case marginal cost of investment, $c_x'(\delta) + \gamma_x$, is greater than or equal to the benefit from building further conflict capacity; (iii) those in which the party in question has conflict capacity equal to 1, in which case marginal

[13] Under Assumption 1, there is no benefit in reaching the $x^* = s^* = 1$, since the capacities of the state and society do not contribute to the size of the social surplus. This will be relaxed below.

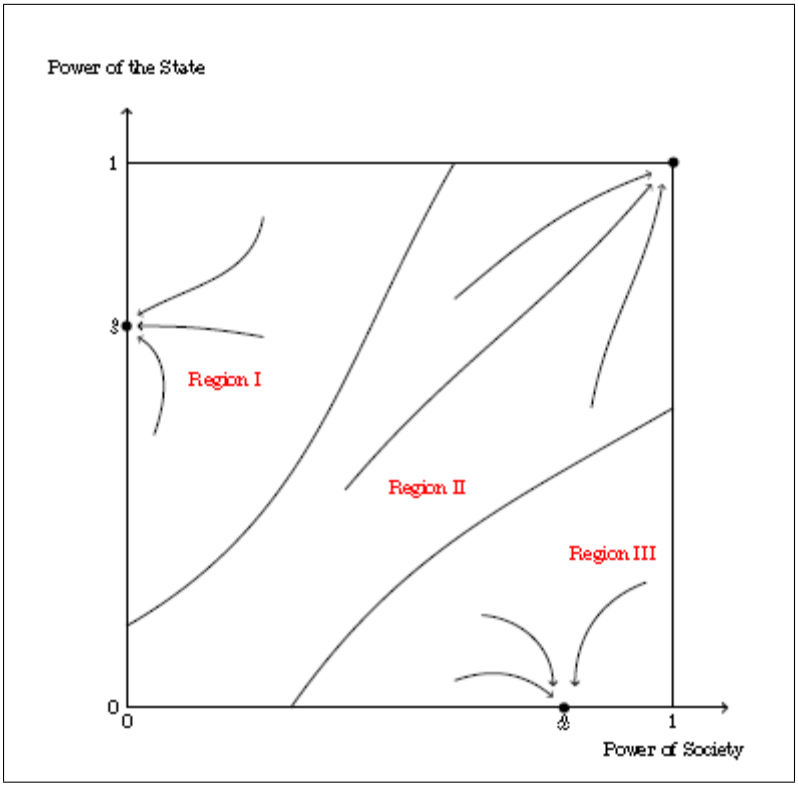

Figure 2: Steady states and their local dynamics.

cost of investment, $c'_x(\delta)$, is less than or equal to the benefit from building additional conflict capacity.

Lemma 1 *There exists a (locally) asymptotically stable steady state with $x^* = s^* = 1$.*

Proof. At $x^* = s^* = 1$, the marginal cost of investment for player $z \in \{x, s\}$ is $c'_z(\delta)$, while the marginal benefit starting from this point is $h(0)$, so Assumption 3 ensures that the marginal benefit strictly exceeds the marginal cost, and neither player has an incentive to reduce its investment, and because 1 is the maximum level of investment, neither party has the ability to increase it.

We turn next to asymptotic stability of this steady state. First note that the laws of motion of x and s in the neighborhood of the state state $(x^* = 1, s^* = 1)$ are given by

$$\begin{aligned} c'_x(\dot{x} + \delta) &= h(x - s) \\ c'_s(\dot{s} + \delta) &= h(s - x), \end{aligned}$$

where we are exploiting the fact that once we are away from the steady state, there cannot be an immediate jump and thus the first-order conditions have to hold in view of Assumption 2.

We have also used the information that we are in the neighborhood of the steady state $(1,1)$ in writing the system for $x > \gamma_x$ and $s > \gamma_s$. The dynamical system in (6) then simplifies to

$$\begin{aligned} \dot{x} &= (c_x')^{-1}(h(x-s)) - \delta \\ \dot{s} &= (c_s')^{-1}(h(s-x)) - \delta. \end{aligned} \tag{7}$$

Now to establish asymptotic stability, we will show that

$$L(x,s) = \frac{1}{2}(1-x)^2 + \frac{1}{2}(1-s)^2$$

is a Lyapunov function in the neighborhood of the steady state $(1,1)$. Indeed, $L(x,s)$ is continuous and differentiable, and has a unique minimum at $(1,1)$. We next verify that in is sufficiently small neighborhood of $(1,1)$, $L(x,s)$ is decreasing along solution trajectories of the dynamical system given by (8). Since L is differentiable, for $x \in (\gamma_x, 1)$ and $s \in (\gamma_s, 1)$, we can write

$$\frac{dL(x,s)}{dt} = -(1-x)\dot{x} - (1-s)\dot{s}.$$

First note that since $h(x-s) > c_x'(\delta)$ and $h(s-x) > c_s'(\delta)$ for x and s in a sufficiently small neighborhood of $(1,1)$, we have both $\dot{x} > 0$ and $\dot{s} > 0$. This implies that, in this range, both terms in $\frac{dL(x,s)}{dt}$ are negative, and thus $\frac{dL(x,s)}{dt} < 0$. Moreover, the same conclusion applies when $x = 1$ (respectively when $s = 1$), with the only modification that $\frac{dL(x,s)}{dt}$ will not only have the $\dot{s}$ (respectively the $\dot{x}$) term, which continues to be strictly negative. Then the asymptotic stability of $(1,1)$ follows from LaSalle's Theorem (which takes care of the fact that our steady state is on the boundary of the domain of the dynamical system in question, see, e.g., Walter, 1998). ■

This lemma shows that under our maintained assumptions, both parties investing at their maximum conflict capacity is a steady-state equilibrium. Intuitively, this proposition exploits the fact that when the two players are "neck and neck," they both have strong incentives to invest. If instead we had, say, x much larger than s, then from part 1 of Assumption 3, both $h(x-s)$ and $h(s-x)$ would be smaller than $h(0)$, reducing the investment incentives of both parties. The stronger investment incentives around $x^* = s^* = 1$ are key for maintaining this combination as a steady state — combined with part 2 of Assumption 3, which ensures that these strong incentives are sufficient to guarantee a corner solution. If this inequality did not hold, $x^* = s^* = 1$ could not be a steady-state equilibrium, and in this case, the only possible steady-state equilibria would be those identified in Lemma 2 below.

The local stability of this steady state is then established by constructing a Lyapunov function. The use of this method is necessitated by the fact that $x^* = s^* = 1$ is at the corner, and thus dynamics around it cannot be characterized by using linearization methods.

Our next result identifies two additional locally asymptotically stable steady states.

Lemma 2 *There exist two additional (locally) asymptotically stable steady states:*

1. *one with $x^* = 0$ and $s^* \in (\gamma_s, 1)$, and*

2. *one with $s^* = 0$ and $x^* \in (\gamma_x, 1)$.*

Proof. We start with the first statement. Suppose first that $x^* = 0$. Then from (5) an interior steady-state level of investment for the state requires

$$h(s) = c_s'(\delta) + \max\{0; \gamma_s - s\}.$$

Note that Assumption 3 implies that at $s = 1$, $h(1) < c_s'(\delta)$, and at $s = \gamma_s$, $h(\gamma_s) > c_s'(\delta)$, thus by the intermediate value theorem, there exists s^* between γ_s and 1 satisfying

$$\begin{aligned} h(s^*) &= c_s'(\delta) \\ h(-s^*) &\leq c_x'(\delta) + \gamma_x. \end{aligned}$$

Moreover, because h is single peaked and symmetric around 0, $h(s)$ is decreasing in $s \geq$, and thus only a unique s^* satisfying this relationship exists.

We next verify that $x^* = 0$ is indeed consistent with the optimization of civil society. This follows immediately since

$$h(-s^*) = h(s^*) = c_s'(\delta) < c_x'(\delta) < c_x'(\delta) + \gamma_x,$$

where the first equality follows from the symmetry of h, the second one simply replicates the first condition, and the inequality follows from Assumption 2. This implies that the second condition for a steady state also holds, and in fact holds as a strict inequality.

The local stability is established using a Lyapunov argument as in the proof of Lemma 1. With a similar argument, the laws of motion of x and s in the neighborhood of the state state $(x = 0, s = s^*)$ are given by

$$\begin{aligned} c_x'(\dot{x} + \delta) &= h(x - s) + \gamma_x - x \\ c_s'(\dot{s} + \delta) &= h(s - x), \end{aligned}$$

where we are now using the fact that we are in the neighborhood of $(0, s^*)$ so that $x < \gamma_x$ and $s > \gamma_s$. The dynamical system in (6) in this case can be written as

$$\begin{aligned} \dot{x} &= (c_x')^{-1}(h(x - s) + \gamma_x - x) - \delta \\ \dot{s} &= (c_s')^{-1}(h(s - x)) - \delta. \end{aligned} \tag{8}$$

We now choose the Lyapunov function

$$L(x,s) = \frac{1}{2}x^2 + \frac{1}{2}(s-s^*)^2,$$

which is again continuous and differentiable, and has a unique minimum at $(0, s^*)$. We will next verify that in the neighborhood of $(0, s^*)$, $L(x,s)$ is decreasing along solution trajectories of the dynamical system given by (8). Specifically, since L is differentiable, for $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$, we can write

$$\frac{dL(x,s)}{dt} = x\dot{x} + (s-s^*)\dot{s}.$$

First note that as $h(-s^*) < c_x'(\delta) + \gamma_x$, for x and s in the neighborhood of $(0, s^*)$,

$$\dot{x} = (c_x')^{-1}(h(x-s) + \gamma_x - x) - \delta < 0. \tag{9}$$

Then, using a first-order Taylor expansion of (8) in this neighborhood, we obtain

$$(s-s^*)\dot{s} = \frac{1}{c_s''(\delta)}h'(s^*)(s-s^*)(s-x-s^*) + o(\cdot), \tag{10}$$

where $o(\cdot)$ denotes second-order terms in x and $s - s^*$.

The desired result follows from the following arguments: (i) for $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$, $|x\dot{x}| > |(s-s^*)\dot{s}|$, regardless of the sign of $(s-s^*)\dot{s}$, as $x \to 0$ and $s \to 0$, $(s-s^*)(s-x-s^*)/x \to 0$, because in the neighborhood of the steady state $(0, s^*)$, $\dot{s}$ is of the order $s - s^*$, while $h(-s^*) < c_x'(\delta) + \gamma_x$, ensuring that $\dot{x} < 0$). Therefore, in the range where $x \in (0, \gamma_x)$ and $s \in (0, \gamma_s)$, $\frac{dL(x,s)}{dt} < 0$. (ii) when $x = 0$, (10) implies that $(s-s^*)\dot{s} < 0$ in view of the fact that $h'(s^*) < 0$, and thus we have $\frac{dL(x,s)}{dt} < 0$. (iii) when $s = s^*$, (9) ensures that $\dot{x} < 0$, so that we again have $\frac{dL(x,s)}{dt} < 0$. Then in all three cases, the asymptotic stability of $(0, s^*)$ follows from LaSalle's Theorem (e.g., Walter, 1998).

The proof of the existence, uniqueness and asymptotic stability of the steady state with $s^* = 0$ and $x^* \in (\gamma_x, 1)$ is analogous, and is omitted. ■

These two additional steady states have a very different flavor than the steady state in Lemma 1. Now both parties have a lower level of conflict capacity, and one of them is in fact at zero. The intuition is again related to the incentives for investment in conflict capacity: when one party is at zero capacity, $h(\cdot)$ is small for both players, which encourages the first player to state with low capacity, and discourages the other player from building further capacity.

Assumptions 2 and 3 play an important role in this lemma also. Without the boundary conditions in Assumption 3, there could be other steady states with some of them including investments below γ_x and γ_s. Though these steady states would be locally unstable (with the

same argument as in Lemma 4 below), in this case it also becomes harder to ensure that there exists a locally stable steady state, making us prefer these assumptions.

The next lemma rules out several types of steady states.

Lemma 3 *There is no steady state with (i) $x^* = s^* = 0$; (ii) $x^* = 0$ and $s^* \in (0, \gamma_s)$, or $s^* = 0$ and $x^* \in (0, \gamma_x)$; and (iii) $x^* \in (\gamma_x, 1)$ and $s^* \in (\gamma_s, 1)$.*

Proof. Claim (i) follows immediately, since Assumption 3 we have $h(0) - \gamma_s > c_s'(\delta)$, so that when $x^* = 0$, the elite will deviate from $s = 0$. Claim (ii) follows directly from the proof of Lemma 2. Finally, for claim (iii), note that a steady state with $x^* \in (\gamma_x, 1)$ and $s^* \in (\gamma_s, 1)$ would necessitate

$$\begin{aligned} h(s^* - x^*) &= c_s'(\delta) \\ h(x^* - s^*) &= c_x'(\delta), \end{aligned} \tag{11}$$

but then from the symmetry of the h function around zero, we have that $h(s^* - x^*) = h(x^* - s^*)$, so that

$$c_s'(\delta) = h(s^* - x^*) = c_x'(\delta),$$

which contradicts part 2 of Assumption 2. ■

There are other types of steady states that could exist, but next lemma shows that when they do, they will all be locally asymptotically unstable.

Lemma 4 *All other (possible) steady states are asymptotically unstable.*

Proof. We will prove this lemma by considering three types of steady states, which exhaust all possibilities.

Type 1: $x \in (0, \gamma_x)$ and $s \in (0, \gamma_s)$.

The optimality conditions in such a steady state are

$$\begin{aligned} h(s - x) &= c_s'(\delta) + \gamma_s - s \\ h(x - s) &= c_x'(\delta) + \gamma_x - x. \end{aligned}$$

The dynamical system (6) now becomes

$$\begin{aligned} \dot{x} &= (c_x')^{-1}(h(x - s) + \gamma_x - x) - \delta \\ \dot{s} &= (c_s')^{-1}(h(s - x) + \gamma_s - s) - \delta. \end{aligned}$$

Since the steady-state levels of state and civil society strength are defined by equality conditions in this case, local dynamics can be determined from the linearized system, with characteristic matrix given by

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(s-x)+1] & -\frac{1}{c_s''(\delta)}h'(s-x) \\ -\frac{1}{c_x''(\delta)}[h'(x-s) & \frac{1}{c_x''(\delta)}[h'(x-s)+1] \end{pmatrix}.$$

Using the fact that from Assumption 3, $h'(s-x) = -h'(x-s)$, the determinant of this matrix can be computed as $\frac{1}{c_s''(\delta)c_x''(\delta)} > 0$. Moreover, from part 2 of Assumption 2, we can show that the trace of this matrix is

$$\frac{1}{c_s''(\delta)}[h'(s-x)+1] + \frac{1}{c_x''(\delta)}[h'(x-s)+1].$$

Once again using Assumption 3, this expression is positive provided that

$$h'(s-x)(c_s''(\delta) - c_x''(\delta)) \leq c_x''(\delta) + c_s''(\delta). \tag{12}$$

Assumption 2 ensures that

$$\left|c_s''(\delta) - c_x''(\delta)\right| \leq \frac{c_x''(\delta)}{|h'(s-x)|},$$

which is a sufficient condition for (12), establishing that both eigenvalues are positive, and we have asymptotic instability.

Type 2: $x \in (\gamma_x, 1)$ and $s \in (0, \gamma_s)$, or $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$. Consider the first of these,

$$\begin{aligned} h(s-x) &= c_s'(\delta) + \gamma_s - s \\ h(x-s) &= c_x'(\delta). \end{aligned}$$

Now once again, local dynamics can be determined from the linearized system, with characteristic matrix

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(s-x)+1] & -\frac{1}{c_s''(\delta)}h'(s-x) \\ -\frac{1}{c_x''(\delta)}[h'(x-s) & \frac{1}{c_x''(\delta)}h'(x-s) \end{pmatrix}.$$

The trace of this matrix is

$$\frac{1}{c_s''(\delta)}[h'(s-x)+1] + \frac{1}{c_x''(\delta)}h'(x-s),$$

which is positive provided that

$$h'(s-x)(c_s''(\delta) - c_x''(\delta)) \leq c_x''(\delta).$$

The same argument as in the proof of Type 1 establishes that this condition follows from Assumption 2, and thus both eigenvalues are positive and the steady state in question is asymptotically unstable. The argument for the case where $x \in (0, \gamma_x)$ and $s \in (\gamma_s, 1)$ is analogous.

Type 3: $x = 1$ and $s < 1$ or $s = 1$ and $x < 1$.

Let us prove the first case. Such a steady state would require

$$\begin{aligned} h(1-s) &\geq c'_x(\delta) \\ h(s-1) &= c'_s(\delta) + \gamma_s - s. \end{aligned}$$

Here we have exploited the fact that in this first type of steady state we cannot have $s \geq \gamma_s$, since otherwise we would have $c'_x(\delta) \geq h(1-s) = h(s-1) = c'_s(\delta)$, which contradicts Assumption 2. But then consider a perturbation to $s + \varepsilon_s$ for $\varepsilon_s > 0$ (since to establish instability it is sufficient to do so for a specific perturbation, we consider only the ones that keep x constant). Then the local dynamics of s are given by:

$$\dot{s} = \frac{1}{c''_s(\delta)}[h'(s-1)+1]\varepsilon_s - \delta.$$

From Assumption 3, $h'(s-1) > 0$, the conflict capacity of the state locally diverges from this steady state, establishing asymptotic instability.

Moving next to the second claim, we now need

$$\begin{aligned} h(1-x) &\geq c'_s(\delta) \\ h(x-1) &= c'_x(\delta) + \max\{\gamma_x - x\}. \end{aligned}$$

Note that in this case we cannot rule out the case where $x > \gamma_x$ (since $c'_s(\delta) \leq h(1-x) = h(x-1) = c'_x(\delta)$, which is verified by Assumption 2). This necessitates that we distinguish between $x \leq \gamma_x$ and $x > \gamma_x$. Consider the first one of these. Then an analysis entirely analogous establishes that for a perturbation to $x - \varepsilon_x$ for $\varepsilon_s > 0$:

$$\dot{x} = -\frac{1}{c''_x(\delta)}[h'(x-1)+1]\varepsilon_x - \delta,$$

which implies that x decreases away from the steady state in question, establishing asymptotic instability. Consider finally the second possibility. In this case, for $x + \varepsilon_x$, we have

$$\dot{x} = \frac{1}{c''_x(\delta)}h'(x-1)\varepsilon_x - \delta,$$

which is also locally asymptotically unstable. This completes the proof of the lemma. ■

Proposition 1 then follows straightforwardly by combining these lemmas.

3.4 Global Dynamics

We next partially characterize the global dynamics. In particular, we will determine three regions, as shown in Figure 3, separating the phase diagram into basins of attraction of the

three asymptotically stable steady states characterized in the previous subsection. For example, starting from Region I, equilibrium dynamics converge to the steady state with $x^* = 0$ and $s^* \in (\gamma_s, 1)$; from Region II, convergence is to the steady state with $x^* = s^* = 1$; and from Region III, convergence will be to the steady state with $x^* \in (\gamma_x, 1)$ and $s^* = 0$. Unfortunately, it is not possible to determine the boundaries of these regions analytically, but we will be able to characterize subsets thereof explicitly.

Consider first Region II, which is the basin of attraction of the steady state $x^* = s^* = 1$. Recall that the dynamical system for the behavior of the conflict capacity of civil society and state take the form given in (6) above. We proceed by first noting that any subset of $[0,1]^2$ over which some $L(x,s)$ is a Lyapunov function for the steady state $x^* = s^* = 1$ is part of the basin of attraction of this steady state. Take the same Lyapunov function as in the proof of Lemma 1, $L(x,s) = \frac{1}{2}(1-x)^2 + \frac{1}{2}(1-s)^2$. Whenever x and s are differentiable functions of time, we have

$$\frac{dL(x,s)}{dt} = -(1-x)\dot{x} - (1-s)\dot{s}.$$

Thus any (x,s) combination such that $\dot{x} \geq 0$ and $\dot{s} \geq 0$, with one of them holding as strict inequality, is in the interior of the basin of attraction of this steady state. Let us first define $\bar{x}$ such that $c_x'(\delta) = h(1-\bar{x})$. Clearly, from Assumption 2 $c_s'(\delta) < h(1-\bar{x})$. This ensures that $\mathcal{R}_{II}' = \{(x,s) : x \geq \max\{\gamma_x, \bar{x}\} \text{ and } s \geq \max\{\gamma_s, \bar{x}\}\}$ is in the interior of this basin of attraction; this follows since in $\mathcal{R}_{II}'$, we have both players in the region without increasing returns and thus their marginal costs are given by $c_x'(\delta)$ and $c_s'(\delta)$ respectively, and consequently, $\dot{x} \geq 0$ and $\dot{s} \geq 0$. This region can be further extended by noting that any combination of (x,s) such that $(c_x')^{-1}(h(s-x) - \max\{\gamma_x - x, 0\}) - \delta \geq 0$ and $(c_s')^{-1}(h(s-x) - \max\{\gamma_s - s, 0\}) - \delta \geq 0$ is also in this region. Let us define $\bar{s}(x)$ such that $h(\bar{s}(x) - x) - \max\{\gamma_s - \bar{s}(x), 0\} - c_s'(\delta) = 0$. Similarly, define $\bar{x}(s)$ such that $h(s - \bar{x}(s)) - \max\{\gamma_x - x, 0\} - c_x'(\delta) = 0$. Both $\bar{s}(x)$ and $\bar{x}(s)$ are upward sloping, and in fact correspond to lines with slope 1 when $s \geq \gamma_s$ and $x \geq \gamma_x$, respectively. Then we have that starting within $\mathcal{R}_{II} = \{(x,s) : s \leq \bar{s}(x) \text{ and } x \leq \bar{x}(s)\}$, we also have $\dot{x} \geq 0$ and $\dot{s} \geq 0$ (and in fact, $\mathcal{R}_{II}' \subset \mathcal{R}_{II}$). This region, as well as $\mathcal{R}_{II}'$, is depicted in Figure 3. The shape of the region is intuitive. In addition to the rectangular area corresponding to $\mathcal{R}_{II}'$, it involves combinations of points where s is large and x is in turn not too small so that $h(s-x)$ is still large enough to encourage $\dot{x} \geq 0$ and $\dot{s} \geq 0$. Conversely, it also includes combinations where x is large and s is not too small relative to that to again ensure $\dot{x} \geq 0$ and $\dot{s} \geq 0$. It also includes combinations of x and s that are very close to each other, extending into the region where there are increasing returns for both players (and thus their marginal costs are $c_s'(\delta) + \gamma_s - s$ and

$c'_x(\delta) + \gamma_x - x)$.

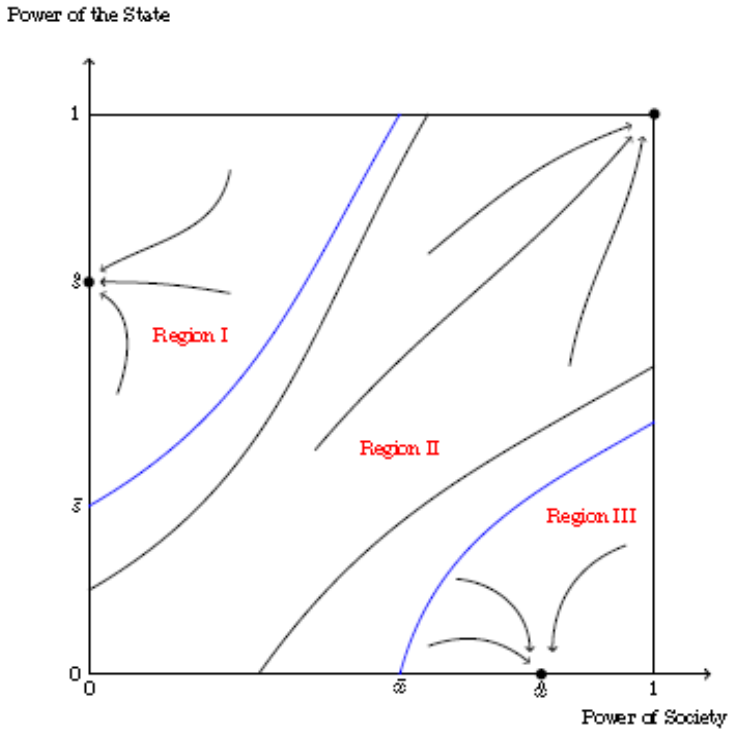

Figure 3: Global dynamics.

Consider next the asymptotically stable steady state $x = 0$ and $s^* \in (\gamma_s, 1)$. Recall that when $x \leq \gamma_x$, we have

$$\dot{x} = (c'_x)^{-1}(h(x - s) - \gamma_x + x) - \delta.$$

Define $\underline{x}'(s)$ such that $h(\underline{x}'(s) - s) = c'_x(\delta) + \gamma_x - \underline{x}'(s)$ and $\underline{x}'(s) < s$ (this last requirement ensures that of the two solutions for $\underline{x}'(s)$, we pick the right one). Thus for any $(x, s) \in \mathcal{R}_I = \{(x, s) : x \leq \underline{x}'(s) \text{ and } x \leq \gamma_x\}$, we have $\dot{x} < 0$. This follows since when $x \leq s$, a further decrease in x reduces $h(x - s)$, inducing $\dot{x} < 0$. Since from Lemma 4 in this region there are no other steady states with a negative eigenvalue, x cannot approach any other rest point and must reach 0. Note also that when $x = 0$,

$$\dot{s} = (c'_s)^{-1}(h(s) - \max\{\gamma_s - s, 0\}) - \delta$$

is negative whenever $s > s^*$ and is positive whenever $s < s^*$, ensuring that starting anywhere in $\mathcal{R}_I$, there will be convergence to $(0, s^*)$, establishing that $\mathcal{R}_I$ is contained in the basin of attraction of this steady state.[14] Finally, the analysis of the dynamics of the asymptotically stable steady state with $x^* \in (\gamma_x, 1)$ and $s = 0$ are similar and are also depicted in Figure 3.

[14] An alternative, equivalent proof is to consider the Lyapunov function $L(x, s) = x$, which by the same con-

We also verify numerically that dynamics take the form shown in Figures 2 and 3. In Figure 4, we depict the vector field for a specific parameterization of the model where we take the cost functions of the state and civil society to be

$$0.342 \times i + 0.336 \times i^2,$$

and in addition set the values of parameters as $\gamma_x = 0.3$, $\gamma_s = 0.6$, and $\delta = 0.05$. The figure verifies the qualitative characterization provided so far.

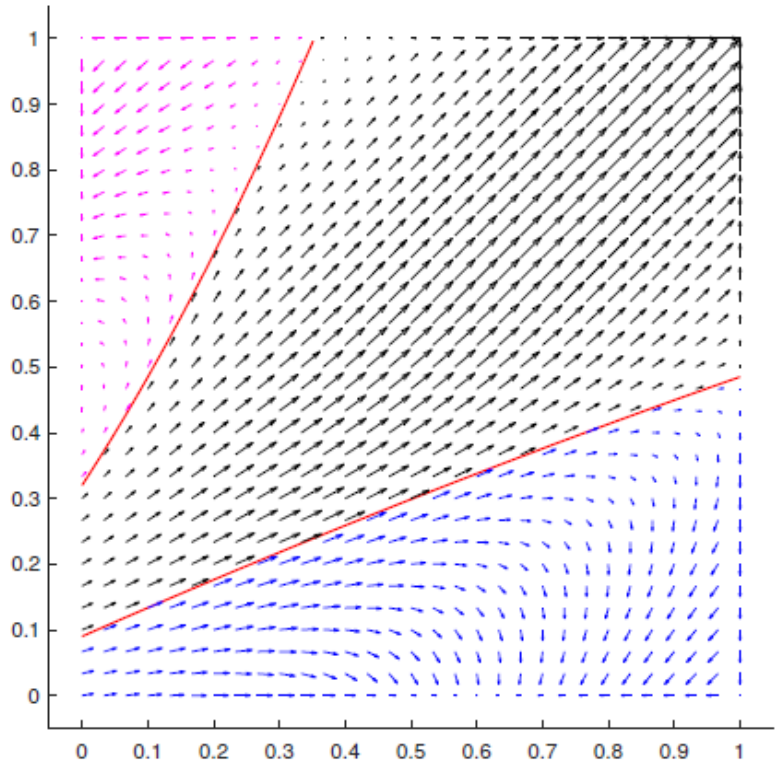

Figure 4: The direction of change of the power of state and society in a simulated example.

3.5 The "Conditional" Effects of Changes in Initial Conditions

Though we will discuss comparative statics (or "comparative dynamics") in greater detail in Section 5, here we undertake a simple exercise: change the initial conditions and trace the effects of these on equilibrium dynamics. The immediate but important conclusion is that the

struction satisfies

$$\frac{dL(x,s)}{dt} = \dot{x} < 0$$

on $\mathcal{R}_I$, and thus again by LaSalle's Theorem converges to the largest invariant subset of the set where $L(x,s) = 0$, which is the steady state $(0, s^*)$.

same change in initial conditions, starting from different parts of the state space, can have drastically different implications.

Consider an increase in s_0 to $s_0 + \bar{s}$. This can leave us in the same region as before, in which case the equilibrium trajectory will be shifted uniformly up, but the long-run outcome will remain unchanged. Alternatively, this increase can shift us from, say, Region III to Region II, in which case not only the equilibrium trajectory but also the long-run outcome will change, and in fact it will involve greater state capacity. However, depending on the exact value of (x_0, s_0), the same increase of $\bar{s}$ could also shift us from Region III to Region I, in which case the impact on the long-run state capacity will be negative instead of positive. This illustrates, and also provides a simple proof, that the effects of changes in initial conditions in this model are *conditional* — they depend exactly where we start.

This discussion thus establishes:

Proposition 2 *The effects of changes in the initial conditions (x_0, s_0) on equilibrium dynamics and the long-run outcome of the economy are conditional in the sense that these depend on which region we move out of and into.*

4 Equilibrium with Forward-Looking Players

In this section, we analyze our general framework with long-lived, forward-looking players. After briefly describing preferences, we first show that for high rates of discounting, equilibrium behavior converges to behavior with short-lived players, which we characterized in the previous section. We then numerically study the dynamics of the equilibrium for a range of discount rates. In this section, we continue to impose Assumption 1, and then relax it in the next section.

4.1 Preferences

We start with the discrete time model. The technology of investment and conflict are the same as in our general framework. The only difference is that now both civil society and state are long-lived and forward-looking. To maximize the parallel with the model with short-lived players, we assume that both players again correspond to sequences of non-overlapping generations, but each generation has an exponentially-distributed lifetime or equivalently, a Poisson end date with parameter $\beta = e^{-\rho\Delta}$. We assume that this random end date is the only source of discounting. Clearly, this specification guarantees that as the period length Δ shrinks, discounting between periods will also decline (and the discount factor will approach 1). Again to maximize the parallel with our static model, we also assume that in expectation, there is one instance of

conflict between the two players during the lifetime of each generation. Since with this Poisson specification, the expected lifetime of his generation is $1/(1-\beta)$, this implies that a conflict arrives at the rate $1-\beta$.[15]

4.2 Main Result

The main result we prove in this forward-looking model is provided in the next proposition.

Proposition 3 *Suppose Assumptions 1, 2 and 3 hold. Then there exist discount rate $\bar{\rho} \geq \underline{\rho} > 0$ such that for all $\rho > \bar{\rho}$, there are three (locally) asymptotically stable steady states:*

1. $x^* = s^* = 1$.
2. $x^* = 0$ *and* $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ *and* $s^* = 0$.

Moreover, for all $\rho < \underline{\rho}$, there exists a unique globally stable steady state $x^* = s^* = 1$.

This result thus shows that the main insights from our analysis apply provided that players, though forward-looking, are sufficiently impatient. We note that this result is not a simple consequence of the fact that as we consider larger and larger values of ρ, players are becoming closer to myopic. It necessitates establishing properties of the the relevant value functions and their derivatives in the limit. In addition, we also observe that for values of ρ between $\underline{\rho}$ and $\bar{\rho}$, one of the two corner steady states disappears while the other one may still exist.

4.3 Proof of Proposition 3

With the specification introduced above, we can straightforwardly represent the maximization problem of each player as a solution to a recursive, dynamic programming problem, written as

$$V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) = \max_{x_t \geq 0} \left\{ (1-\beta)H(x_t - s_t) - \Delta \cdot C_x\left(x_t, x_{t-\Delta}\right) + \beta V_x(x_t, s'^*(x_{t-\Delta}, s_{t-\Delta}; \Delta), \beta; \Delta) \right\}, \tag{13}$$

and

$$V_s(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) = \max_{s_t \geq 0} \left\{ (1-\beta)H(s_t - x_t) - \Delta \cdot C_s\left(s_t, s_{t-\Delta}\right) + \beta V_s(x'^*(x_{t-\Delta}, s_{t-\Delta}), s_t, \beta; \Delta) \right\}. \tag{14}$$

[15] An alternative specification of the model with long-lived players which leads to identical equations, but eschews the parallel with the static model, is to assume that both players are infinitely lived and discount the future at the rate $\beta = e^{-\rho\Delta}$ and there is a conflict during each interval of length Δ. Recall from footnote 12 that in this case there will be no investment when $\Delta \to 0$ with short-lived players (because they do not take into account the benefit from increasing future conflict capabilities), but incentives for investment do not disappear with long-lived, forward-looking players even as $\Delta \to 0$ (because they do take into account the benefit from increasing future conflict capabilities).

Several things are important to note. First, as anticipated in the previous section, we multiply the benefits and costs with Δ, since these are flow benefits, and we have conditioned on Δ in writing the value functions for emphasis. Second, notice that we have already imposed the boundary conditions in the maximization problems. Third, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are the policy functions, which give the next period's values of the state variables as a function of this period's values (and are explicitly conditioned on $\Delta > 0$ two indicate that this is the period length).

A dynamic equilibrium in this setup is given by a pair of policy functions, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ which give the next period's values of the state variables as a function of this period's values (for $\Delta > 0$), and each solves the its corresponding value function taking the policy function of the other party is given. Once these policy functions are determined, the dynamics of civil society and state strength can be obtained by iterating over these functions.

Since these are standard Bellman equations, the following result is immediate (throughout this proof we take $(x,\ s,\ \Delta) \in [0, 1]^3$).

Lemma 5 *For any $\Delta > 0$, $V_x(x, s, \beta; \Delta)$ and $V_s(x, s, \beta; \Delta)$ exist and are continuously differentiable in x, s and Δ. Moreover, $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ and are continuous in x, s and Δ.*

In particular, from (13) and (14), as $\Delta \to 0$, $V_x(x, s, \beta; \Delta) \to V_x(x, s, \beta = 1; \Delta)$ and $V_s(x, s, \beta; \Delta) \to V_s(x, s, \beta = 1; \Delta)$. But since $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are maximizers of the continuous (and bounded) functions, (13) and (14), we can apply Berge's maximum theorem to conclude that $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are also continuous, particularly in Δ, and thus $x'^*(x, s, \beta; \Delta) \to x'^*(x, s, \beta = 1; \Delta)$ and $s'^*(x, s, \beta; \Delta) \to s'^*(x, s, \beta = 1; \Delta)$, and thus for β sufficiently close to 1, we have that $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ are approximately the same as their myopic values. Therefore, there exists $\bar{\beta} < 1$, such that for all $\beta > \bar{\beta}$, a steady state of the dynamical system given by $x'^*(x, s, \beta; \Delta)$ and $s'^*(x, s, \beta; \Delta)$ exists and is locally stable if and only if it is a locally stable steady state of the myopic model.

This argument establishes that the forward-looking, discrete-time dynamics when the discount factor is sufficiently close to 1 will have the same locally stable steady states as the myopic, discrete-time dynamics. In the previous section, we approximated the discrete-time dynamics with their continuous-time limit, and it is also convenient to do the same here, and to maximize the parallel, this is how we have stated the proposition. The next subsection discusses the continuous-time limit and also derives the continuous-time Hamilton-Jacobi-Bellman (HJB) equations, which can be used for characterization of equilibrium more generally.

4.4 Continuous-Time Approximation

For characterizing the equilibrium for any value of the players' impatience, we can once again use the continuous-time approximation by taking the limit $\Delta \to 0$, which shrinks the period length (and correspondingly adjusts the discount factor, so that the discount rate remains constant at ρ). This yields:

Lemma 6 *As $\Delta \to 0$, the value functions $V_x(x, s, \beta; \Delta)$ and $V_s(x, s, \beta; \Delta)$ converge to their continuous-time limits $V_x(x, s)$ and $V_s(x, s)$, and the policy functions $x'^*_\Delta(x, s)$ and $s'^*_\Delta(x, s)$ converge to their continuous-time limits $x'^*(x, s)$ and $s'^*(x, s)$.*

Proof. This follows given the continuous differentiability of $V_x(x, s, \beta; \Delta)$ and $V_s(x, s, \beta; \Delta)$ and of $x'^*_\Delta(x, s)$ and $s'^*_\Delta(x, s)$ for all $\Delta > 0$. ■

The continuous-time Hamilton-Jacobi-Bellman (HJB) equations can be obtained as follows. First rearrange (13) evaluated at the optimal choices and divide both sides by Δ to obtain

$$\begin{aligned} & \frac{1-\beta}{\Delta} V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta) \\ = \quad & \max_{x_t \geq 0} \left[\frac{1-\beta}{\Delta} H(x_t - s_t) - C_x\left(x_t, x_{t-\Delta}\right) + (1-\beta) \frac{V_x(x_t, s^*_\Delta(x_{t-\Delta}, s_{t-\Delta}), \beta; \Delta) - V_x(x_{t-\Delta}, s_{t-\Delta}, \beta; \Delta)}{\Delta} \right]. \end{aligned}$$

Now note that as $\Delta \to 0$, $(1-\beta) \to 0$ and $(1-\beta)/\Delta \to \rho$. Moreover the last term in the previous expression tends to the total derivative of the value function with respect to time. Therefore, the continuous-time HJB equation for civil society is

$$\rho V_x(x, s) = \rho H(x - s) + \max_{\dot{x} \geq -\delta} \left\{ -C_x(x, \dot{x}) + \frac{\partial V_x(x, s)}{\partial x} \dot{x} \right\} + \frac{\partial V_x(x, s)}{\partial s} \dot{s}^*(x, s).$$

where we have used the notation $C_x(x, \dot{x})$ to denote the continuous-time cost function as a function of the change in the conflict capacity of civil society, while $\dot{x}^*(x, s)$ and $\dot{s}^*(x, s)$ designate the continuous-time policy functions, conveniently written in terms of the time derivative of the conflict capacities of the two parties. We have also imposed that $\dot{x}$ cannot be less than $-\delta$.

Applying the same argument to (14) and denoting the continues-time cost function for the state by $C_s(s, \dot{s})$, we also obtain

$$\rho V_s(x, s) = \rho H(x - s) + \max_{\dot{s} \geq -\delta} \left\{ -C_s(s, \dot{s}) + \frac{\partial V_s(x, s)}{\partial s} \dot{s} \right\} + \frac{\partial V_s(x, s)}{\partial x} \dot{x}^*(x, s),$$

The first-order optimality conditions for civil society are given by

$$\begin{aligned} \frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} &= \frac{\partial V_x(x, s)}{\partial x} && \text{if} \quad -\delta < \dot{x}(x, s), \text{ and } x \in (0, 1), \\ \frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} &\leq \frac{\partial V_x(x, s)}{\partial x} && \text{if} \quad x = 1, \\ \frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} &\geq \frac{\partial V_x(x, s)}{\partial x} && \text{if} \quad \dot{x}(x, s) = -\delta \text{ or } x = 0. \end{aligned} \tag{15}$$

In the first case, when we have an interior solution, we can also write

$$\dot{x} = \begin{cases} (c_x')^{-1}\left(\frac{\partial V_x(x,s)}{\partial x} - \gamma_x + x\right) & \text{if } x \leq \gamma_x \\ (c_x')^{-1}\left(\frac{\partial V_x(x,s)}{\partial x}\right) & \text{if } x > \gamma_x \end{cases}. \tag{16}$$

The first-order conditions for state are also similar, and for interior solution, they yield

$$\dot{s} = \begin{cases} (c_s')^{-1}\left(\frac{\partial V_s(x,s)}{\partial s} - \gamma_s + s\right) & \text{if } s \leq \gamma_s \\ (c_s')^{-1}\left(\frac{\partial V_s(x,s)}{\partial s}\right) & \text{if } s > \gamma_s \end{cases}. \tag{17}$$

4.5 Numerical Results

We next provide a numerical characterization of the dynamics in the forward-looking model. We use the same formulation of the cost function and parameter values as above. The critical threshold for ρ is computed as $\bar{\rho} = 500$, and for discount rates above this value, the vector field is identical to that shown in Figure 4, confirming that for high discount rates the equilibrium dynamics of the model with forward-looking agents coincides with the equilibrium of the model with myopic agents as claimed in Proposition 1.

5 General Characterization

In this section, we relax Assumption 1. Since we have established the equivalence of the myopic and forward-looking models when the discount rate is sufficiently large in the latter (which is a result that does not depend in any way on Assumption 1), here we focus on a model with forward-looking players. We also simplify the analysis throughout by assuming that f is linear as specified in the next assumption, which replaces Assumption 1.

5.1 Modified Assumptions

Assumption 1′ $f(x,s) = \phi_0 + \phi_x x + \phi_s s$, where $\phi_0 > 0$, $\phi_x > 0$ and $\phi_s > 0$.

Our other two assumptions also require some minor modifications, which are provided next.

Assumption 2′

1. c_x and c_s are continuously differentiable, strictly increasing and weakly convex over $[0, 1+\delta]$, and satisfy $\lim_{x\to\infty} c_x(x) = \infty$ and $\lim_{x\to\infty} c_s(s) = \infty$.
2. $$c_s'(\delta) \neq c_x'(\delta).$$

$$\frac{|c_s''(\delta) - c_x''(\delta)|}{\min\{c_x''(\delta), c_s''(\delta)\}} < \inf_z \frac{2h(z)(\phi_s + \phi_x)}{|h'(z)|\,(\phi_0 + \phi_s + \phi_x)}.$$

3.

$$c_s'(\delta) + \gamma_s \geq c_x'(\delta) \text{ and } c_x'(\delta) + \gamma_x > c_s'(\delta).$$

The minor modifications in parts 3 and 4 are in view of the fact that marginal benefits of investment are different between the state and civil society. For same reason, we also modify Assumption 3 as follows.

Assumption 3′ 1. h exists everywhere, and is differentiable, single-peaked and symmetric around zero.

2. For each $z \in \{x, s\}$,

$$c_z'(\delta) > h(1)(\phi_0 + \phi_z) + H(1)\phi_z.$$

3. For each $z \in \{x, s\}$,

$$\min\{h(0)\phi_0 + H(0)\phi_z - \gamma_z; h(\gamma_z)(\phi_0 + \phi_z\gamma_z) + H(\gamma_z)\phi_z\} > c_z'(\delta).$$

Under these assumptions, the first-order optimality conditions with short-live players (in continuous time) are modified in the following straightforward fashion:

$$\begin{array}{ll} h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x \leq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } \dot{x}_t = -\delta \text{ or } x_t = 0, \\ h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x \geq c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{if } x_t = 1, \\ h(x_t - s_t)(\phi_0 + \phi_x x + \phi_s s) + H(x_t - s_t)\phi_x = c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} & \text{otherwise.} \end{array}$$

and

$$\begin{array}{ll} h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s \leq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } \dot{s}_t = -\delta \text{ or } s_t = 0, \\ h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s \geq c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{if } s_t = 1, \\ h(s_t - x_t)(\phi_0 + \phi_x x + \phi_s s) + H(s_t - x_t)\phi_s = c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} & \text{otherwise,} \end{array}$$

5.2 Main Result

We have the following straightforward result.

Proposition 4 *Suppose that Assumptions 1′, 2′ and 3′ hold. Then Propositions 1 and 3 apply.*

Proof. The proof of this proposition follows directly from the proofs of Propositions 1 and 3, with only minor changes to Lemma 4, which we provide next ruling out the stability of three different types of steady states. We again treat each type separately.

Type 1: $x \in (0, \gamma_x)$ and $s \in (0, \gamma_s)$.

The optimality conditions in such a steady state are

$$\begin{aligned} h(s - x)(\phi_0 + \phi_x x + \phi_s) + H(s - x)\phi_s &= c_s'(\delta) + \gamma_s - s \\ h(x - s)(\phi_0 + \phi_x x + \phi_s) + H(x - s)\phi_x &= c_x'(\delta) + \gamma_x - x. \end{aligned}$$

Local dynamics are in turn given by

$$\begin{aligned} h(s-x)(\phi_0+\phi_x x+\phi_s)+H(s-x)\phi_s &= c_s'(\dot{s}+\delta)+\gamma_s-s \\ h(x-s)(\phi_0+\phi_x x+\phi_s)+H(x-s)\phi_x &= c_x'(\dot{x}+\delta)+\gamma_x-x. \end{aligned}$$

Since the steady-state levels of state and civil society strength are defined by equality conditions in this case, local dynamics can be determined from the linearized system, with characteristic matrix given by

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1] & \frac{1}{c_s''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_x-\phi_s)] \\ \frac{1}{c_x''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_s-\phi_x)] & \frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x+1] \end{pmatrix},$$

where we wrote $h(\cdot)$ or $h'(\cdot)$ instead of $h(s-x)$ and $h'(s-x)$ in order to save space (and we will adopt this shorthand whenever we write matrices or long expressions below). From part 2 of Assumption 3′, we can show that the trace of this matrix is positive. In particular, the trace is given by

$$\frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1]+\frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x+1].$$

Using Assumption 3′, this expression is positive if

$$h'(s-x)(c_s''(\delta)-c_x''(\delta))(\phi_0+\phi_s s+\phi_x x)\leq(c_x''(\delta)+c_s''(\delta))(1+2h(s-x)(\phi_s+\phi_x)). \tag{18}$$

Assumption 2′ ensures that

$$\left|c_s''(\delta)-c_x''(\delta)\right|\leq\frac{c_x''(\delta)(1+2h(s-x)(\phi_s+\phi_x))}{|h'(s-x)|\,(\phi_0+\phi_s+\phi_x)},$$

which is a sufficient condition for (18), establishing that at least one of the eigenvalues is positive, and we have asymptotic instability.

Type 2: $x\in(\gamma_x,1)$ and $s\in(0,\gamma_s)$, or $x\in(0,\gamma_x)$ and $s\in(\gamma_s,1)$. Consider the first of these,

$$\begin{aligned} h(s-x)(\phi_0+\phi_x x+\phi_s)+H(s-x)\phi_s &= c_s'(\delta)+\gamma_s-s \\ h(x-s)(\phi_0+\phi_x x+\phi_s)+H(x-s)\phi_x &= c_x'(\delta). \end{aligned}$$

Now once again, local dynamics can be determined from the linearized system, with characteristic matrix

$$\begin{pmatrix} \frac{1}{c_s''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_s+1] & \frac{1}{c_s''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_x-\phi_s)] \\ \frac{1}{c_x''(\delta)}[-h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+h(\cdot)(\phi_s-\phi_x)] & \frac{1}{c_x''(\delta)}[h'(\cdot)(\phi_0+\phi_x x+\phi_s s)+2h(\cdot)\phi_x] \end{pmatrix}.$$

The trace of this matrix can now be computed as

$$\frac{1}{c_s''(\delta)}[h'(s-x)(\phi_0+\phi_x x+\phi_s s)+2h\phi_s+1]$$
$$+\frac{1}{c_x''(\delta)}[h'(x-s)(\phi_0+\phi_x x+\phi_s s)+2h(x-s)\phi_x].$$

which is positive if

$$h'(s-x)(c_s''(\delta)-c_x''(\delta))(\phi_0+\phi_s s+\phi_x x)\leq(c_x''(\delta)+c_s''(\delta))(2h(s-x)(\phi_s+\phi_x))+c_x''(\delta).$$

The same argument as in the proof of Type 1 establishes that this condition follows from Assumption 2', and thus at least one of the eigenvalues is positive and the steady state in question is asymptotically unstable. The argument for the case where $x\in(0,\gamma_x)$ and $s\in(\gamma_s,1)$ is analogous.

Type 3: $x=1$ and $s<1$ or $s=1$ and $x<1$.

Let us prove the first case. Such a steady state would require

$$\begin{aligned} h(1-s)(\phi_0+\phi_x x+\phi_s)+H(x-s)\phi_x &\geq c_x'(\delta) \\ h(s-x)(\phi_0+\phi_x x+\phi_s)+H(s-x)\phi_s &= c_s'(\delta)+\max\{0,\gamma_s-s\}. \end{aligned}$$

We distinguish between $s\leq\gamma_s$ and $s>\gamma_s$. Consider the first one of these. Consider a perturbation to $s+\varepsilon_s$ for $\varepsilon_s>0$ (it is sufficient to consider perturbations that maintain x constant). Then the local dynamics of s are given by:

$$\dot{s}=\frac{1}{c_s''(\delta)}[h'(s-1)(\phi_0+\phi_x+\phi_s s)+2h(s-1)\phi_s+1]\varepsilon_s.$$

From Assumption 3', $h'(s-1)>0$, the conflict capacity of the state locally diverges from this steady state, establishing asymptotic instability. Consider next the second possibility. In this case, for $s+\varepsilon_s$, we have

$$\dot{s}=\frac{1}{c_s''(\delta)}[h'(s-1)(\phi_0+\phi_x+\phi_s s)+2h(s-1)\phi_s]\varepsilon_s,$$

which is also locally asymptotically unstable. The other case is proved identically. ■

5.3 Comparative Statics

In this subsection, we discuss how changes in parameters affect the steady states and the dynamics of equilibrium. We focus on the effects of changes in the parameters ϕ_x, ϕ_s, γ_x and γ_s as well as the cost functions c_x and c_s. The effects of changes in initial conditions are identical to those already discussed in Section 3.5

Assumption 3′ guarantees that $x^* = 1$ and $s^* = 1$ is a steady state. There are also at least two interior steady states. These steady states are one of two types. The first type is given by $x^* = 0$ and any s^* that satisfies the following equation:

$$h(s)(\phi_0 + \phi_s s) + H(s)\phi_s = c_s'(\delta).$$

The second type is given by $s^* = 0$ and any x^* that satisfies the following equation

$$h(x)(\phi_0 + \phi_x x) + H(x)\phi_x = c_x'(\delta).$$

Assumption 3′ guarantees that at least one steady state of each type exists. We impose the following assumption to make sure that only one steady state of each type exist:

Assumption 4 $h(y)(\phi_0 + \phi_z y) + H(y)\phi_z$ is a decreasing function of $y \geq 0$ for $z \in \{s, x\}$.

This assumption is fairly mild. The following two conditions would be sufficient to guarantee it: (i) ϕ_z is small, in which case the fact that, from Assumption 3′, $h(y)$ is decreasing for $y \geq 0$ ensures that this assumption is also satisfied, or that (ii) the elasticity of the h function is greater than 1/2, in which case for any value of ϕ_0, Assumption 4 is satisfied.

Let us focus on the comparative statics of the steady state with $x^* = 0$ and $s^* \in (\gamma_s, 1)$. The other case is identical. s^* solves the following equation:

$$h(s^*)(\phi_0 + \phi_s s^*) + H(s^*)\phi_s = c_s'(\delta). \tag{19}$$

ϕ_x does not directly appear in this equation. Therefore,

$$\frac{\partial s^*}{\partial \phi_x} = 0.$$

Implicitly differentiating with respect to ϕ_0, we get

$$h'(s^*)\frac{\partial s^*}{\partial \phi_0}(\phi_0 + \phi_s s^*) + h(s^*)\left(1 + \phi_s \frac{\partial s^*}{\partial \phi_0}\right) + h(s^*)\phi_s \frac{\partial s^*}{\partial \phi_0} = 0.$$

Therefore,

$$\frac{\partial s^*}{\partial \phi_0} = \frac{-h(s^*)}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} > 0,$$

where the inequality is by Assumption 4. Implicitly differentiating equation (19) with respect to ϕ_s, we get

$$h'(s^*)\frac{\partial s^*}{\partial \phi_s}(\phi_0 + \phi_s s^*) + h(s^*)\left(s^* + \phi_s \frac{\partial s^*}{\partial \phi_s}\right) + h(s^*)\frac{\partial s^*}{\partial \phi_s}\phi_s + H(s^*) = 0.$$

Therefore,

$$\frac{\partial s^*}{\partial \phi_s} = \frac{-h(s^*)s - H(s^*)}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} > 0,$$

where again the inequality is a consequence of Assumption 4. Let us next focus on comparative statics with respect to the cost function. Clearly γ_s, γ_s, and $c_x(\cdot)$ do not affect the solution of equation (19). Therefore,

$$\frac{\partial s^*}{\partial \gamma_s} = \frac{\partial s^*}{\partial \gamma_x} = \nabla_{c_x(\cdot)} s^* = 0.$$

But the marginal cost of increasing capacity affects the location of the steady state. To quantify this effect, let us implicitly differentiate equation (19) with respect to $c_s'(\delta)$:

$$h'(s^*)\frac{\partial s^*}{\partial c_s'(\delta)}(\phi_0 + \phi_s s^*) + h(s^*)\frac{\partial s^*}{\partial c_s'(\delta)} + h(s^*)\frac{\partial s^*}{\partial c_s'(\delta)}\phi_s = 1.$$

Therefore,

$$\frac{\partial s^*}{\partial c_s'(\delta)} = \frac{1}{h'(s^*)(\phi_0 + \phi_s s^*) + 2h(s^*)\phi_s} < 0.$$

Figure 5 illustrates how the steady states and the basins of attraction change when we increase ϕ_s, making the capacity of the state more important for overall output. To draw this figure, we use exactly the same parameterization is in the simulation reported in Figure 4, which corresponds the case in which $\phi_x = \phi_s = 0$ in terms of the model of this section. We then show how the steady states and dynamics are affected when we increase ϕ_s to 0.25. Particularly noteworthy are the shifts in the boundaries between the regions, which show that the same type of conditional comparative statics highlighted in Section 3.5 in response to shifts in initial conditions now apply when we consider changes in parameters such as the sensitivity of aggregate surplus to the capacity of the state.

5.4 Numerical Results

Here we briefly show that the same results as those provided above generalized even when f is concave. Figure 6 shows the dynamics in the case where

$$f(x, s) = 0.5x^{0.8} + 0.5s^{0.8}.$$

We can see that in this case with concave surplus function, the dynamics are very similar to the ones studied in the section where the surplus function is linear.

6 Direct Transitions between Region I and Region III

Figure 2 demonstrates how in our main model, the state space is divided into three regions, and Region II always lies between Regions I and III. However, throughout much of pre-modern history (though notably not in Ancient Greece which we discuss in the next section), we have many examples of societies approximating our Regions I and III, but relatively fewer examples of

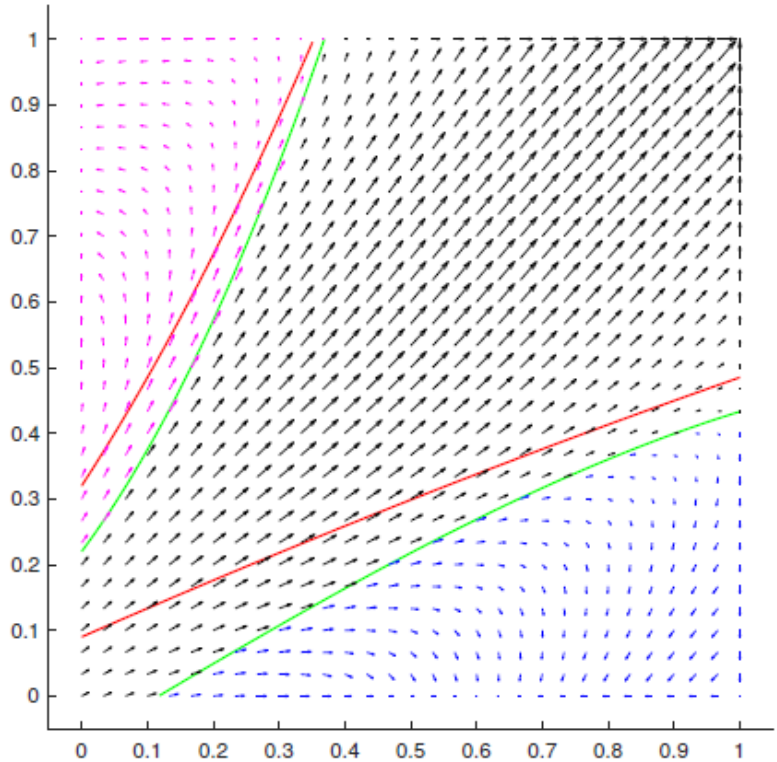

Figure 5: Changes in steady states and dynamics in response to an increase in ϕ_s. The red curves depict the boundaries between the basis of attractions of the different steady states when $\phi_s = 0$ and the green curves show the same boundaries when $\phi_s = 0.25$.

Region II. Perhaps more challengingly for our model, we observe several transitions from Region I directly into Region III, which would not be possible in our baseline model, since Region II is in-between and should be traversed. Here we present a simple modification of the model where Region II shrinks, and creates a subset of the state space (with low levels of state and civil society strength) where Regions I and III are adjacent. The basic idea is to modify the model such that the economies of scale in the cost of investment function becomes dependent on relative strengths.

Suppose that the cost function for the two players take the form

$$C_x(x_t, x_{t-\Delta}) = c\left(\frac{x_t - x_{t-\Delta}}{\Delta} + \delta\right) + [\max\{\gamma - x_{t-\Delta}, 0\} - \max\{\gamma - s_{t-\Delta}, 0\}]\,\frac{x_t - x_{t-\Delta}}{\Delta},$$

and

$$C_s(s_t, s_{t-\Delta}) = c\left(\frac{s_t - s_{t-\Delta}}{\Delta} + \delta\right) + [\max\{\gamma - s_{t-\Delta}, 0\} - \max\{\gamma - x_{t-\Delta}, 0\}]\frac{s_t - s_{t-\Delta}}{\Delta},$$

where we have made two changes relative to our baseline model. First, we have made c and γ the same for the two players, which is just for simplicity's sake. Second and more important, we

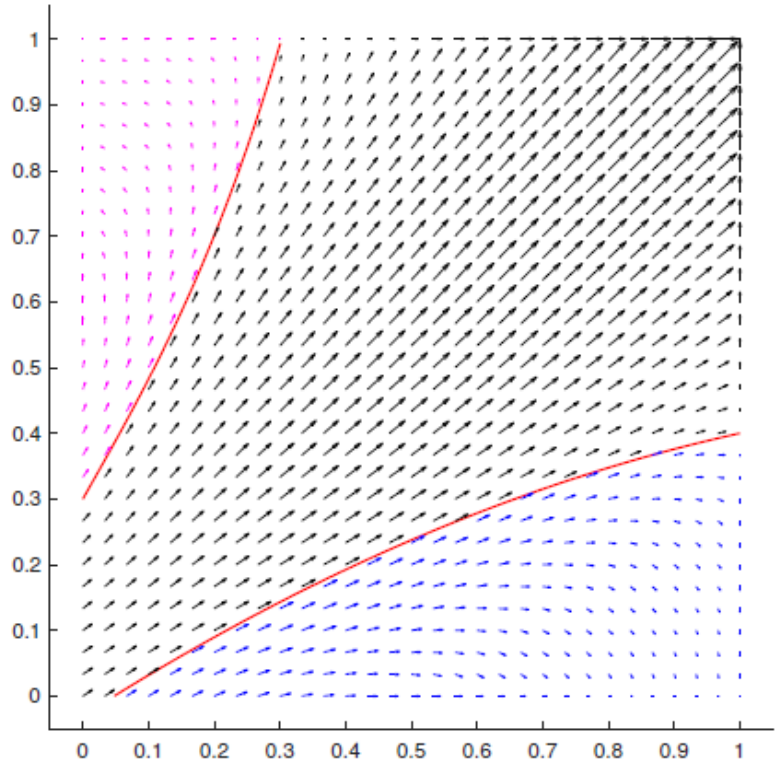

Figure 6: Dynamics when the aggregate output function, $f(x, s)$, is concave.

have changed the formulation of economies of scale in conflict, so that it is the relative strength of the two players that matters. In particular, when both x and s are less than γ, the second term in the cost function becomes simply a function of the gap between x and s. Clearly this leaves the dynamics when $x_t > \gamma$ and $s_t > \gamma$ unchanged. Consider the case in which $x_t < \gamma$ and $s_t < \gamma$. The differential equations for the strength of society and strength can now be written as

$$\begin{aligned} \dot{x} &= (c')^{-1}(h(x-s)+x-s)-\delta \\ \dot{s} &= (c')^{-1}(h(s-x)+s-x)-\delta. \end{aligned}$$

Therefore, defining a new variable $z = x - s$, we have

$$\dot{z} = (c')^{-1}(h(z)+z) - (c')^{-1}(h(z)-z).$$

Or approximating this around $z = 0$, we have

$$\dot{z} = \frac{2z}{c''(\delta)}.$$

Thus regardless on whether $x \gtrless s$, we will have the gap between these two variables grow, with either x or s increasing. Moreover, with x and s sufficiently small, this implies that we

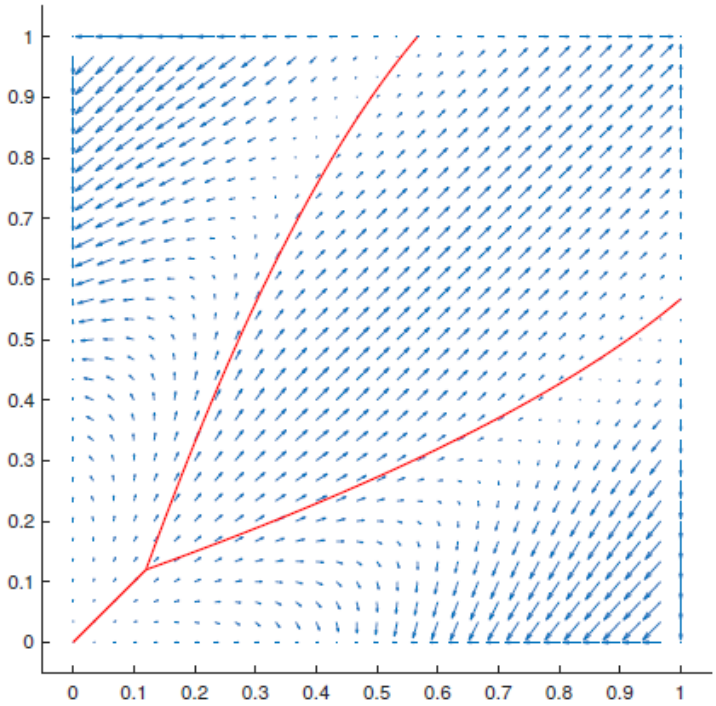

Figure 7: Dynamics with the relative formulation of increasing returns to scale in the cost function for investing in capacity for the state and society. We see that shrinking of Region II and the possibility of direct transitions between Regions I and III.

converge to one of these two variables being zero. Therefore, we can conclude that there exists a neighborhood of (0, 0), such that starting in this neighborhood, Region II is absent, and the economy will go to either of the two steady states in Regions I or III. This is depicted in Figure 7, where we use exactly the same parameterization as in Figure 4, except that we use the cost functions in this section and also set $\gamma_x = \gamma_s = 0.4$. This pattern implies that starting with low values of state and civil society strength, a society that starts with a weak state could transition directly into one on the path to a despotic state. However, when we consider societies with sufficiently developed states and civil societies, transitions from despotic or weak states could take us towards an inclusive state.

7 Weak, Despotic and Inclusive States in Classical Greece

A revealing example of the divergence of state capacity and institutions is that which took place in Classical Greece. Existing evidence suggests that at least in the Greek Dark Ages,[16] there were few differences in the institutions of different Greek societies. In the 'Catalogue of Ships'

[16] The standard periodization of Greek history is the Dark Ages, 1200-750 BC, the Archaic Period, 750-480 BC, and the Classical Age from 480 to 323 BC (Morris, 2010, p. 100).

in the *Iliad*, for example, Homer recounts the forces that united to sail to attack Troy. These involved; Athens; Sparta (Lacedaemonia); and a collection of cities in the Peloponnese also led by Menelaus: Messe, Augeae's, Amyclae and Laas, part of an area known as Mani. Leaders and people gathered from all over mainland Greece to join the hunt for Helen and the subsequent siege of Troy. They shared the same language and religion, thought that the Gods lived on Mount Olympus and that Zeus was their King. Although there was plenty of rivalry between Achilles and Agamemnon, they both, as did all the Greeks, traced their ancestry back to the mythical ancestor Hellen. The Greeks all grew the same crops, the triad of wheat, olives and grapes, and used the same iron agricultural technology. They lived in a land the different parts of which were very similar to each other geographically. Moreover, they had similar military technologies such as the famous hoplite warfare. Homer's poem does not suggest there were significant cultural, ethnic or institutional differences between the warriors from Athens, Sparta of the Mani.[17]

Yet during the Archaic and Classical periods, there was a dramatic divergence in the nature of their states and societies. On the one hand, many Greek city states, epitomized by Athens, developed inclusive states. Others, like Sparta, created despotic states. Still others, like those polities inhabiting the Mani peninsular of the Peloponnese, never developed states at all, and indeed fought tooth and nail against other states and empires extending their influence into the Mani. These outcomes bear a striking resemblance to our three steady states.

7.1 Athens: An Inclusive State

In the century after 600 BC Athens developed a centralized state with a great deal of popular involvement and control. This state was forged in the context of a conflict between elites and citizens as in our model. Aristotle notes that it emerged in the context of "an extended period of discord between the upper classes and the citizens" (*The Constitution of Athens*, II.1), while Plutarch discusses the "long-standing political dispute, with people forming as many different political parties as there were different kinds of terrain in the country. There were the Men of the Hills, who were the most democratic party, the Men of the Plain, who were the most oligarchic, and thirdly, the Men of the Coast, who favored an intermediate, mixed kind of system" (Plutarch, *Solon*, 13).

For Athens, the pivotal period in the creation of a far stronger state is bracketed by the

[17] Though Homer's poem is supposedly describing Bronze Age society, the standard view amongst scholars is that it more accurately represents late Dark Age society with which Homer was familiar. For example, we know Bronze Age polities were quite bureaucratized with writing and 'Palace economies' but this is nowhere mentioned in Homer (see Finley, 1954).

reforms of Solon in 594 BC and those of Cleisthenes in 508/07 BC. Aristotle records that there were eleven constitutions of Athens up to his time (probably around 330 BC), with Solon's being the fourth after the mythical ones of the Ionians and Theseus, and Draco's first written constitution of 621 BC (*The Constitution of Athens*, XLI.2). Solon was made Archon, one of the chief executive positions in Athens, for a year in 594 BC, with a mandate to try and re-configure institutions. Solon himself observed, in a fragment of his writings which is preserved, that his institutional design was intended to create a balance of power between the rich and the poor

> "To the people I gave as much priviledge as was sufficient for them, neither reducing nor exceeding what was their due. Those who had power and were enviable for their wealth I took good care not to injure. I stood casting my strong shield around both parties and allowed neither to triumph unjustly" Solon quoted in Aristotle's *The Constitution of Athens*, XII.1

Solon both increased the strength of the state as well as that of society. From the perspective of the state first in significance were his judicial reforms. Solon abolished all of Draco's laws except one, his homicide law. As Osborne puts it, the homicide law was "not a law about homicide as such, but about the extent of the family which can pursue vendetta, and the ways in which conditions can be made safe for the homicide's return to the community. It is a law which aims to end the situation where any killing leads the perpetrator to a life of wandering exile" (Osborne, 2009, p. 176).[18] Thus Draco's 'constitution' was really a codification of feuding regulations, something more akin to the Albanian *Kanun*, than a real constitution.[19] Solon's laws were very different, and Hall (2013, p.) points out an interesting feature of the homicide law in that it states that guilt is to be judged by a *basileis*, an archaic Homeric word for a ruler which Hall compares to "Big Man" in the ethnographic literature on political institutions. Hall shows how significant it is in terms of state formation that a "feature of the earliest laws is the appearance of named offices and magistracies in place of the generic term *basileis*." Bureaucracy began to emerge at the same time as Solon's laws to implement them and Solon also re-organized the main Athenian judicial institution the *Areopagus*.

Second in importance was the reform of the executive. Athens' political system had previously been oligarchic with the choice of the Archons being obscure and likely determined by powerful families. Solon stipulated that there were to be nine *Archons*. He divided the population into four classes based on their incomes from land and only men from the top two classes

[18] For the text of the law: https://www.atticinscriptions.com/inscription/IGI3/104

[19] Which is why it became notorious for punishing every crime with death, something quite common in codes like the *Kanun*.

could be chosen as *Archons* (chosen by lot from a list of people nominated by the four traditional 'tribes' of Athens). After serving as *Archon*, which they could do only once and for a year, a man could serve in the *Areopagus*.

Solon's laws made society stronger at the same time. First, he restored many citizens their rights. At the time many had been forced into debt peonage since "all loans were made on the security of the person of the debtor until the time of Solon" (*The Constitution of Athens*, II.2). Solon made enserfing an Athenian citizen illegal, eliminating debt peonage, and implemented a land reform by the act of uprooting the boundary markers of fields. Osborne suggests "the boundary markers will .. have recorded ... the obligation to pay a sixth of the produce, and in uprooting them Solon would have been freeing the tenants from landowners, giving them the land they owned, and turning Attica into the land of small farmers which it was in the classical period" (2009, p. 211). He also eliminated restrictions on movement and location within Athens.

Second, Solon increased popular control over the newly strengthened state. A popular Assembly, the *Ekklesia*, pre-dated Solon and he ruled that all Athenian citizens (non-slave, male) could attend this it. Freeing enserfed Athenians was a critical part of making this institution more democratic since as a consequence of serfdom Aristotle noted that "the mass of the people .. had virtually no share in any aspect of government" (*The Constitution of Athens*, II.3) since they lost their citizenship. However, while democratizing the Assembly, the *Archons*, as we noted, were elites. The agenda for the Assembly was drawn up by a Council of 400, the *Boule*, equally representing the four Athenian tribes, whose membership excluded the lowest income class. Though the poorest people were only represented in the Assembly Aristotle observes that "These three seem to be the features of Solon's constitution which most favored the people: first and greatest, forbidding loans on security of a person's body; second, the possibility of a volunteer seeking justice for one who was wronged; third, and they say that this particularly strengthened the people, appeal to the court" (*The Constitution of Athens*, IX.1). Thus he emphasizes that Solon's constitution of courts which guaranteed popular membership of juries and openness to everyone was a key aspect of citizen's control of the state since "when the people have the right to vote in the courts they control the constitution" (*The Constitution of Athens*, IX.1-2).

Third, Solon institutionalized social norms which helped to contain elites. The most important example is the Hubris Law (Ober, 2005, Chapter 5). This forbade any act of hubris, behavior aimed at humiliation and intimidation, against any resident of Attica (the broader region in which Athens lay). Importantly, people could be charged with acting hubristically

towards slaves, who were thus also protected and people were executed for repeated violations of the law.

Solon's reforms made the state and society stronger, but they did not stop the contest. After he left power a series of 'tyrants', such as Peisistratos, staged coups and took power, but who also engaged in state building. Peisistratos took a series of measures to integrate Athens with the countryside in Attica. These included the establishment of rural circuit judges, a system of roads centered on Athens and the introduction of processions linking Athens with rural sanctuaries as well as the Great Panathenaea festival. Peisistratos also coined the first Athenian money.

Ultimately tyranny collapsed and Cleisthenes was brought to power by a mass popular uprising against his opponents and their Spartan backers. The reforms he implemented again had the feature of both strengthening the state and strengthening society. With respect to the state, firstly he developed an elaborate fiscal system (see Ober, 2015b, van Wees, 2013, Fawcett, 2016), which levied a poll tax on metics (resident foreigners), direct taxes of the wealthy who had to pay for festivals or for outfitting warships, a variety of customs tolls and charges, particularly at the port of Piraeus, and taxes on the silver mines of Attica. Second, the state began to provide an array of public goods, not just security or coinage, but infrastructure in the forms of wall, roads and bridges, relief for orphans and the handicapped and prisons. Third, the state became more bureaucratic and was run by various types of functionaries. Aristotle claims that in the days of Aristides, probably 480-470 BC, there were 700 men working for the state in Attica and 700 abroad and in addition 500 guards in the docks and 50 on the Acropolis. The *Boule* had authority over expenditure decisions and there were a series of boards of magistrates (usually 10) which implemented policy. Though these were chosen by lot and served annually, they were aided by professional and state owned slaves.[20] Fourthly, he abolished the four tribes that had provided the people for Solon's *Boule* of 400 and replaced it with a new *Boule* of 500 composed of people chosen by lot from ten new political units, called *demes*, which were regionally based within Attica. There were now no class restrictions on membership. The creation of the regional units in itself was a state building measure which consolidated the initiatives of Peisistratos. Aristotle notes that Cleisthenes "made fellow demesmen of those living in each deme so they would not reveal the new citizen by using a man's father's name, but would use his deme in addressing him" (*The Constitution of Athens*, XXI.4)). Thus Cleisthenes tried to underpin his

[20] The Athenian state at this time had no professional state prosectors or police force. In fact prosecutions for violations of laws had to be brought by private citizens and guilty verdicts had to be collectively enforced (Lanni, 2016). Gottesman (2014) shows that to be implemented, laws passed by the Assembly had to generate a great deal of consensus more broadly in society and be implemented by popular force. 'Popular' included women, slaves and non-citizens. One reason this worked so well was the vibrancy of civil society.

new political institutions with new non-kin based identities.[21]

Cleisthenes then deepened popular control over the state. Membership of the *Boule* was restricted to citizens over the age of 30 and any person could only serve for a year and at most twice in your lifetime. This implies that most Athenian citizens served at some point in their life. The *Boule* president was randomly chosen and served for 24 hours, again giving agenda setting power to any citizen of Athens irrespective of their wealth or social background. As Aristotle puts it "The people had taken control of affairs" (*The Constitution of Athens*, XX.4).

Cleisthenes also formalized the informal institution of 'ostracism'. Every year the Assembly could take a vote as to whether or not to ostracize someone. If at least 6,000 people voted in favor of an ostracism then each citizen got to write the name of a person who they wanted ostracized on a shard of pottery. Whichever name was written on the most shards was ostracized - banished from Athens for 10 years. This law, like Solon's Hubris law, seems to have formalized existing social norms which were used to discipline elites. Indeed, Aristotle notes about the law "it had been passed by a suspicion of those in power" (*The Constitution of Athens*, XXII.3). Even Themistocles, the genius behind the Athenians victory at Salamis over the Persians, and probably the most powerful man in Athens at the time, was ostracized for 10 years sometime around 476 BC. Ostracism was used very sparingly, however, only 15 people were ostracized over the 180 year period when the institution was in full force, but the threat of ostracism "off the equilibrium path" was a powerful way for citizens to discipline elites.

The evolution of the Athenian constitution did not stop with Cleisthenes, this was only the sixth of Aristotle's eleven constitutions, but it moved steadily both towards greater empowerment of citizens and also a stronger state. This did not happen without conflict. During the Persian wars the aristocratic *Aeropagus*, which Cleisthenes had left alone, took upon itself more power. In response reforms by Ephialtes in 462/1 BC stripped it of most of its power. Democracy was overthrown and restored twice during the Peloponnesian wars, but the trend was towards "increasing power being assumed by the people. They have made themselves supreme in all fields; they run everything by decrees of the *Ekklesia* and by decisions of the *dikasteria* (courts) in which the people are supreme" (*The Constitution of Athens*, XL.3).

Apart from the formal political institutions and direct measures to strengthen the power of society relative to elites, there is evidence for social change over time in Athens in the direction of a greater "public sphere" and also greater social capital (Jones, 1999, Kierstead, 2013). Gottesman (2014, p. 50). discusses the emergence of what he calls "mixed associations" which

[21] An aspect of state building first studied by Weber (1976).

became institutionalized after 306 BC when a right of association emerged "for many groups that before could not express their solidarity publicly." An earlier institutional innovation, which occurred between 353 and 330 BC was that of "supplication" whereby people had the right to petition the Assembly and ask for their action on a particular issue. This practice arose earlier but after this time fully one quarter of Assembly meetings were given over to dealing with supplicants. Gottesman (2014, p. 103) surveying the existing inscriptions which resulted from these supplications concludes "they appear to involve only non-citizens" (see also Forsdyke, 2012).

More broadly there is a lot of evidence that state and society evolved together in Athens. The nature of Athenian democracy did not remain fixed after 508 BC, and neither did the state. As late as 337 BC it passed the tyranny law (Teegarden, 2013) to provide citizens with another instrument to fight against despotism. The dynamics of state and society in Classical Athens fit very well into the inclusive state development path in Region II of our model.

7.2 Sparta: A Despotic State

Like Athens, Sparta also had a pivotal moment in the emergence of a strong state, the so-called Great Rhetra initiated by Lycurgus, and probably at around the same time, circa 600 BC (see Finlay, 1982, on the dates). Yet the consolidation of the Spartan state came with a very different relationship with society.

Like Athens's reforms under Solon, the Great Rhetra took place in a contest over power and political institutions. Herodotus says (speaking of the period before the Lycurgian reforms) that Sparta "had the most disorderly state of all the Greeks" (Herodotus, 1.65.2) and Thucydides notes that Sparta was in a "state of civil unrest" (Thucydides, 1.18.1) in the same period. Forsdyke concludes her analysis by noting (2005, p. 292) "It is safest to conclude, therefore, that both conflict within the elite and tensions between elites and non-elites were driving forces in the development of the Spartan political system".

The reforms of Lycurgus that definitively organized Spartan society into three groups. Most important were the Spartan citizens, the *homoioi* (equals), also known as Spartiates. These were adult males over the age of 30 who has gone through the system of *agoge* (meaning leading or guidance), whereby at the age of 7 they were put into collective male age groups and trained to be warriors. During adulthood they were members of particular messes and to maintain ones rights as a Spartan citizen, one had to provide a certain amount of food to the mess. The other classes were the *helots*, now reduced to state owned serfs, and the *perioikoi* who were settled in

villages and made manufactured goods for the Spartiates.[22]

The institutionalization of this class structure went along with a land reform which divided the land, particularly that of Messenia, and the *helots* with it, between Spartan citizens, though the *helots* were the property of the state and could not be bought or sold by individual citizens. The produce of these lands in what Spartans used to provide for their mess.

The state was ruled by the two hereditary kings of the Agiad and the Eurypontid families, who had religious, judicial, and military roles. They communicated with the oracle at Delphi, presided over various types of legal cases and led the army into battle. The democratic element of the constitution was the council of five *ephors*, elected democratically by the citizens. A ephor could serve for one year and only once in a lifetime. The *ephors* monitored the kings and could depose them for misconduct. There was another council of 28 elders over the age of 60 plus the two kings known as the *Gerousia*. Members of the *Gerousia* were elected for life and usually seem to have consisted of part of the royal households in addition to the kings themselves. Finally there was the analogy to the Athenian Assembly, the *Apella* which was an assembly of all Spartan citizens.

The type of state that emerged from this process was very different from the Athenian state however. Though the Great Rhetra created a Spartan state which was strong militarily (holding back the Persians at Thermopylae and eventually winning the Peloponnesean Wars) it was less strong than the Athenian state is several obvious ways. First, it provided far less public goods. It coined no money and made no effort to support trade or mercantile activities which instead were actively discouraged.[23] Second, Sparta had neither the type of bureaucracy that Athens built, nor the fiscal system. Weapons, for example, were procured directly by fiat from *perioikoi* while the Athenians taxed rich people to pay for their fleet. Sparta did not build fiscal capacity to finance military power and instead constructed the Peloponnesean league where they went into a coalition with other states in the Peloponnese so that they provided troops during wartime. As Morris (2010, p. 155) puts it

> "the Spartans chose a low cost way to concentrate coercive power, outsourcing war rather than building state capacity, then spent the next century and a half quarreling over whether or accept its limitations or to restructure their society to build state capacity and overcome them. The two key issues were the relationship

[22] Finlay describes them as "free men probably enjoying local self-government" but "were subject to Sparta in military and foreign affairs" (1982, p. 25)

[23] There was a type of money consisting of iron bars but it was so unwieldy that it was useless as a medium of exchange.

between citizens and helots and oliganthropia, the decline in citizen numbers."

Morris here emphasizes a third issue, rather than broadening the citizenship base as Solon and Cleisthenes had done and trying to free it from kinship to build an Athenian identity, the Great Rhetra narrowed citizenship. This narrowing of citizens was not just about the definition of helots and perioikoi. Over time the concentration of wealth meant that fewer and fewer Spartiates could actually provide the food required to stay members of their mess. Indeed, Aristotle noted (*Politics*, 1270a - 1271a) noted that, "some Spartans have come to have far too many possessions, while others very few indeed; as a result, the land has fallen into the hands of a small number ... although the land was sufficient to support 1500 cavalry and 30,000 hoplites, the number [of Spartans] fell to below 1000."

A key to thinking about why this might be is that the Great Rhetra did not institutionalize the power of society the way that Solon or Cleisthenes did. First, and most obvious, the vast mass of the population, probably around 90% were reduced to hereditary slavery. Athens of course had slaves as well, but this was around 25% of the population and as we noted slaves were protected by the rule of law and seem to have even played a role in enforcing the law. In contradistinction, the *ephors* declared war of the helots every year and Spartiates were even encouraged to murder them. Second, though there was an assembly in Sparta, and though this had to vote on policies formulated by the kings or the *Gerousia*, "unlike the Athenian Assembly the Spartans rarely discussed proposals: they normally only voted (by shouting)" (Morris, 2010, p. 123). The Great Rhetra did not create the same sorts of democratic institutions which Athens had. Indeed, while the Great Rhetra did endorse "the final decision-making authority of the dêmos, it also made it fairly explicit that its role was primarily ratifying proposals formulated by the kings and the aristocratic council ... the nominal supremacy of the Spartan *dêmos* was little more than an illusion" Hall (2013, p. 219). Aristotle describes the election to the *Gerousia* as 'oligarchical' (Aristotle,) while Herodotus calls the members 'among the first by birth'. Leading scholar of Sparta Hodgkinson (1983, p. 280) sums up his view by stating "for all the uniqueness of the Spartiates upbringing and way of life [the political system] perpetuated the existence of a typical Greek aristocracy."

In terms of our model, in Sparta a state emerged which was much more despotic in the way it treated a society which was far less strong and organized than Athenian society. Again there was a contest for power, but it evolved dynamically in a very different way. In terms of our model Sparta fits into Region I.

7.3 The Mani: A Weak State

While Greece experienced different patterns of state formation in the Classical era, not all parts of it experienced such dynamics of political institutions. Both Thrace and Schythia to the northeast seem to have not experienced it until much later and instead stayed as stateless societies based on kinship. Hall (2013, p. 91) distinguishes between different parts of Greece where the *polis*, the city state, emerged first, possibly as a consequence of greater cultural and settlement continuity with Mycenean Greece. In contradistinction to the *polis* was the *ethnos* which he describes as "a group of people - or more generally, a population. It's common identity resided in the bonds of kinship, however fictive, that were recognized by its members, bolstered no doubt by shared rituals and customs ... while the population of a *polis* take their name from an urban center, those populations that are described as *ethnê* typically give their name to the general region they inhabit."

Within the heart of Greece, therefore some societies, some *ethnê*, stayed on the fringe of the states attempting to maintain their independence and statelessness. One was the Mani, located on the Tainaron peninsular in the Southern Peloponnese. Mani was on the borders of Laconia in Sparta and it is probable that during Sparta's heyday, its citizens were mostly *perioikoi*, though probably with some *helot* communities as well (Mexis, 2006, pp. 43-45). As we noted, they are described by Homer as having been mustered by Menelaus at the time of the Trojan War.

Though they may have lived under the shadow of Sparta historically, the Maniates managed to subsequently avoid the creation of centralized authority. They maintained this equilibrium not just in the Classical period of Greece, but also subsequently under Roman, Byzantine, Venetian and Ottoman rule. None of these empires were able to exert their authority or rule over the Mani. During the Roman period they were recognized as the 'Commonwealth of the Lacedaemonians' and Mexis notes "on the Tainaron Peninsular Roman colonizers never established themselves" (2006 p. 92). With respect to the Ottomans "The armed Maniati populace fought til the last towards two ends. The first was to keep the Turks out of Mani. The second was for the individual producer-cultivator himself personally not to have a bond of subservience with the Turkish feudalist. And on both scores they were successful" (Mexis, 2006, p. 352).

In its stead the society was governed in a very decentralized way by clans and authorities developing a system of conflict resolution based on feuding and vendetta of a type very similar the one we described in Montenegro in the introduction and many other stateless societies. Mexis notes "During the period of Turkish occupation courts did not exist in Mani; neither did they exist during the period of the national revolutions of 1821. And neither were there any after

independence" (2006, p. 409). Instead clans resolved disputes and also engaged in prolonged conflicts (many documented in Mexis, 2006, see also Fermor, 1958).

7.4 Comparison

As we mentioned above, during the Greek Dark Ages there seems to have been little to distinguish Athens, Sparta and Mani, even if they ended up in dramatically different places. Moving closer to Solon or the Great Rhetra, the similarities are again evident.[24]

Yet they diverged and both elites and citizens made different decisions which led to and intensified that divergence. For example, while the Spartans were consolidating the dependent status of *perioikoi*, "the situation in early sixth century Athens may have been fairly close to the situation in contemporary Laconia" (Hall, 2013, p. 256) to the extent that something close to the *perioikoi*, the dependent artisans of Sparta, existed in Attica. Nevertheless, "In the final decade of the sixth century .. the city made a choice concerning its perioikic neighbors that was not taken by Sparta" (Hall, 2013, p. 257).

Just as in Athens the statebuilding reforms of Cleisthenes were designed to break down kinship ties and replace them with new identities, the Great Rhetra was designed to "transfer allegiance away from the family or kinship group to various male groups .. the family, in sum, was minimized as a unit of either affection or authority, and replaced by overlapping male groupings" (Finlay, 1982, p. 28). But the lost role of the family was taken over by the *agoge*, something controlled by elites.

A final interesting point of comparison and divergence is the differing fates of social norms bolstering the control of society over the state. We discussed above Solon's Hubris law and Cleisthenes' Ostracism law. There was no such law in Sparta after the Great Rhetra. But the

[24] Most existing scholarly work focuses not on such variation but on why Classical Greece was so different from Bronze Age (Mycenean Greece) or other parts of the Mediterranean basin or the Near East (such as Persia). Seminal work by Morris (1987, 1996) has argued that the deconcentration of political power after 1200 BC led to the emergence of much more democratic societies where citizens had relatively greater power compared to elites. Morris tracked the consequences of this for grave goods and the location and number of graves in addition to the distribution of house sizes. Though the evidence is compelling, it does less well at explaining the variation within Greece. This is also true of other arguments often used in this context such as the democratizing impact of iron technology (Childe, 1942, and Snodgrass, 1980) since iron was widely used everywhere. Similarly, the argument that the broader access to writing and literacy which came with the transition from Linear B to Greek, had a democratizing effect (Ober, 2015a) cannot explain the variation we are interested in. A further argument is that elites did not exercise religious power, which while true again does not distinguish our three cases. Finally, the same applies to the argument that the creation of hoplite warfare redistributed power towards citizens. Both the Athenians and the Spartans used hoplites, but significantly in the Spartan case, because of the organization of the economy, the military equipment of the hoplites was provided by the state, not by the individual hoplite (Finlay 1982, p. 30). Here the political and social organization of the state probably trumped any empowering impact military technology or tactics might have had.

See Hodgkinson ed. (2009) for a recent collection of essays by Classical scholars that does focus on the differences between Sparta and other Greek states.

evidence is that such social norms were widespread in Archaic Greece. Forsdyke, for example, argues "In general, consideration of the evidence from outside Athens suggests that Athenian ostracism was simply one elaboration of a more generalized Greek practice of using written ballots - whether leaves or potsherds - as a means of determining a penalty (removal from public office or exile)" (Forsdyke, 2005, p. 285).[25] Though Sparta did not use the institution of ostracism in the same way there is evidence of the legacy of similar institutions which partially lived on even after the Great Rhetra. Forsdyke (2005, Appendix Three) points out how both Athens and Sparta used exile as a judicial punishment and in Sparta it was used several times to discipline kings. For example, Leotychidas in 476 BC and Pleistoanax in 446/5 BC, both of whom were exiled for accepting bribes. Given Forsdyke's arguments about the common origins or ostracism and exile it seems likely that this use of exile to discipline kings is a residue of the types of social norms which led to the laws of hubris and ostracism in Athens. That such norms did not perpetuate themselves after the Great Rhetra may be due to the massive re-organization of society that this implied. Thus it is not that Sparta lacked the social norms the Athenians had, rather they were present in a situation where elites were initially more powerful. Hence after the Great Rhetra, instead of these social norms being institutionalized and thus strengthened, as in Athens, they were instead significantly eroded, living on only in the judicial instrument of exile.

Just as there were many similarities between Athens and Sparta, so there were between these two societies and the Maniates. We noted for example that Draco's Homicide Law was a codification of feuding regulations suggesting that the institutional equilibrium in Athens at that time was not so different from that of Mani. Yet Solon's reforms initiated a transition towards a much more state like organization of authority. This process is in fact captured in Aeschylus' trilogy *The Orestia*, which is the classic depiction of the transition from a stateless society where conflicts are resolved by feuding and revenge (see Finlay, 1954, for a characterization) to one based on law, modelled after Aeschylus' own Athens. In the first play Clytaemnestra murders her husband Agamemnon after his return from the Trojan wars. In the second play, *The Libation Bearers*, Agamemnon's son, Orestes, murders his mother in revenge. The chorus eggs him on with the words "stroke for bloody stroke be paid, The one who acts must suffer, Three generations strong the word resounds." (*The Libation Bearers*, 315-321).

Here Aeschylus depicts a society based on the feud, on 'retribution' and 'stroke after bloody stroke', lacking centralized authority. But in the final play, *The Eumenides*, Orestes is sent by

[25] Archaeological evidence attests to ostracism being used as an institution outside of Athens in Argos, Cyrene (in Libya), Megara, Syracuse and Tauric Chersonesus (in the Crimea), see Robinson (2011).

the god Apollo to Athens pursued by the Furies seeking revenge for his killing of Clytaemnestra. But in Athens, the patron goddess Athena breaks the cycle of revenge by creating a court which judges Orestes. Mexis points out the Mani vendetta, or 'chosia' bares a strong resemblance to surviving depictions of the Dorian 'krypteia'

> "Maniati 'chosia' has to be a survival of the methodology of a 'krypteia' that was preserved in the traditions of the armed clan" (Mexis, 2006, p. 382).

Moreover, "Such a supposition is supported also by the fact that the vendetta in Mani was a means for the solution of the differences between the "powerful". Thus he argues that there are very deep roots of the vendetta in Mani stretching back to the Classical period.

According to our theory one can understand the divergence of Athens, Sparta and the Mani as a consequences of conflict between the state and citizens, but in a situation where there were initially differences, possibly small ones, in the balance of power between state and citizens. To understand the divergence of Athens and Sparta, for example, our model suggests, one should look for differences in the balance of power between citizens and elites and the hypothesis would be that citizens were relatively more powerful in Athens and even more powerful in Mani.

There seem to be a number of reasons for believing that this was indeed the case. Consider first Athens and Sparta.

First, potentially important is that the Spartans were Dorians who migrated into the Peloponnese from central Greece at some point in the early Dark Ages. A consequence of this migration was the enslavement of helots as they expanded out of Laconia where they first settled. Thus one can imagine the Spartans a bit like colonial societies elsewhere in the world, for example in Latin America after 1492. Coming from the outside, like the Spanish in Latin America, the Spartans found densities of indigenous peoples and created institutions to exploit them. As everywhere in history, this seems to have created the basis for hierarchical elite dominated societies. It is possible that the problem of controlling and exploiting a large population of slaves made the society more militarized and gave more power to kings and elites who were useful in organizing the militarized suppression of the helots. Indeed, Rhodes (2011, p. 4) notes that "Originally Sparta's culture had been like its' neighbours" and he goes on to argue that one source of Spartan institutional distinctiveness was indeed that because of "the conquest of Messenia" there was a "need to keep the subject population under control". Like Rhodes, Hall (2013, p. 230) notes "Sparta ... had not originally been so distinct from other Greek *poleis*. There must have been some sort of turning point" and he proposes what distinguishes Sparta was the fact that the territory over which it ruled was "exceptionally large" (p. 232).

Second, though the government of Athens before Solon appears to have been oligarchic Athens did not have the type of hereditary kings that Sparta did and elites already could not dominate society (this is evident in the failure of Kylon to establish a tyranny in 632 BC, see Ober 2015a, p. 148). That elites were more powerful in Sparta is attested to in Plutarch's life of Lycurgus, he notes that Soos, supposedly an ancestor of Lycurgus, was the Spartan king who enslaved the helots. Soos' son, Eurypon

> "appears to have been the first king to relax the excessive absolutism of his fellow-lords, seeking favor and popularity with the multitude" (*Lycurgus*, 2).

There was no such "excessive absolutism" to relax in Archaic Athens, nor anything like a king who could have implemented a mass enslavement.[26]

Another final potentially important factor is that the Spartan capital never really became an urban center like Athens, but remained an agglomeration of four, subsequently five, separate villages (see Cartledge, 2002). This perhaps made the type of mass democratic participation built in the Athenian state less feasible and harder to organize and again strengthened the power of elites.

Thus the evidence is consistent with the despotic state emerging in Sparta based on the exploitation of helots, and to a lesser extent perioikoi, as a consequence of the initial balance of power favoring elites. This allowed a relatively pro-elite constitution to be constructed by Lycurgus, albeit with important elements of checks and balances. Equally significant was the social organization and socialization of Spartiates via the *agoge* which seems to have eliminated the types of resources the citizens of Athens had available to discipline elites.

What distinguished the Mani from these other cases to which it seems to have been so historically similar? In our theory this is a consequence of society being more powerful than in the Athenian case. There were no elites with institutionalized power in Mani like the Spartan kings or even Athenian oligarchs who filled the positions of Archons prior to their reform under Solon. Rather there was a balance of power between different clans and families. Centralization would have involved one clan dominating the others and as is well documented in ethnographically observed societies, such a move would have been stringently opposed by other clans who feared being dominated. In some circumstances of course such state formation does take place, often

[26] Plutarch describes another telling incident. The Spartan assembly, known as the *Apella*, which existed prior to the Great Rhetra, used its' right to sanction the Spartan kings (of which there were two). During the First Messenian War of 743-722 BC, in the midst of a revolt by the *helots*, the slave class of Sparta, the kings Polydoros and Theopobos attacked citizens' rights by promulgating a law such that "if the people should adopt a distorted motion, the senate and kings shall have power of adjournment" (*Lycurgus*, 6) in effect shutting down the *Apella* if it did something the kings didn't like.

when some clan gains a military or other advantage over the other. But in many parts of the world a stateless stalemate occurs. The long persistence of this in Mani is not unusual. For example, in West Africa at the time of the scramble for Africa possibly one third of people lived in stateless societies (Curtin, Feierman, Thompson and Vansina, 1995). Many of these, such as the Tiv in Nigeria studied by Bohannan (1958), share many features with the Mani or Montenegrins.

8 Conclusion

There is a great deal of diversity in the nature of states in the world today, in particular in the extent to which they have capacity to fulfill basic functions, such as raise tax revenues, establish a monopoly of violence or effectively regulate society. But societies, not just states, also differ enormously. Some are highly mobilized and organized collectively, with high levels of 'social capital' while others are not. In this paper we have developed a simple model to understand the variation in state capacity, arguing that states endogenously acquire capacity in a dynamic contest with society. At the heart of our model is the notion that elites that control states must contest with society (non-elites) for control over political power, resources and rents. If the state accumulates capacity, what we called 'strength' then this helps it win this contest. But in response society can also accumulate strength, which we associated with collective organization and social norms, which helps it contest against the state.

We showed that a simple model based on this intuition had three very distinct stable steady-states with very different constellations of state society relations. In one steady-state, which we called a despotic state, the state acquired far more strength than society, in a sense dominating it. In another the reverse situation arose where society accumulated far more strength than the state. Finally, there was a steady-state which was much more balanced where both society and the state acquired high levels of strength. We associated these steady-states with three different sorts of states, despotic, weak and inclusive. Interestingly, of the three, it is inclusive states, with strong societies, which have the most capacity. This is because state elites need to be pushed by society to accumulate strength and thus capacity.

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Appendix

A Model of Economic and Political Investments

In this part of the Appendix, we provide a more detailed model meant to clarify what the strength of state and society stand for, and show that this model can be mapped to the more reduced-form set up we use in our main analysis.

Suppose that society consists of a state (ruler) and a number of small producers, each with the production function

$$F(g_t, k_{it}),$$

where g_t is a measure of public good provision (such as infrastructure, bureaucratic services or law enforcement) at time t, and k_{it} designates the capital investment of producer i.

The cost of public good investment by the state depends on what sociologist Michael Mann refers to "infrastructural power" of the state, or simply the "presence" of the state, denoted by

s_t. Suppressing time indices when this causes no confusion, we write this cost as

$$\Gamma_g(g \mid s).$$

This dependence captures the fact that investing in public good provision will be much more difficult for the state when it is not present or it is not otherwise powerful. There is also a separate cost of increasing the infrastructural power of the state as specified in the text. In addition, as we discuss below, this infrastructural power of the state will also determine the state's relationship with society.

The producers, on the other hand, individually choose their capital level, but also jointly choose their coordination, which we denote by x. A higher degree of coordination among the producers might (but need not) impact their costs of investing in capital, which we write as

$$\Gamma_k(k \mid x),$$

and this dependence might reflect the fact that a greater degree of coordination among the producers enables them to help each other or develop greater trust in production relations or internalized some externalities. More importantly, as discussed in the Introduction, this degree of coordination impacts how they can deal with the state's demands. More broadly, this degree of coordination may also stand for certain social norms that society develops for managing political hierarchy or outside control as our historical cases also emphasize. We assume that the cost of investing in x is as specified in the text.

Note that the assumptions that only s and x, and not g and k, build on their non-depreciated stock is for simplicity, and facilitates the comparison with our reduced-form model in the text.

The political game takes the following form: the state announces a tax rate τ on the output of the producers. If the producers accept this tax rate, it is collected and the remainder is kept by the producers. If they refuse to recognize this tax rate, there will be a conflict between state and society, the outcome of which will be determined by s and x in a manner similar to the conflict in the text. In particular, state will win this conflict if

$$s - x > \sigma$$

and can extract the entire output of producers, while if the inequality is reversed, society wins, and the state will not be able to collect any taxes. We assume, as in the text, that σ has a distribution given by the distribution function H.

Here we focus on the economy in discrete time for simplicity and discuss the equilibrium in a single period. We also suppress time arguments to simplify the notation. The equilibrium can

be solved by backward induction within the period, starting from the tax decision of the state. Given the conflict technology we have just specified, it is clear that if the tax rate τ is greater than the likelihood of the state winning the conflict, $H(s-x)$, then there will be a conflict. We may thus focus, without loss of any generality, on the case in which $\tau = H(s-x)$.

Then the state's maximization problem can be written as

$$H(s-x)F(g,k) - \Gamma_g(g \mid s) - \tilde{C}_s(s, s_{-\Delta}),$$

where $\tilde{C}_s$ is a cost function for the power of the state similar to the one specified in the text, $s_{-\Delta}$ denotes last period's state strength, and k is the common physical capital investment level of all agents. The solution to this problem for g can be summarized as

$$g = g^*(x, k, s).$$

Note that even though $s_{-\Delta}$ influences s, it does not directly impact the choice of g.

Similarly, recalling that $1 - H(s-x) = H(x-s)$, the maximization problem of citizens can be written as

$$H(x-s)F(g,k) - \Gamma_k(k \mid x) - \tilde{C}_x(x, x_{-\Delta}),$$

with solution

$$k = k^*(x, g, s).$$

Solving this equation together with the equation for g, we can eliminate dependence on the economic decision of the other party, and obtain an equilibrium (which may not be unique), expressed as

$$g = g^{**}(x, s),$$

and

$$k = k^{**}(x, s).$$

Substituting these into the payoff functions, we obtain a simplified maximization problem for both players, essentially replicating our reduced-form model in the text. In particular,

$$H(s-x)f(x,s) - C_s(s, s_{-\Delta}),$$

and

$$H(x-s)f(x,s) - C_x(x, x_{-\Delta}),$$

where

$$f(x,s) = F(g^{**}(x,s), k^{**}(x,s)),$$

$$C_s(s, s_{-\Delta}) = \Gamma_g(g^{**}(x, s) \mid s) + \tilde{C}_s(s, s_{-\Delta})$$

and

$$C_x(x, x_{-\Delta}) = \Gamma_k(k^{**}(x, s) \mid s) + \tilde{C}_x(x, x_{-\Delta}),$$

with the only complication relative to the model in the text that the cost functions may depend on the equilibrium action choices of the other player.

Alternative Contest Functions

As noted in the text, the specification of the contest between the state and citizens we have used so far is not special, and its main properties are shared with general contest functions. To see this, consider a contest function such that the payoff of the state is

$$\frac{k(s)}{k(s) + k(x) + \eta},$$

while that of society is

$$\frac{k(x)}{k(s) + k(x) + \eta},$$

where $k(\cdot)$ is an increasing, differentiable function, and $\eta \geq 0$ is a constant. In this case, the marginal return to increasing investment for the state is (going directly to the continuous time to economize on space)

$$\frac{k'(s_t)(k(x_t) + \eta)}{(k(s) + k(x) + \eta)^2},$$

and the expression for society is also similar. The cross-partial derivative of this expression, showing us how this marginal return changes when society increases its investment, is

$$\frac{k'(x_t)k'(s_t)(k(s_t) - k(x_t) - \eta)}{(k(s) + k(x) + \eta)^3}.$$

Notice that when $\eta = 0$, this has the same property as our main specification; it is positive when $s_t > x_t$, and negative when $s_t < x_t$. When $\eta > 0$, the same result holds provided that s_t is sufficiently larger than x_t.