Capital Specificity, the Distribution of Marginal Products and Aggregate Productivity

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Abstract

This paper studies the role of capital specificity and investment irreversibility on the distribution of marginal products of capital and aggregate TFP. We use a methodology new to the misallocation literature, based on the study of "mobility" across quantiles of a distribution. In a panel of Peruvian firms, we show that persistent dispersion in marginal products is explained to an important extent by the persistence of low marginal products. That is, by unproductive firms that take a long time to downsize. Using a quantitative general-equilibrium model of firm dynamics with idiosyncratic shocks, calibrated to match key features of our data, we argue that the persistence of low marginal products suggests that irreversibility frictions are large. Moreover, it is inconsistent with theories of misallocation based only on financing constraints.

PRELIMINARY AND INCOMPLETE

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1 Introduction

There is extensive evidence of large dispersion of marginal products across productive units. This has been shown for developing countries in general (e.g. Hsieh and Klenow (2009)) and specifically in Latin America (e.g., Restuccia and Rogerson (2013); Pages (2010)). Furthermore, as we also document in this paper, the relative rankings in the distribution of marginal products appear to be highly persistent. Both small, high productivity firms take years to grow, and large, low productivity firms take years to downsize.

These observations suggest that there may be large "frictions" in capital reallocation. If these frictions were to be removed, there could be large TFP gains by facilitating reallocation. This will allow not only high productivity firms to grow faster but, importantly, let unproductive firms downsize faster. Yet, to the best of our knowledge, little is known about the specific frictions preventing reallocation. Are they mainly policy distortions? Or financial constraints? Or are they technological constraints, such as adjustment costs? Moreover, do frictions affect high- and low-productivity firms equally, or do they have asymmetric effects?

In this paper, we make a first step towards identification of frictions in capital reallocation. We propose a new methodology to study the dynamic properties of the distribution of marginal products of capital. By using a quantitative general equilibrium model of firm dynamics as well as micro investment data on Peruvian firms, we show that the *mobility* within the distribution of marginal products[1] is a key factor to identify frictions. It allows us to distinguish between frictions that primarily affect firms on a positive growth path (e.g., financial frictions) from frictions that mainly affect firms on a negative growth path (e.g., capital specificity and irreversibility).

The literature has largely explained persistent misallocation as the effect of financial constraints. [2] Using a general model that accomodates many frictions, we argue that these theories predict that high marginal products should be more persistent than low ones. This is because financial frictions slow down the positive growth path of high-productivity firms . Meanwhile, they do not affect (at least, directly) low-productivity firms trying to

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[1] That is, how likely it is to move across different quantiles of the distribution.

[2] See the next section for a detailed discussion of the related literature.

downsize. Yet, we show that this is not true in the data. The persistence of dispersion in marginal products is left-skewed. That is, it is mostly explained by the large persistence of low-marginal products.

We show that this observation is instead consistent with an important role for capital specificity, inducing partial investment irreversibility.

Firms need specific assets to produce their output. Expanding units buy bundles of investment goods. Some of them are fairly easy to redeploy (e.g., property, computers), while others are relatively tied to a specific business line (e.g. textile machinery). The presence of some capital specificity implies the following. When firms are hit by negative profitability shocks and need to downsize or exit, the resale value of their assets is often substantially lower than their cost, leading to partial irreversibility of investment. This technological friction is present both when capital is reallocated across firms in the same industry, because differentiated products often require different types of equipment, and even more when assets are reallocated across different industries. In a dynamic environment with profitability shocks, irreversibility generates dispersion in the cross-sectional distribution of marginal products of capital. Relatively large and unprofitable firms cannot downsize as fast as they would, absent this friction. Hence, their marginal product remains persistently low, relative to adjusting firms.

Besides the issue of identifying different frictions, we argue that understanding which tail of the distribution of marginal products is more affected by frictions is also important to correctly compute aggregate TFP. Using both our quantitative model and a simpler environment that allows us to derive analytical results, we show that the dispersion of marginal products is not, in general, a sufficient statistic for TFP. Consider two economies with the same degree of dispersion, but where frictions affect different tails of the distribution. When high-productivity firms are distorted in their capital choice (as in models of financial frictions), TFP is lower than when low-productivity firms face distortions (as in models of partial irreversibility). This result, together with our discussion of identification of frictions, suggests that paying attention to moments of the distribution of marginal products beyond the dispersion is a necessary step to learn about the drivers of the allocation of capital at the micro level. Furthermore, it shows that if we erroneously attribute the empirical dispersion to financial frictions only, we are likely to overstate the TFP losses relative to an efficient benchmark.

2 Related Literature

This paper relates to several strands of literature. First, it relates to the growing literature on capital misallocation that aims to explain the large dispersion of marginal product of capital (MPK). A large number of studies have focused on distortions as explanations for the static dispersion observed in the data. Among others, the distortions emphasized include credit contraints (Buera et al., 2011; Buera and Shin, 2013), government policies (Guner et al., 2008), trade barriers (Epifani and Gancia, 2011; Pavcnik, 2002) and lack of insurance (Adamopoulos and Restuccia, 2014). In addition, recent work by Asker et al. (2014) shows that some of this dispersion come from optimal dynamic firm-level decisions subject to time to build and adjustment costs. If capital takes time to accumulate, a firm's capital stock will seem statically inefficient relative to current TFP shocks. In either case, we worry about capital misallocation to the extent that it is persistent (Banerjee and Moll, 2010). If MPK dispersion was transitory, we should observe fast reversals in the ranking of marginal products. As it is known, this is at odds to what empirical studies find (Banerjee and Duflo, 2005). To explain this persistence, the literature has focused on the role of financial frictions (Erosa, 2001; Amaral and Quintin, 2010; Buera et al., 2011; Midrigan and Xu, 2014; Banerjee and Moll, 2010).

The main contribution of the present paper is to dig deeper into this persistence. By doing that, we depart from and extend this literature in two ways. First, we establish a new fact on MPK mobility by considering a previously unexplored time dimension. While we observe large and persistent MPK dispersion, we document that persistence is asymmetric. In particular, MPK mobility is lower for firms in the low quantiles of MPK distribution. Using methodological tools from the social mobility literature (Chetty et al., 2014; Charles and Hurst, 2003), we calculate MPK transition matrices. For every manufacturing industry in the data, they are always asymmetric, with more persistence in the tail of low marginal product firms. This suggests, somewhat in contrast to existing work, that one main reason for MPK persistence is firms not being able to downsize quickly.

Second, we lay a microfoundation for this result by incorporating capital irreversibility in a model with financial frictions and time to build. We build on a firm dynamics model as in Hopenhayn (1992), adding frictions in downsizing as in Cooper and Haltiwanger (2006) and Lanteri (2016). This is also motivated by the evidence of capital specificity and irreversibility in the work of Medina (2017) and Gavazza (2011). We find that, while all frictions lead to dispersion, only capital irreversibility matches the patterns of MPK mobility. Thus, the model provides theoretical support for the empirical identification of this type of friction.

In addition, the paper relates to the work that emphasizes that misallocation of resources across firms have important effects on aggregate TFP. Starting with the pioneering work by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), there is a wide consensus that these effects could be so large that will help explain differences in TFP across rich and poor countries. This paper contributes to this discussion by showing that the type of friction matters for aggregate TFP calculations. Both financial frictions and irreversibility lead to TFP losses. However, the quantitative effects are very different. If TFP losses are calculated assuming there are only financial constrains, while there is also capital irreversibility, total productivity costs will be overestimated. Thus, a third contribution of this paper is to highlight the importance of identifying specific frictions to calculate aggregate effects of misallocation, while providing an empirical method to identify one of those frictions.

3 Data and Empirical Patterns

3.1 Data

Our main data source is the Peruvian Economic Survey (EEA). It is a representative panel sample of manufacturing industries and spans from 2003-2012. This dataset contains 18,906 firm-year observations.

For every firm, the EEA contains net sales, value added, material, energy, labor expenditures, among other variables. Moreover, the EEA provides detailed information on capital. Besides total capital stock and expenditure, it records two capital measures rarely found in other datasets. First, it includes information on the stock and flow of capital. This includes capital additions and retirements. Additions refer to purchases, own construction, and revaluations. Retirements refer to sales and withdrawals. Second, it disaggregates capital in different types. Types are land, fixed installations, buildings, machinery and equipment, furniture, computers, and transportation. For every type, both stock and flows of capital are listed.

3.2 Stylized Facts

3.2.1 MRPK Dispersion

MRPK is not equalized across firms. This fact has been extensively documented for many developing countries, with Peru not being an exception. In the Peruvian manufacturing industry, the static dispersion of MRPK is 1.70, which is larger than similar estimates for countries like Chile, Mexico or India. Moreover, as shown by Figure 1, this dispersion is not driven by a particular manufacturing industry. Rather, it is high for any manufacturing industry in the sample. This is in line with results from Bartelsman et al. (2013) that document misallocation also occurs within narrowly defined industries.

While these findings suggest the existence of frictions preventing firms to efficiently allocate resources, they could also come from optimal dynamic decisions of the firm. As first pointed by Asker et al. (2014), capital decisions taken in the past might not seem optimal as current productivity shocks are realized. If this is the case, MRPK dispersion will be positively correlated with TFP volatility. This relationship holds in the Peruvian data, as shown in Figure 2. Thus, some of the static MRPK dispersion is explained by capital formation.

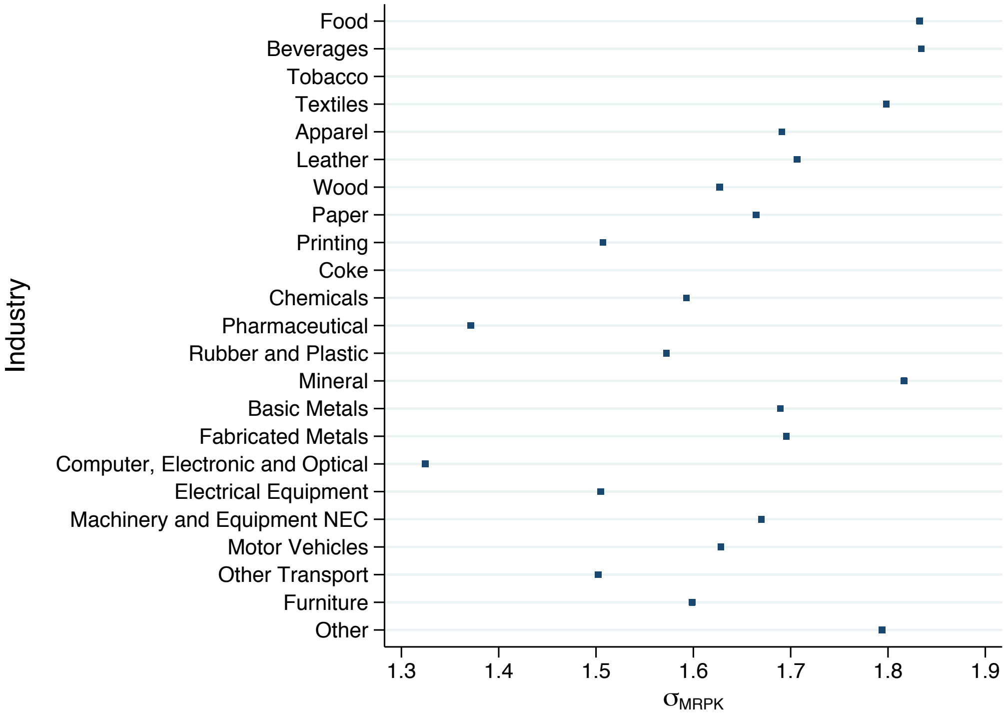

Figure 1: Dispersion of MRPK by Industry

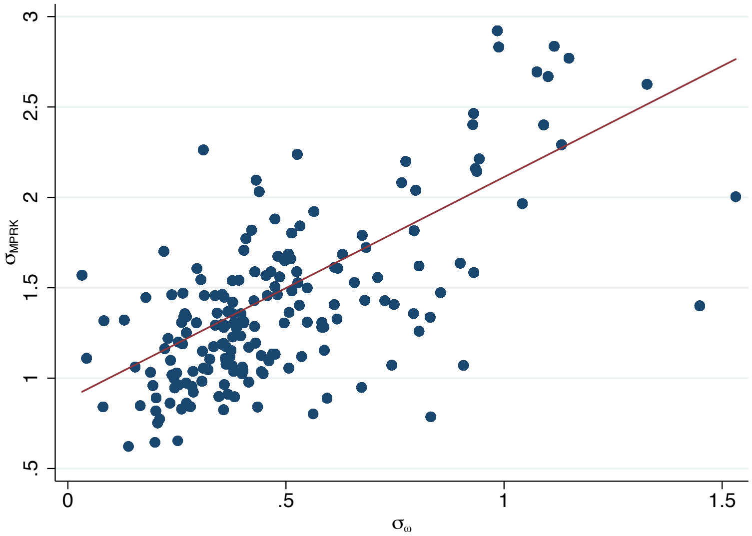

Figure 2: Dispersion of MRPK and TFP Volatility

3.2.2 Persistence

Dispersion in MRPK is not only large but it shows high persistence. As shown in Table 1, for firms that survive all the sample, the median within-firm dispersion is 0.34. Moreover, the median of within-firm autocorrelation is 0.51, while pooled autocorrelation is higher, at 0.8. These results corroborate the findings by Midrigan and Xu (2014) and feature even more stark patterns. The high level of persistence in the aggregate and firm-level is worrisome. It suggests the existence of frictions that, even in a 10-year span, are not eliminated.

Table 1: MRPK Persistence

Variables	MRPK	TFPR
Year FE	0.008	0.034
2-digit Industry FE	0.002	0.009
Firm FE	0.84	0.76
Autocorrelation (pooled)	0.80	0.85
Median within-firm dispersion	0.34	0.28
Median within-firm autocorrelation	0.51	0.44

Not only MRPK but also TFPR is highly persistent in all industries. As shown in Table 1, it is very significant yet lower than the one for MRPK. This is also consistent throughout all industries where TFPR autocorrelation coefficients go from 0.49 to 0.96.

3.2.3 MRPK Mobility

A novel finding is that MRPK is asymmetrically persistent, with higher levels observed for firms with low MRPK. While persistence is high for all firms, firms on the left tail of the distribution are consistently more likely to remain with low MRPK. To fix ideas, we compare the four more important Peruvian manufacturing industries. We compute annual transition matrices of the terciles of MRPK distribution by industry. Table 2 shows those transition matrices.

Table 2: Transition Matrices: MRPK Persistence

	1	2	3
1	0.70	0.20	0.10
2	0.24	0.57	0.19
3	0.13	0.34	0.53

(a) Food

	1	2	3
1	0.71	0.18	0.11
2	0.26	0.57	0.17
3	0.07	0.29	0.63

(b) Apparel

	1	2	3
1	0.83	0.13	0.03
2	0.14	0.68	0.18
3	0.04	0.19	0.77

(c) Chemicals

	1	2	3
1	0.75	0.17	0.08
2	0.21	0.53	0.25
3	0.16	0.35	0.48

(d) Electrical Machinery

Asymmetric persistence is important since it provides a moment to identify the nature of the frictions leading to capital misallocation. Moreover, this empirical fact is at odds with the standard reasons proposed to explain persistence. For example, financial frictions will particularly affect firms on the right tail of MRPK distribution. Any reduction of financial frictions will lead productive firms to invest in additions of capital stock. As capital increases, these firms should converge to the mean of the MRPK distribution over time. The same is true for models including time to build and capital adjustment costs.

However, frictions related to the specificity of assets will have an impact on the left tail. Irreversibility of capital will imply that firms that need to downsize will not be able to so. Thus, those firms will likely be on the low percentiles of the MRPK distribution. If these frictions are large, models including irreversibility will be more suitable to explain this asymmetry. Despite its importance, to our knowledge, this time dimension has not been explored yet in the literature.

This fact is also is consistent with the dynamics of capital accumulation observed in the data. Table 3 shows the distribution of scaled investment for our main four industries.

The median investment ratio is around 10%, with a large dispersion. The fraction of firms that disinvest or do not adjust their capital is low in every industry. Yet, when firms do disinvest, the average disinvestment is significantly large.

Table 3: Distribution of $\frac{I}{K}$

Variables	Food	Apparel	Chemical	Electronics
Mean	0.24	0.27	0.07	0.17
Median	0.10	0.12	0.11	0.11
SD	0.81	0.83	0.48	0.73
Fraction ≤ 0	0.04	0.06	0.11	0.09
Fraction $\leq \frac{mean}{4}$	0.10	0.10	0.11	0.11
$E[\frac{I}{K} \vert \frac{I}{K} < 0]$	-0.61	-0.54	-0.61	-0.58
Inaction [Window = 1%]	0.00	0.00	0.00	0.00
Inaction [Window = 5%]	0.01	0.02	0.03	0.02
Inaction [Window = 10%]	0.13	0.10	0.17	0.16
Inaction [Window = 20%]	0.20	0.20	0.34	0.30

One natural question is whether this asymmetric persistence is driven by TFP shocks. While models often assume these shocks are normally distributed, actual distributions of these shocks could be skewed. If this is the case, asymmetry in shocks will lead to differences in mobility for firms in different parts of the MRPK distribution. To rule out this case, Table 4 shows the transition matrices for the innovation term of TFP in each of our four industries.

Indeed, TFP innovation shocks are not normally distributed. However, their distribution is right rather than left skewed. That is, positive TFP shocks are more persistent that negative ones. Because of this, firms in the right tail of the MRPK distribution should show more persistence. Given continuous positive shocks, firms will want to invest on capital. Due to financial frictions, some of them will not be able to do it. Thus, there will be a high persistence on the right tail of the MRPK distribution. This is the opposite to what is observed in the data. While they help us understand the levels of right tail persistence, TFP shocks cannot explain the patterns of MRPK mobility.

Table 4: Transition Matrices: TFP Persistence

	1	2	3
1	0.60	0.33	0.07
2	0.21	0.58	0.21
3	0.02	0.17	0.81

(a) Food

	1	2	3
1	0.57	0.34	0.09
2	0.18	0.56	0.26
3	0.05	0.22	0.73

(b) Apparel

	1	2	3
1	0.72	0.25	0.02
2	0.17	0.58	0.25
3	0.02	0.20	0.78

(c) Chemicals

	1	2	3
1	0.57	0.39	0.04
2	0.24	0.58	0.18
3	0.00	0.19	0.81

(d) Electrical Machinery

3.2.4 Utilization Margin

Standard models assume that all capital is used in production. Yet, if firms firm it difficult to downsize, capital could be under utilized. Idle capital has important implications to our measures misallocation. Figure 3 shows MRPK dispersion when controlling capital by energy consumption or material use. In almost all cases, dispersion of MRPK decreases when a corrected measure of effective capital is used. It does not eliminate dispersion; however, it suggests a potential overestimation of the degree of capital misallocation.

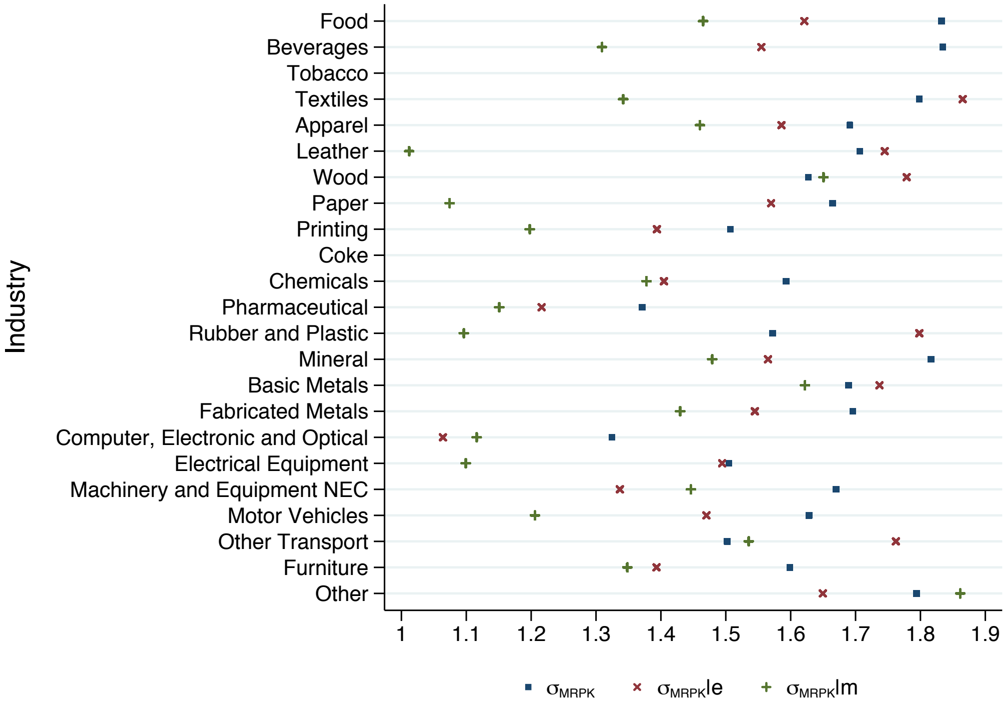

Figure 3: Corrected Dispersion of MRPK

In addition, as shown in Tables 5 and 6, when capital is corrected by utilization rate, MRPK mobility is not longer skewed. This is true for both the correction with materials inputs and energy use. This also points to the fact that capital is being under utilized by firms wanting to downsize, for which MRPK is low. Thus, if capital has some irreversibility, it is not surprising to see this effect on the distribution of persistence.

Table 5: Transition Matrices: MRPK Persistence with Energy Correction

	1	2	3
1	0.57	0.26	0.17
2	0.23	0.51	0.26
3	0.11	0.26	0.62

(a) Food

	1	2	3
1	0.61	0.24	0.15
2	0.15	0.56	0.29
3	0.14	0.23	0.62

(b) Apparel

	1	2	3
1	0.63	0.23	0.13
2	0.22	0.60	0.17
3	0.14	0.21	0.65

(c) Chemicals

	1	2	3
1	0.63	0.16	0.21
2	0.24	0.49	0.27
3	0.12	0.34	0.54

(d) Electrical Machinery

Table 6: Transition Matrices: MRPK Persistence with Materials Correction

	1	2	3
1	0.66	0.29	0.05
2	0.24	0.55	0.21
3	0.05	0.19	0.76

(a) Food

	1	2	3
1	0.55	0.32	0.13
2	0.21	0.52	0.27
3	0.03	0.25	0.72

(b) Apparel

	1	2	3
1	0.79	0.19	0.03
2	0.13	0.74	0.13
3	0.02	0.12	0.86

(c) Chemicals

	1	2	3
1	0.68	0.29	0.03
2	0.21	0.59	0.20
3	0.05	0.25	0.70

(d) Electrical Machinery

4 Model

In this section we introduce a general equilibrium model of firm dynamics with frictions in capital reallocation and we use it to establish our results on the implications of capital specificity for our moments of interest. Throughout our exposition, we focus on a stationary equilibrium, hence all aggregates and prices are constant.

4.1 Firm problem

There is continuum of firms $i \in [0, 1]$ producing a homogeneous good with production (value added) function

$$y_{it} = s_{it} k_{it}^{\alpha_k} l_{it}^{\alpha_l} \tag{1}$$

where s_{it} is the (Markov) composite productivity-demand shock, which we refer to as productivity for simplicity and $\alpha_k, \alpha_l > 0$, $\alpha_k + \alpha_l < 1$.[3]

We assume that there are no frictions in the labor market. This implies that we can rewrite output net of the wage bill as

$$y_{it} - wn_{it} = A(w) s_{it}^{\theta} k_{it}^{\alpha_k \theta} \tag{2}$$

with $A \equiv \left[(\alpha_l/w)^{\frac{\alpha_l}{1-\alpha_l}} - w(\alpha_l/w)^{\frac{1}{1-\alpha_l}} \right]$ and $\theta \equiv \frac{1}{1-\alpha_l}$, where w is the wage.

We introduce three sources of dispersion in marginal products:

1. capital is subject to a one-period time-to-build
2. investment is partially irreversible
3. there are limits to external finance available for firms (collateral constraint)

We will argue that these features of the model have different implications for the shape and the dynamic properties of the distribution of marginal products.

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[3] For simplicity, we use a competitive model where decreasing returns are a technological feature. Alternatively, we could assume constant returns in production and a CES demand structure. Given that we do not observe firm-level prices, these models are observationally equivalent in our data. Hence, we choose to abstract from markups in the model to make our point in a simpler environment and we use the terms “marginal product” and “marginal revenue product” interchangeably.

Because of partial irreversibility, the unit value of capital Q_{it} is a function of the desired investment, in particular it is equal to the output price (1) for positive investment, and less than 1 for disinvestment:

$$Q_{it} = \begin{cases} 1 & if \ k_{i,t+1} \geq (1-\delta)k_{it} \\ q & otherwise \end{cases} \tag{3}$$

with $q \in [0,1]$.

We now introduce frictions in firms' financing. Let b_{it} be outstanding one-period debt, with interest rate r. We can express the firms' dividend as follows

$$d(k_{it}, b_{it}, k_{i,t+1}, b_{i,t+1}; w, r) \equiv A(w)s_{it}^{\theta}k_{it}^{\alpha_k\theta} - Q_{it}\left[(k_{i,t+1} - (1-\delta)k_{it})\right] + b_{i,t+1} - b_{it}(1+r) \tag{4}$$

Debt is subject to a collateral constraint $b_{i,t+1} \leq \lambda k_{i,t+1}q(1-\delta)$, expressed in terms of resale value of future depreciated capital, with $\lambda \in [0,\infty)$. In order to make this constraint meaningful, we also impose that dividends have to be non-negative (no equity issuance).

Turning to recursive notation, dropping (constant) factor prices to simplify notation and suppressing i's, the firm solves the following dynamic program

$$v(k,b,s) \equiv \max_{k',b'} \left\{d(k,b,k',b') + \beta\mathbb{E}\left[v(k',b',s')|s\right]\right\} \tag{5}$$

subject to $k' \geq 0$, $b' \leq \lambda k' q(1-\delta)$ and $d(k,b,k',b') \geq 0$.

4.2 Equilibrium

To close the model, we assume that there is a representative household with utility from consumption and disutility from labor effort

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left[\log(C_t) - \psi L_t\right] \tag{6}$$

and budget constraint

$$C_t = w_t L_t + \Pi_t \tag{7}$$

where Π_t are aggregate profits.[4] The labor supply schedule is defined by the first order condition that equates the marginal rate of substitution between hours and consumption to the wage w_t

$$\psi C_t = w_t. \tag{8}$$

and the interest rate is given by $r_t = \frac{1}{\beta} - 1$ (in stationary equilibrium).

A **stationary competitive equilibrium** is defined as a value function v (with associated decision rules), a distribution of individual states $\mu(k, s)$ and prices w and r such that individual optimality holds, markets clear and μ perpetuates itself given the shocks transitions and the decision rules.

4.3 Calibration

Parameter values are reported in Table 7. Our strategy to parametrize the model is as follows. The parameters governing the production technology (factor shares) and the productivity shock process are taken from our estimates for the four largest industries in the Peruvian data.[5]

Preference parameters are standard in the literature: a period corresponds to a year and labor disutility implies that hours are approximately equal to a third.

As far as the frictions are concerned, in our baseline calibration we first abstract from financial frictions ($\lambda = \infty$) and calibrate q to match the (average) standard deviation of log marginal products in the four largest industries considered (1.65). We then use the persistence properties of the distribution of marginal products to evaluate the performance of our model. Finally, we re-introduce financial frictions and provide a comparison of the results implied by the different type of frictions in reallocation.

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[4] Alternatively, one could write this budget constraint including the household's choice of buying and selling shares in all firms. In equilibrium, its portfolio would have to coincide with the distribution of firms in the economy, and stock prices would be given by the firms' value functions below. The distinction between these two formulations is immaterial in terms of competitive equilibrium allocations and prices.

[5] In the model, there is a single industry. However, these coefficients are remarkably similar in the data across the considered industries, hence the simplifying assumption of a single industry does not appear to be very consequential.

β	δ	ψ	α_k	α_l	$\sigma_{inn,s}$	ρ_s	q
0.96	0.065	2.15	0.3	0.45	0.52	0.9	0.37

Table 7: Parameter values

4.4 The distribution of marginal products: asymmetric persistence

We solve our general equilibrium model and study its implications for our moments of interest. In particular, we focus on the implications of the model for the equilibrium distribution of marginal products and its dynamic properties. Figure 4 shows a histogram of the distribution of marginal products of capital in the calibrated economy, while Table 8 illustrates the model-implied estimates for the transition probabilities across terciles of this distribution. Despite not being targeted, these transition probabilities are close to their empirical counterparts in Table 2. In particular, notice that the model captures the asymmetry in persistence: the probability of staying in the left tail (low marginal product) is higher than the probability of staying in the right tail (high marginal product).

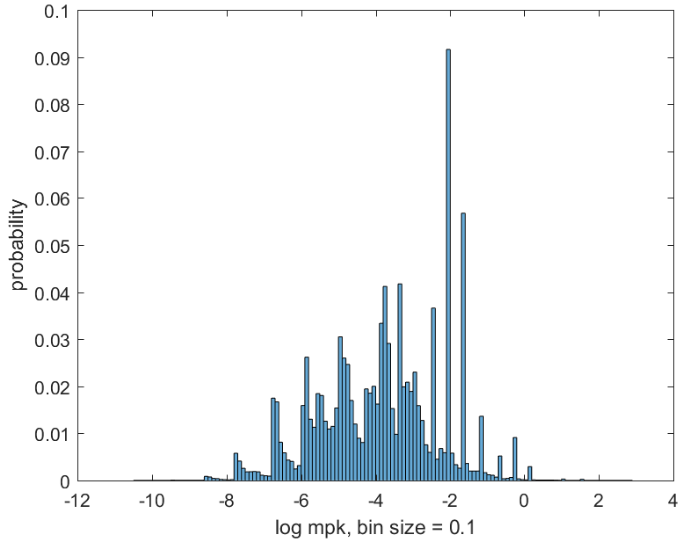

Figure 4: Distribution of MPK: baseline calibration

In order to inspect the mechanism that generates this result, we solve the model in

Table 8: Transition Matrices for MPK terciles: baseline calibration

	1	2	3
1	.85	.12	.03
2	.13	.74	.14
3	.03	.14	.83

absence of any irreversibility ($q = 1$), while mantaining the assumption of one-period time to build. In this case, the standard deviation of marginal products is reduced by 41%. The transition probabilities for the terciles of the distribution are reported in Table 9 . As can be seen, while time to build induces static dispersion, it cannot generate asymmetry in persistence. This exercise confirms the role of partial irreversibility in explaining this important feature of the data. Firms hit by adverse productivity shocks find it hard to downsize quickly because of this technological friction, hence their marginal product stays low for several periods. On the other hand, time to build is the only friction slowing down the growth path of firms hit by positive shocks, in the model without financial frictions.

Table 9: Transition Matrices for MPK terciles: time to build only

	1	2	3
1	.33	.33	.33
2	.33	.33	.33
3	.33	.33	.33

4.5 Financial frictions

In this subsection, we compare our results with the outcomes of a model with financial frictions. To do so, we specialize our general model as follows. We remove the irreversibility friction (by setting $q = 1$) and introduce a constraint that prevents firms from access to any external finance ($\lambda = 0$). This can be considered an upper bound on the role of collateral constraints, as firms can only finance investment out of their own retained earnings. This version of the model generates a standard deviation of marginal products of capital equal to 1.23. In order to provide a fair comparison across model specifications, we re-calibrate the partial irreversibility model to match this same moment, and obtain a new estimate $q = .71$.

Both financial frictions and irreversibility can generate static dispersion in marginal products. Hence, this moment is insufficient to identify them from one another. However, these two frictions have opposite implications for the asymmetry of persistence of marginal products. This is easily seen by comparing Table 10 (financial frictions) with Table 11 (partial irreversibility). With financial frictions, it is actually more likely for firms to remain in the right tail of the distribution than to remain in the left tail. This is in contrast with the empirical evidence discussed in Section 3.

Table 10: Transition Matrices for MPK terciles: financial frictions

	1	2	3
1	.71	.13	.16
2	.19	.69	.12
3	.10	.17	.72

Table 11: Transition Matrices for MPK terciles: partial irreversibility (recalibrated)

	1	2	3
1	.76	.15	.09
2	.12	.69	.19
3	.12	.17	.72

To the best of our knowledge, ours is the first paper that shows this (counterfactual) implication of this class of models that the literature (e.g., Banerjee and Moll (2010)) has used to study the persistence of capital misallocation. Going forward, our analysis suggest that we may need to rethink about the role of financial constraints in inducing persistence dispersion of marginal products. Models that incorporate frictions in downsizing appear to be more promising in matching this feature of the data.

One potential limitation of this argument is given by the presence of right skewness in the empirical distribution of MRPK. In our model, assuming log-normal productivity shocks, this feature arises under financial frictions, but not under partial irreversibility. However, the productivity shocks we recover from our empirical estimates appear to be right-skewed. Hence, it appears that this feature of the MRPK distribution may be induced by the nature of the shocks, rather than by the frictions. We also checked whether asymmetric persistence was driven by the stochastic process for productivity, but found no evidence of it, consistently with our findings on the persistence of marginal products of labor and other factors.

4.6 Aggregate TFP and the source of dispersion in marginal products

Identifying different types of frictions is not only key for our understanding of the mechanisms preventing reallocation at the microeconomic level, but also, importantly, for the measurement of aggregate productivity. In this subsection, we use our quantitative model to show that aggregate TFP takes very different values depending on the underlying friction, even for a same amount of dispersion in marginal products. In particular, TFP losses are overestimated under the assumption of financial frictions, relative to their size in the economy with partial irreversibility. In the next subsection, we provide an analytical example, where we prove that, consistent with the numerical result, the TFP losses arising from distortions in the right tail of MRPK are larger than those induced by distortions in the left tail.

Hsieh and Klenow (2009) show that, when marginal products and productivity are jointly log-normally distributed, the dispersion of log MRPK is a sufficient statistic for the level of aggregate TFP (and for its ratio relative to its efficient level). However, marginal products are unlikely to be log-normally distributed and we argue that the deviations from this benchmark distribuition are quantitatively important for the aggregation exercise that allows to compute TFP losses. In particular, we argue that for a given standard deviation of log MRPK, TFP losses are larger if the distortions are affecting the right tail (as in the case of financial frictions) than if they are affecting the left tail (as in the case of partial irreversibility).

To make this point in the context of our model, we compare again the economy with financial frictions and the economy with partial irreversibility calibrated to match the same dispersion of MPRK. We compute aggregate TFP in both economies and compare these two values with the level of TFP that arises if time to build is the only reason why marginal products cannot be equalized. We find that the TFP loss associated with the financial frictions model is 11%, while the loss induced by partial irreversibility is only 3%. This suggests that static measures of MPK dispersion are far from being sufficient to make inference about aggregate TFP. Specifically, attibuting the observed dispersion to financial frictions leads to a substantial negative bias in the estimate of TFP (or upward bias in the estimate of the TFP loss from misallocation).

4.7 Aggregate TFP and asymmetric distortions: an analytical result

To further inspect the difference between distortions that affect firms with high or low marginal products, we now provide an analytical result for a simple economy that has a closed-form solution. Our analytical result supports the numerical result described in the previous subsection. There is a continuum of firms of total mass equal to three. Capital is the only input in production. The production function is $y = sk^\alpha$ and the firm productivity level is constant and uniformly distributed on the discrete support $\{e^{-\sigma}, 1, e^{\sigma}\}$, with $\alpha \in (0, 1)$ and $\sigma > 0$.

As in our main model, let β be the discount factor, δ be capital depreciation and (without loss of generality, given that productivity is constant) let there be one period time to build for capital. The constant firm size that solves a standard firm value maximization is

$$k^*(s) = \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \tag{9}$$

In the frictionless equilibrium, marginal products are equalized and all (log) marginal products are equal to $-\log(\frac{\beta}{1-\beta(1-\delta)})$. Aggregate TFP is defined as the ratio $\frac{Y}{K^\alpha}$ where Y and K are aggregate output and aggregate capital respectively. In the frictionless equilibrium this equals $TFP^* = (e^{\frac{-\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})^{(1-\alpha)}$.

We now consider two distortions. In the first case, we give e^Δ, with $\Delta \geq 0$, extra capital to firms with productivity level $e^{-\sigma}$ (that is, low productivity firms). We denote the TFP level associated with this distortion with a prime. In the second case, we take away e^Δ capital from firms with productivity level e^σ (that is, high productivity firms). We denote the TFP level associated with this distortion with a double prime. Clearly, both distorted economies coincide with the undistorted one for $\Delta = 0$. We will consider small deviations away from this undistorted benchmark. Notice that these distortions do not lead to the same size for the aggregate capital stock. In this sense we depart from the related analysis of Hopenhayn (2014), which considers distortions that preserve the total amount of productive factors.

First, notice that it is easy to microfound these distortions with associated wedges in the cost of capital, along the lines used in the framework of Hsieh and Klenow (2009). The first distorted economy mimics the effect of partial irreversibility, by making unproductive firms larger than at the optimum, while the second distorted economy mimics the effect of a financial frictions that makes productive units smaller than at the optimum. Second, it is easy to verify that these two distortions give rise to the same standard deviation of log marginal products. This is because of the symmetry of two implied distributions of log marginal products. Despite having the same degree of dispersion in log marginal products, these two economies have different TFP levels, and hence feature different TFP losses relative to the frictionless benchmark TFP^*. More specifically, we now prove the following result: distorting high productivity firms gives rise to larger TFP losses, consistently with our numerical findings on the difference between financial frictions and irreversibility.

Proposition 1 *For $\Delta > 0$ sufficiently small, $TFP' > TFP''$.*

Proof The two distorted TFP levels are

$$TFP' = \frac{e^{\alpha\Delta - \frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}}}{(e^{\Delta - \frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})^\alpha} \tag{10}$$

and

$$TFP'' = \frac{e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\alpha\Delta}}{(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\Delta})^{\alpha}} \tag{11}$$

Let $r(\Delta) \equiv \frac{TFP'}{TFP''}$. We prove the following steps that suffice for our proposition: (i) $r(0) = 1$, (ii) $r'(0) = 0$ and (iii) $r'(\Delta) > 0$ for Δ sufficiently small. This establishes that $r(\Delta)$ is bounded below by 1 in a neighborhood of the undistorted economy, that is, for sufficiently small distortions Δ.

Step (i) is immediate and is easily verified by setting the distortion to zero. For steps (ii) and (iii), we compute the derivative of the ratio r with respect to the distortion Δ and, after some algebra, we get

$$r'(\Delta) = A\left[\frac{1}{(e^{\Delta-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\Delta})} - \frac{1}{(e^{\alpha\Delta-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\alpha\Delta})}\right] \tag{12}$$

where $A = \alpha\frac{(1+\frac{e^{\Delta}-1}{(e^{\Delta-\frac{\sigma}{1-\alpha}}+1+e^{\frac{\sigma}{1-\alpha}})})^{\alpha}}{(1+\frac{e^{\alpha\Delta}-1}{(e^{\alpha\Delta-\frac{\sigma}{1-\alpha}}+1+e^{\frac{\sigma}{1-\alpha}})})}(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})$

Notice that $\alpha\frac{A^{\alpha}}{B}(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}}) > 0$, hence the sign of r' equals the sign of the expression in the square bracket, which in turn equals the sign of

$$\frac{(e^{\alpha\Delta-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\alpha\Delta})}{(e^{\Delta-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}})(e^{-\frac{\sigma}{1-\alpha}} + 1 + e^{\frac{\sigma}{1-\alpha}-\Delta})} - 1$$

Clearly, this expression equals zero for $\Delta = 0$, proving (ii). For step (iii), it is convenient to adopt the following change of variable. Let $x \equiv e^{\Delta}$. It follows that $e^{\alpha\Delta} = x^{\alpha}$. Consider the denominator of the expression above. Using our change of variable, we can express it as $\frac{ax^2+bx+c}{x}$ where $a = e^{-\frac{2\sigma}{1-\sigma}} + e^{-\frac{\sigma}{1-\sigma}}$, $b = 3 + e^{-\frac{\sigma}{1-\sigma}} + e^{\frac{\sigma}{1-\sigma}}$, $c = e^{\frac{\sigma}{1-\sigma}} + e^{-\frac{\sigma}{1-\sigma}}$. The numerator can be expressed similarly, in particular with the same quadratic equation, except with x^{α} instead of x. Hence, the sign of r' equals the sign of

$$f(x) \equiv \frac{x}{x^{\alpha}}\frac{(x^{\alpha}-\rho_1)}{(x-\rho_1)}\frac{(x^{\alpha}-\rho_2)}{(x-\rho_2)} - 1$$

where ρ_1 and ρ_2 are the roots of the above quadratic equation.

Consider the derivative $f'(x)$ evaluated at $x = 1$. We get $f'(1) = \frac{1-\alpha}{(1-\rho_1)(1-\rho_2)}(\rho_1\rho_2 - 1)$.

The two roots satisfy $\rho_1 + \rho_2 = -\frac{b}{a} < 0$ and $\rho_1 \rho_2 = \frac{c}{a} > 1$, hence $f'(1) > 1$. This implies that the sign of r' is positive in a neighborhood of $\Delta = 0$, which is step (iii). QED.

This result shows that understanding which tail of the distribution of marginal products is more affected by frictions is not only important for identification of the underlying frictions, but also in order to correctly compute aggregate productivity. An implication of our result is that if we attribute all the dispersion in marginal products to financial frictions (i.e., to the right tail of the distribution of marginal products), we are likely to underestimate aggregate TFP, and hence overestimate the TFP losses from misallocation.

5 Conclusions

While large attention has been devoted to mechanisms that generate persistent capital "misallocation" because productive firms face obstacles on their growth path (such as financing constraints), much less attention has been directed to explore the importance of frictions faced by unproductive firms in their downsizing process. This paper represents a step to fill this gap in the literature, both empirically and theoretically.

Using a panel of Peruvian firms, we have documented that persistent dispersion in marginal products of capital is explained to a large degree by persistent low marginal products. Next, we have shown that this feature of the data arises naturally in a model where capital is partially specific at the firm level, making investment partially irreversible. We have also argued that distinguishing what friction is preventing equalization of marginal products is important in order to assess the degree of TFP losses, for a given degree of dispersion of marginal products.

More work remains to be done in order to fully explore the importance of capital specificity for the distribution of marginal products and, ultimately, for the level of aggregate productivity of an economy. First, we will dig deeper into the role of capital specificity, by using a unique feature of the Peruvian data. By matching them with import data, we will be able to observe the capital composition at the firm level, hence establishing a more direct link between the degree of capital specificity and the properties of marginal products.

Second, we will use our methodology, based on the study of the mobility of marginal products across quantiles, to establish whether the key facts of this paper hold in our datasets that the literature has used, both across countries and over time. This will allow us to address more general questions such as: is persistence in the left tail a characteristic of developing countries, or is it a universal fact?

Finally, in this paper we have focused on stationary equilibrium. In the future, we will study the importance of (endogenous) investment irreversibility in the context of transitional dynamics after shocks that drive changes in the industry composition of the economy. Industries that are negatively affected downsize slowly and underutilize their capital, as the resale value of their specific capital is low. In this sense, the persistence of low marginal products may induce persistence in the dynamics of aggregate TFP.

References

Adamopoulos, T. and D. Restuccia (2014): “The Size Distribution of Farms and International Productivity Differences,” *American Economic Review*, 104, 1667–97.

Amaral, P. S. and E. Quintin (2010): “LIMITED ENFORCEMENT, FINANCIAL INTERMEDIATION, AND ECONOMIC DEVELOPMENT: A QUANTITATIVE ASSESSMENT*,” *International Economic Review*, 51, 785–811.

Asker, J., A. Collard-Wexler, and J. D. Loecker (2014): “Dynamic Inputs and Resource (Mis)Allocation,” *Journal of Political Economy*, 122, 1013–1063.

Banerjee, A. V. and E. Duflo (2005): “Chapter 7 Growth Theory through the Lens of Development Economics,” Elsevier, vol. 1, Part A of *Handbook of Economic Growth*, 473 – 552.

Banerjee, A. V. and B. Moll (2010): “Why Does Misallocation Persist?” *American Economic Journal: Macroeconomics*, 2, 189–206.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013): “Cross-Country Differences in Productivity: The Role of Allocation and Selection,” *American Economic Review*, 103, 305–34.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): “Finance and Development: A Tale of Two Sectors,” *American Economic Review*, 101, 1964–2002.

Buera, F. J. and Y. Shin (2013): “Financial Frictions and the Persistence of History: A Quantitative Exploration,” *Journal of Political Economy*, 121, 221–272.

Charles, K. K. and E. Hurst (2003): “The Correlation of Wealth across Generations,” *Journal of Political Economy*, 111, 1155–1182.

Chetty, R., N. Hendren, P. Kline, E. Saez, and N. Turner (2014): “Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility,” *American Economic Review*, 104, 141–47.

Cooper, R. W. and J. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 73, 611–633.

Epifani, P. and G. Gancia (2011): "Trade, markup heterogeneity and misallocations," *Journal of International Economics*, 83, 1 – 13.

Erosa, A. (2001): "Financial Intermediation and Occupational Choice in Development," *Review of Economic Dynamics*, 4, 303 – 334.

Gavazza, A. (2011): "Leasing and Secondary Markets: Theory and Evidence from Commercial Aircraft," *Journal of Political Economy*, 119, 325–377.

Guner, N., G. Ventura, and Y. Xu (2008): "Macroeconomic implications of size-dependent policies," *Review of Economic Dynamics*, 11, 721 – 744.

Hopenhayn, H. (1992): "Entry and Exit in Long Run Equilibria," *Econometrica*, 60.

——— (2014): "On the Measure of Distrotions," *NBER Working Paper 20404*.

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India*," *The Quarterly Journal of Economics*, 124, 1403.

Lanteri, A. (2016): "The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle," Mimeo, Duke University.

Medina, P. (2017): "Import Competition, Quality Upgrading and Exporting: Evidence from the Peruvian Apparel Industry," Mimeo, University of Toronto.

Midrigan, V. and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104, 422–58.

Pages, C. (2010): *The Age of Productivity*, New York: Palgrave Macmillan US, 1–21.

Pavcnik, N. (2002): "Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants," *The Review of Economic Studies*, 69, 245–276.

Restuccia, D. and R. Rogerson (2008): "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 11, 707 – 720.

——— (2013): "Misallocation and productivity," *Review of Economic Dynamics*, 16, 1 – 10.