Two-sided Search in International Markets

(preliminary draft)

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1 Introduction

To break into global markets, either as an exporter or an importer, firms must first identify foreign business partners. And since most of international partnerships are short-lived, trading firms must continually seek new connections if they wish to maintain or expand their foreign market presence. The resulting patterns of international buyer-seller connections are surprisingly fluid, and they largely determine the dynamics of firm-level trade flows.

Herein we develop a new empirical model of these search and matching processes, quantify the associated costs, and explore their implications for trade dynamics and welfare. Specifically, we develop a dynamic model of trade in consumer goods with three types of agents: foreign exporters, domestic retailers, and domestic consumers. Heterogeneous exporters and retailers engage in costly search for one another, taking stock of their current situation and the structure of the buyer-seller network. The resulting matching patterns determine which retailers carry which varieties of goods. Consumers then choose how to allocate their expenditures across retailers and the individual goods that they offer. When a retailer and an exporter form a new business relationship, they divide they associated rents in a forward-looking Nash bargaining game, thereby determining the wholesale prices at which trade occurs. The retailer then passes the goods on to domestic consumers after adding an optimal mark-up.

Fit to customs records on Colombian footwear imports, our model speaks to a variety of empirical issues.[1] First, it provides estimates of the value of international business connections for different types of agents with different portfolios of business partners. Second, it allows us to decompose trade and welfare changes into two basic driving forces: market entry by

[1] An application to U.S. apparel importers is in progress.

Chinese firms, and reductions in search costs. Similary, it quantifies the capital gains and losses induced by these two types of shocks for different types of firms. Third, it characterizes the effects of search costs and foreign competition on firm dynamics. Finally, since firms with more clients find it less expensive to meet additional business partners, and since the rate at which firms acquire connections is partly due to luck, it quantifies the extent to which large firms owe their success to fortuitous events early in their life cycles.

Our model is related to a wide variety of earlier contributions. First, speaking broadly, it follows in the tradition of papers that analyze trade with information frictions, beginning with work by Rauch (2001), Rauch and Trindade (2002), and Rauch and Watson (2003). In this literature, firms face uncertainty about the appeal of their products to foreign buyers (Rauch and Watson, 2003; Albornoz *et al.*, 2012; Eaton *et al.*, 2014), about prices in a foreign location at a particular time (Allen, 2014; Steinwender, 2014), or about the identity and location of potential foreign clients (Albornoz *et al.*, 2012; Rauch and Watson, 2003; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Antras and Costinot, 2011; Fernandez-Blanco, 2012). Our analysis focuses on the latter.

Second, it resembles a number of recent trade papers in its emphasis on customer accumulation as a driver for firm dynamics (Albornoz *et al.*, 2012; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Piveteau, 2015; Fitzgerald *et al.*, 2016).[2] We depart from these papers by treating both exporters and importers as choosing their search intensity optimally. This formulation better conforms to actual practices, and allows us to generate richer exporter-importer network

[2]Interest in this approach to firm dynamics is not confined to the trade literature. Recent contributions that focus on the accumulation of domestic customers include Foster *et al.* (2015) and Gourio and Rudanko (2014).

structures than would have been possible with a one-sided search model.

By making retailers a central feature of our model, we also contribute to the literature on intermediated trade. This includes papers that predict which kinds of exporters will use intermediaries (Blum *et al.*, 2009; Ahn *et al.*, 2011), and more relevant to our work, papers on the effects of trade on welfare under different types of intermediation and bargaining (Rauch and Watson, 2004; Antras and Costinot, 2011; Fernandez-Blanco, 2012; Bernard and Dhingra, 2015). Among these latter papers, the one most closely related to ours is Bernard and Dhingra (2015). Therein, exporters bargain with retailers abroad in order to avoid double marginalization and (in some cases) the price-depressing effects of competition among retailers. We too invoke a Nash bargaining game between retailers and exporters, but our focus is not on the endogenous choice of contract form.

Finally, we contribute to the literature on the life-cycle of exporters and importers. As with the earlier literature on firm dynamics, our model is partly motivated by the fat-tails that typically characterize firm-size distributions. One way earlier studies have generated these tails is through stochastic shocks to firm productivity or demand, coupled with either the possibility for entrants to imitate incumbents (Luttmer, 2007) or an exogenous drop in the productivity of old firms (Luttmer, 2011). Another possibility for generating fat tails is to use a matching model and a convenient search cost function (Klette and Kortum, 2004; Eaton *et al.*, 2014). We follow the latter modeling strategy. In particular, we assume that the cost for a firm to find an additional client decreases as the number of its incumbent clients increases.

2 Data and Stylized Facts

Our modelling choices are partly motivated by the stylized facts that have recently emerged concerning international links between buyers and sellers. Studies reporting such facts are now available for the United States and Colombia (Eaton *et al.*, 2008, 2014; Bernard *et al.*, 2014), Chile (Blum *et al.*, 2010), Mexico (Sugita *et al.*, 2014), Norway (Bernard *et al.*, 2014), and Ireland (Fitzgerald *et al.*, 2016). Our formulation is additionally motivated by the dynamics of these buyer-seller relationships.

Below we present these facts for the population of Colombian firms that import footwear and their suppliers abroad. This choice of network reflects several considerations. First, by studying goods that are mainly supplied by foreign producers, we minimize the importance of domestic suppliers, whose connections we are unable to observe. Second, by choosing a sector in which most of the importers are wholesale/retail firms, we are able to keep the buyer side of the market relatively simple. That is, within each wholesale or retail firm, revenue functions are nearly separable across categories of consumer goods, and firms' payoff functions can be reasonably approximated with relatively simple expressions.

2.1 Data Sources

We base our analysis on data obtained from the Colombian customs authority: Dirección de Impuestos y Aduanas Nacionales de Colombia (DIAN). These data describe all merchandise shipments to Colombia. Each record includes a ten-digit Harmonized Schedule (HS) product code, shipment value, shipment quantity, entry or exit port, date of transaction, mode of transportation (land, sea, air), and the domestic firm's tax identification number (NIT).

Critically for our study, each record also includes the name and address of the foreign firm that is party to the transaction.

In order to keep track of foreign suppliers, we construct an alphanumeric foreign exporter ID for each shipment in the database. It is based on the business names and addresses that appear in the customs records. For example, one version of this ID combines the firm's country code, first three letters of the first two main words in the firm's name, the street address, and the first three letters of the city name.[3] These codes are imperfect identifiers because, despite standardization, the same firm may appear in different records with slight differences in its spelling or address. The longer the string identifier, the more frequently this problem is likely to occur. On the other hand, when short string IDs are used, firms with names and street addresses that begin the same may become indistinguishable. Robustness checks and visual inspections of the data (in progress) will give us a sense for the importance of this issue.

2.2 Stylized Facts: Aggregates

We now document some stylized facts that motivate our model, focusing on the period 2006-2013.[4] Table 1 reports time series on the total number of Colombian footwear importers, the total number of footwear exporters serving the Colombian market, the number of importer-exporter matches, and the total value of imports in millions of U.S. dollars.

[3]Before constructing strings, names and street addresses were standardized using the Stata routines "stnd_compname" and "stnd_address." Information on zip codes and states was also used in variants of the ID string.

[4]We begin our sample in 2006 because there is a large drop of the number of importers from 2005 to 2006 with no large economic shocks. This is most likely due to the change of the registration system of importers for the textile/footwear (Decree 1299 of 2006).

Year	Importers	Exporters	Matches	Total Volume
2006	522	892	1847	177
2007	518	1041	2066	215
2008	527	979	1992	251
2009	572	928	1922	251
2010	630	1200	2391	320
2011	822	1602	3341	481
2012	852	1699	3375	575
2013	947	1569	2989	492

Table 1: Number of Importers, Exporters, and Matches

Note first that the aggregates are fairly stable during $2006 - 2009$, so this period serves as a good benchmark. But thereafter Colombian imports grew rapidly, as did the number of matches and the number of exporters supplying the Colombian market. Further, total exports grew more rapidly than the number of exporters or the number of matches, so the typical exporter to Colombia increased its sales per business partner between 2010 and 2013.

The country's rapid post-2009 import growth most likely traces to two main factors. First, a large number of low-cost Chinese footwear producers emerged and penetrated the Colombian market during this subperiod, and second, Colombia's government unilaterally reduced tariffs during $2010 - 2011$. It also aggressively negotiated the formation of free trade areas (FTAs), most notably with U.S. and Panama for footwear in 2013.

To examine the role of Chinese exporters more closely, we next break down Colombian footwear imports by country of origin. Table 2 reports time series on the number of exporters and aggregate export values for the main countries that directly supply Colombian footwear retailers and distributors. Here we note that, by far, the largest surge in export values came from Pamana. This is because Panama operates the largest free trade zone in the western hemisphere, and Chinese exporters of consumer goods have routinely used the trading compa-

Year	China	USA	Panama	Brazil	HK	Ecuador
Number of Sellers						
2006	201	154	202	64	42	7
2007	276	192	184	77	46	10
2008	273	188	144	70	40	5
2009	257	195	145	51	44	7
2010	341	242	180	65	67	6
2011	386	443	234	76	69	6
2012	451	377	245	80	70	5
2013	371	319	226	69	90	13
Value of Exports (Millions of USD)						
2006	49.2	3.93	67.4	16.9	4.35	23.5
2007	71.9	5.42	67.3	18.1	4.97	27.5
2008	79.4	6.84	85.6	16.9	4.70	27.2
2009	65.9	5.58	105	16.1	7.97	27.8
2010	93.7	6.89	136	20.6	8.53	25.5
2011	130	16.0	223	30.2	13.0	27.4
2012	179	19.2	256	36.7	17.5	17.6
2013	114	14.0	234	39.8	15.1	21.1

Table 2: Major Countries of Direct Sellers

nies located therein to reach the Colombian market.[5] Accordingly, while direct exports from China to Colombia did not grow particularly rapidly after 2009, low-cost footwear from China was very much behind the post-2009 import surge.

[5]Colombian customs records show both the "exporting country" and the "country of origin," so it is possible to determine where imports arriving in Colombia from Panama "last underwent substantial transformation." However, since Panamanian trading companies typically take ownership of the goods they sell to Colombian importers, the names of the Chinese manufacturing firms do not appear on the invoices of the Chinese-made goods that arrive in Colombia via Panama. This means we must treat the Panamanian firms as the exporters searching for business partners when we fit our model to the data.

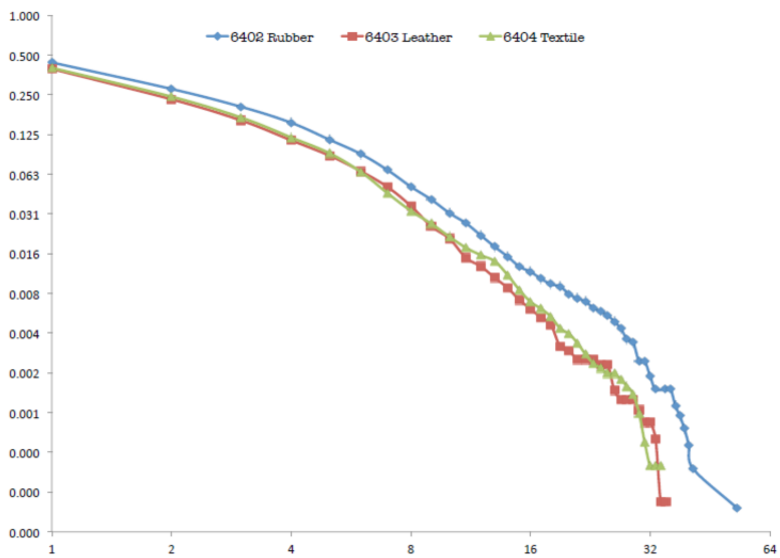

Figure 1: Degree distribution: sellers per buyer, 2009

2.3 Stylized Facts: Buyer-Seller Distributions

We next exploit our buyer and seller IDs to summarize the frequency distributions of the sellers (a.k.a. exporters) per buyer (a.k.a. importer) and buyers per seller for the Colombian footwear's international market.[6] Figure 1 and Figure 2 depict the degree distributions for the three main HS4 categories of shoes: rubber, leather, and textile. On the horizontal axis, we have the number of connections of a buyer or seller. On the vertical axis, we report the inverse empirical CDF. Both axes are in log scale, so if the data were distributed according to a "power law" the lines would be linear.

Several patterns emerge. First, the distributions are quite similar for all types of footwear, so we will not be emphasizing cross-product distinctions much hereafter. Second, the tail of the distribution of sellers per buyer in Figure 1 begins to curve downward almost immediately, suggesting that no portion of the distribution is well-approximated by the Pareto distribution.

[6] We will use "buyer" interchangeably with "importer" and "exporter" interchangeably with "seller."

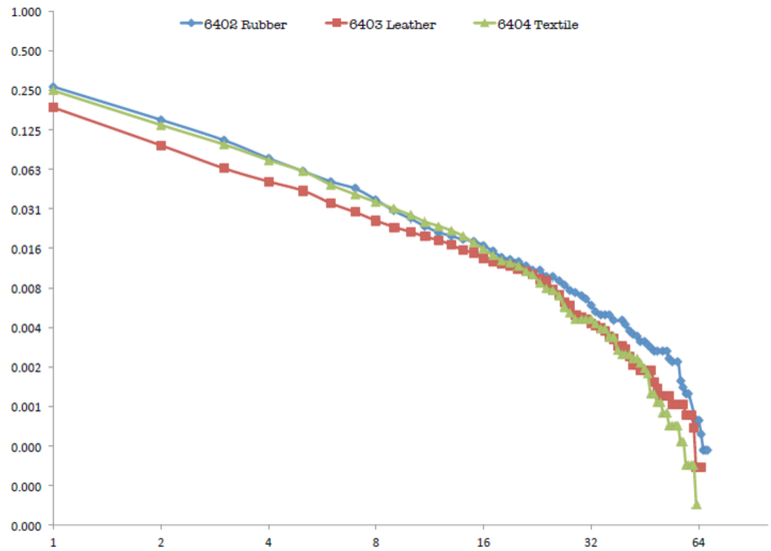

Figure 2: Degree distribution: buyers per seller, 2009

Finally, the distribution of buyers per seller (Figure 2) is approximately power law in the left-hand tail. That is, out to about 20 buyers, the distribution is roughly Pareto.

Have these shapes changed over time? Table 3 reports the coefficients from regressions fit to log-log plots like those in Figure 1 for different products and years. The regression slopes have become flatter as trade has increased, indicating that there are relatively more large buyers in 2013 than there were in 2009.

2.4 Stylized Facts: Transition rates

Finally, given that our model will generate predictions on firm-level matching dynamics, it is useful to examine the overtime transitions rates for seller counts. We report these in Table 4. Several patterns are worth highlighting here. First, there is a non-trivial probability that one buyer's connections get completely eliminated from one year to the next. Some of these transitions to zero sellers reflect exit of the retailer, but most occur because the retailer

	2009	2013	2009	2013	2009	2013
	6402	6402	6403	6403	6404	6404
# sellers	rubber	rubber	leather	leather	textile	textile
1	0.560	0.432	0.608	0.475	0.600	0.446
2	0.163	0.180	0.161	0.176	0.158	0.189
3	0.075	0.100	0.072	0.090	0.074	0.091
4	0.049	0.064	0.046	0.061	0.049	0.062
5	0.039	0.040	0.027	0.047	0.029	0.042
6	0.025	0.035	0.021	0.026	0.025	0.033
7	0.022	0.023	0.016	0.022	0.020	0.031
8	0.017	0.027	0.014	0.022	0.012	0.017
9	0.010	0.017	0.011	0.017	0.007	0.017
10	0.008	0.014	0.005	0.013	0.005	0.013
Regression coef	-2.005	-1.873	-2.176	-1.986	-2.203	-1.949

Table 3: Degree Distribution: Seller's per Buyer

$t \backslash t+1$	0	1	2	3	4	5	6	7	8	9	10	10+
1	0.529	0.316	0.105	0.015	0.010	0.010	0.003	0.004	0.000	0.004	0.001	0.003
2	0.263	0.285	0.219	0.109	0.055	0.022	0.007	0.011	0.007	0.000	0.007	0.015
3	0.227	0.174	0.167	0.152	0.061	0.114	0.008	0.045	0.015	0.008	0.000	0.030
4	0.182	0.156	0.104	0.195	0.104	0.091	0.039	0.052	0.013	0.026	0.026	0.013
5	0.120	0.067	0.107	0.107	0.147	0.107	0.133	0.067	0.027	0.027	0.027	0.067
6	0.149	0.064	0.064	0.021	0.128	0.128	0.043	0.064	0.128	0.085	0.064	0.064
7	0.085	0.021	0.085	0.064	0.043	0.106	0.170	0.128	0.128	0.064	0.043	0.064
8	0.063	0.000	0.063	0.031	0.063	0.125	0.156	0.063	0.188	0.094	0.125	0.031
9	0.069	0.000	0.069	0.103	0.034	0.034	0.138	0.034	0.138	0.103	0.034	0.241
10	0.050	0.000	0.050	0.100	0.000	0.150	0.100	0.000	0.100	0.050	0.100	0.300
10+	0.098	0.009	0.027	0.000	0.027	0.027	0.027	0.036	0.054	0.045	0.045	0.607

Table 4: Transition Matrix of Sellers per Buyer

stopped stocking imported shoe varieties. Second, there is a general tendency for buyers to lose suppliers, on net. This is implied by the fact that, for any row, the probability mass to the left of the diagonal exceeds the mass to the right. Overall, this transition pattern is consistent with the cross-sectional distribution, in that both reflect a large probability mass at the lower numbers of connections.

3 A model of buyer-seller networks

Motivated by the stylized facts described above, we now develop a continuous-time two-sided search model. As depicted in Figure 3, our model is populated by three types of agents: sellers, buyers, and consumers. Sellers provide goods to buyers in the wholesale market,who pass them on to consumers in the retail market. Though we are thinking of sellers as foreign merchandise exporters and buyers as the domestic retailers they supply, our model could be applied in contexts that do not involve international trade.

Consumers acquire goods exclusively through retailers, who offer different but possibly overlapping menus of products, depending upon the set of suppliers they are currently partnered with. Retailers are also vertically differentiated in terms of the amenities they offer, like locational convenience, ambiance, and service. As a group, consumers allocate their expenditures across retailers in a way that reflects their preferences for amenities and product menus.

The dimensions of retailer heterogeneity are publicly observable, so consumers' expenditure patterns are characterized by a standard static optimization problem with full information. However, buyers and sellers in the wholesale market are unable to costlessly match with one another. Rather, each type of agent must invest in costly search to establish new business partnerships.

Because it is costly to find new business partners, buyers and sellers create rents when they meet one another. They bargain continuously and bilaterally over these rents, and the expected outcomes of these bargaining games determine the expected returns to successful search for each party.

Other things equal, the more intensively an agent searches, the higher the hazard rate with which she finds new partners and reaps her share of the associated rents. But these hazard rates depend upon other things as well.

First, matching hazards are influenced by market tightness. For example, when many buyers are searching for new suppliers, but not many suppliers are searching for new buyers, matching hazards will tend to be low for buyers and high for suppliers. As we will discuss shortly, the precise way in which search intensities on both sides of the market influence aggregate market tightness is determined by the matching function in our model, which we adopt from the labor-search literature.

Second, the ease with which agents find new business partners depends upon their previous successes. That is, agents who have already accumulated a large portfolio of business partners find it relatively easy to locate still more. This feature of our model, taken from Eaton *et al.* (2014), helps us to capture the "fat-tailed" distributions of buyers across sellers and sellers across buyers discussed above.

3.1 The Retail Market

Preferences and pricing: We now turn to model specifics. As in Akin *et al.* (forthcoming) and Bernard and Dhingra (2015), we start from a nested CES demand structure in which consumers have preferences over retailers, and within retailers, over products. Specifically, assume the retail market is populated by a measure-B continuum of stores, and suppose consumers view these stores as imperfect substitutes, both because they offer distinct amenities and because they carry different—but not necessarily disjoint—sets of products. More pre-

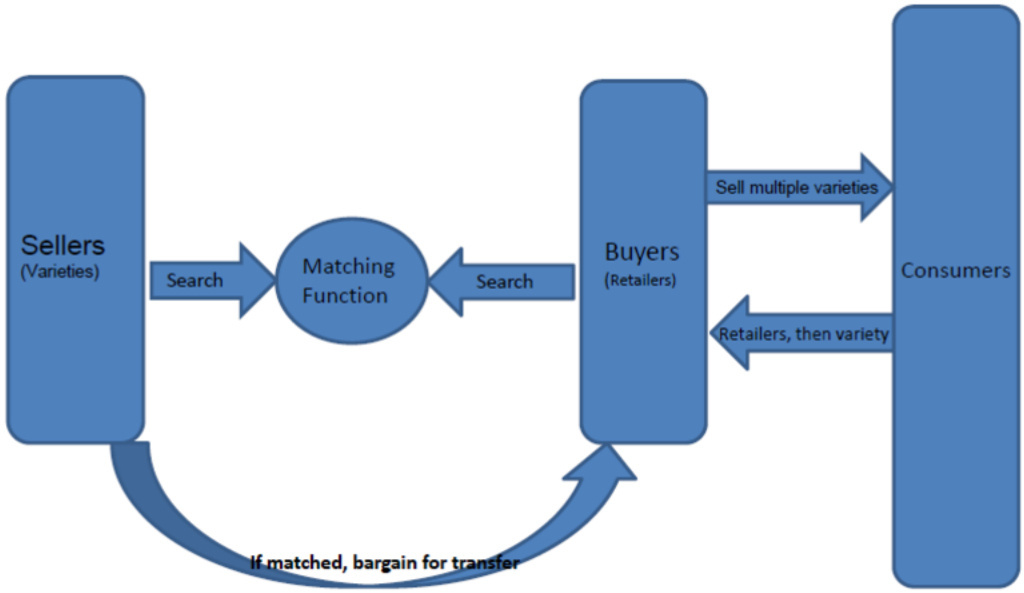

Figure 3: Model diagram

cisely, indexing stores by b and products (or exporting firms) by x, let consumers' preferences over retailers be given by the utility function:

$$C = \left[\int_{b \in B} (\mu_b C_b)^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}},$$

where C_b measures consumption of the set of products, J_b, offered at store b,

$$C_b = \left[\sum_{x \in J_b} (\xi_x C_b^x)^{\frac{\alpha - 1}{\alpha}} \right]^{\frac{\alpha}{\alpha - 1}},$$

and μ_b and ξ_x are exogenous parameters that measure the inherent appeal or quality of retailer b and product x, respectively.[7] This characterization of preferences implies that the exact price index for retailer b is $p_b = \left[\sum \left(\frac{p_{jb}}{\xi_{jb}} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ and the exact price index for retailers as a group

[7]Alternative nesting structrues are possible. In particular, consumers might have preferences over bundles of types of goods, each of which is a CES aggregation over the bundles available from alternative retailers. That is, consumers first allocate spending across product categories, then across retailers in each category.

is $P = \left[\int_b \left(\frac{p_b}{\mu_b}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$.

Because of search frictions, retailers cannot instantaneously adjust the set of products they offer consumers. Rather, at each point in time they take their current offerings as given and engage in Bertrand-Nash competition in the retail market. It follows that the optimal retail prices at store b satisfy

$$q_{xb} + \sum_{j' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b, \tag{1}$$

where $c_{x'b}$ is the marginal cost of supplying product x' to final consumers through retailer b, including the manufacturing and shipping costs incurred by the producer of x' and the retailing costs incurred by b.[8]

Operating profits: Equation (1) implies the standard result that the within-retailer cannibalization effect exactly offsets the cross-store substitution effect, so the mark-up rule is simply (Atkeson and Burstein, 2008; Hottman *et al.*, forthcoming; Bernard and Dhingra, 2015):

$$\frac{p_{xb} - c_{xb}}{p_{xb}} = \frac{1}{\eta}.$$

And since each retailer perceives the elasticity of demand for each of the products it offers to be η, the instantaneous profit flow jointly generated by retailer b and its suppliers is[9]

$$\pi_b^T = \frac{E}{\eta P^{1-\eta}} \left[\sum_{x \in J_b} \left(\frac{\eta}{\eta - 1}\right)^{1-\alpha} \tilde{c}_{xb}^{1-\alpha}\right]^{\frac{1-\eta}{1-\alpha}} \mu_b^{\eta-1}, \tag{2}$$

This formulation is used in Atkin, et al (2015). Which specification is preferable depends upon the importance of transport and shopping time costs to consumers.

[8]Note that since buyer-seller pairs set retail prices to maximize the value of the surplus generated by their business relationships, the double marginalization problem does not arise.

[9]See appendix A for details.

where $\tilde{c}_{xb} = \frac{c_{xb}}{\xi_{xb}}$ is the quality-adjusted marginal cost incurred by buyer-seller pair $x - b$ per unit supplied in the retail market.

3.2 The Wholesale Market and Payoff Functions

Buyer-seller transfers: We can now describe the flow pay-off functions for buyers (retailers) and sellers (foreign exporters) in the wholesale market. Suppose there are I intrinsic buyer types indexed by $i \in \{1, 2, ..., I\}$, so that if buyer b is a type$-i$ retailer, $\mu_b = \mu_i$. Similarly, suppose there are J intrinsic seller types indexed by $j \in \{1, 2, .., J\}$, so that if seller x is type j, matches between this seller and a type$-i$ buyer generates a quality-adjusted marginal cost of $\tilde{c}_{ji}$. Finally, let $\mathbf{s} = \{s_1, s_2, ..., s_J\}$ be a vector of counts of the number of sellers of each type currently matched to a particular buyer. Then by equation (2), the gross profit flow accruing to a type-i buyer and its portfolio of suppliers $\mathbf{s}$ is:

$$\pi_i^T(\mathbf{s}) = \frac{E}{\eta P^{1-\eta}} \left[\sum_j^J \left(\frac{\eta}{\eta - 1} \right)^{1-\alpha} s_j \tilde{c}_{ji}^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_i^{\eta-1} \tag{3}$$

Note that when the elasticity of substitution across retailers exceeds the elasticity of substitution across products ($\alpha > \eta > 1$), this surplus exhibits diminishing returns with respect to the number of suppliers of any type. That is, buyers who add additional sellers reduce total surplus per supplier.

To determine the division of this profit flow between a particular buyer and her portfolio of sellers, we assume that the total surplus associated with a particular buyer-seller match is divided up according to the Stole and Zwiebel (1996) bargaining protocol.[10] As demonstrated

[10]Under the Stole and Zwiebel (1996) bargaining protocol, buyers bargaining continuously with each of the sellers they are matched with, treating each as the marginal supplier.

in the appendix, this implies that at each point in time the profit flow transferred to each type j seller is

$$\begin{aligned}\tau_{ji}(\mathbf{s}) &\approx \beta\frac{\partial \pi_i^T(\mathbf{s})}{\partial s_j} \qquad (4)\\ &= \frac{\beta}{\alpha-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\frac{E}{P^{1-\eta}}\left[\sum_j^J s_j \tilde{c}_{ji}^{1-\alpha}\right]^{\frac{\alpha-\eta}{1-\alpha}}\tilde{c}_{ji}^{1-\alpha}\mu_i^{\eta-1}\end{aligned}$$

where $\beta \in [0,1]$ is a parameter measuring the bargaining strengh of the seller, and the equality is approximate because we have used a derivative to describe a discrete one-unit change in s_j.

Expressing the transfer function in observables: Equation (4) provides a basis for estimating some key parameters of our model, but several tranformations are necessary in order to bring it to the data. First, since $\tau_{ji}(\mathbf{s})$ is not observable, we need to convert it to an expression describing the flow of export payments from a type$-i$ buyer to a type$-j$ seller in state $\mathbf{s}$. Recognizing that exports payments include both exporter profits, $\tau_{ji}(\mathbf{s})$, and compensation for the exporter's production costs, this is straightforward. As shown in the appendix, if some fraction λ of the marginal costs $\tilde{c}_{ji}$ incurred by an $i-j$ partnership is attributable to the seller, her flow export revenues from the partnership are:

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\sum_{\ell=1}^J s_\ell \tilde{c}_{\ell i}^{1-\alpha}\right]^{\frac{\alpha-\eta}{1-\alpha}}\tilde{c}_{ji}^{1-\alpha}\mu_i^{\eta-1}\left[\frac{\beta}{\alpha-1}+\lambda\right]. \qquad (5)$$

Second, neither quality-adjusted marginal costs, $\tilde{c}_{\ell i}$, nor counts of the different types of sellers, s_ℓ, are observable. However, we can eliminate the term in square brackets by using the within buyer i revenue share of a type$-j$ seller:

$$h_{j|i} = \frac{\tilde{c}_{ji}^{1-\alpha}}{\sum_{\ell=1}^J s_\ell \tilde{c}_{\ell i}^{1-\alpha}} \qquad (6)$$

Thus we can rewrite equation (5) in terms of observables and fixed effects:

$$r_{ji}(\mathbf{s}) = (h_{j|i})^{\frac{\alpha-\eta}{\alpha-1}}\frac{E}{P^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\mu_i}{\tilde{c}_{ji}}\right)^{\eta-1}\left[\frac{\beta}{\alpha-1}+\lambda\right] \qquad (7)$$

An even simpler expression obtains in the special case where cost per unit quality does not vary across products within retailers: $\tilde{c}_{ji} = \tilde{c}_i$. Then equation (5) collapses to

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}} \left(\frac{\eta}{\eta - 1}\right)^{-\eta} s^{\frac{\alpha-\eta}{1-\alpha}} \left(\frac{\mu_i}{\tilde{c}_i}\right)^{\eta-1} \left[\frac{\beta}{\alpha - 1} + \lambda\right] \tag{8}$$

where $s = \sum_{\ell=1}^{J} s_\ell$ is the total number of sellers matched to the buyer, an observable variable.

3.3 Search and Matching

3.3.1 Market aggregates and Market Slackness

Next we characterize matching patterns in wholesale markets. For expositional clarity, we focus on the case of a single type of seller, and thereby reduce the vector $\mathbf{s}$ to the scaler, s. The more general case of multiple seller types is treated in our appendix.

First, we introduce variables that measure agents' "visibility." The key feature of these objects is that, for any two agents or groups of agents on the same side of the market, the ratio of their visibilities is also the ratio of their hazards for meeting a new business partner.

Let $M_i^B(s)$ be the measure of type-i buyers with s sellers, and define these buyers' visibility to be:

$$H_i^B(s) = \sigma_i^B(s) M_i^B(s)$$

where $\sigma_i^B(s)$ measures the search intensity of any one of these buyers. Aggregating over types and partner counts, the overall visibility of buyers is measured by:

$$H^B = \sum_{i=1}^{I} \sum_{s=0}^{s_{\max}} H_i^B(s)$$

Analogously, let $M_j^S(n)$ be the measure of type j sellers with n buyers, and suppose each of these sellers searches with intensity $\sigma_j^B(n)$. Then this group's visibility is measured by:

$$H_j^S(n) = \sigma_j^S(n) M_j^S(n),$$

and the overall visibility of sellers to buyers is:

$$H^S = \sum_{j=1}^{J} \sum_{n=0}^{n_{\max}} H_j^S(n)$$

Following much of the labor search literature, we assume a matching function that is homogeneous of degree one in the visibility of buyers and sellers. Specifically we assume that the measure of matches per unit time is given by (Petrongolo and Pissarides, 2001):[11]

$$X = f(H^S, H^B) = H^B \left[1 - (1 - \frac{1}{H^B})^{H^S}\right] \approx H^B \left[1 - e^{-H^S/H^B}\right] \tag{9}$$

From buyers' perspective, we can then define market slackness in manner analogous to random search models:

$$\theta^B = \frac{f(H^S, H^B)}{H^B}.$$

The larger is θ^B, the more matches take place for a given amount of buyer visibility. Likewise, market slackness from sellers' perspective is:

$$\theta^S = \frac{f(H^S, H^B)}{H^S}. \tag{10}$$

Finally, assuming random matching, the share of matches involving buyers of type i with $s > 0$ sellers is:

$$\frac{\sigma_i^B(s) M_i^B(s)}{H^B} \tag{11}$$

[11] Other matching functions are of course feasible here. We have also experimented with $x = \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$.

and the share of matches involving sellers of type j with $n > 0$ buyers is:

$$\frac{\sigma_j^S(n)M_j^S(n)}{H^S}.$$

In the absence of $\tilde{c}$ heterogeneity across seller types, sellers' payoffs do not depend upon j. And if sellers' search cost functions do not depend upon their type either, we can drop the j subscript from $\sigma_j^S(n)$. For the time being we do so.

3.3.2 Optimal search

It remains to characterize the policy functions $\sigma_i^B(s)$ and $\sigma^S(n)$ that maximize the values of agents' expected payoff streams. To do this we introduce buyer and seller search cost functions, which measure the flow cost of sustaining search intensities σ^B and σ^S, respectively:

$$\begin{aligned} k^B\left(\sigma^B, s\right) &= \frac{\left(\sigma^B\right)^{\nu_B}}{(s+1)^{\gamma^B}} \\ k^S\left(\sigma^S, n\right) &= \frac{\left(\sigma^S\right)^{\nu_S}}{(n+1)^{\gamma^S}} \end{aligned}$$

By assumption, search costs are positive and convex in search intensity: $\nu_B, \nu_S > 1$. Also, network effects may reduce the costs of forming new matches as agents' partner counts grow: $\gamma^B, \gamma^S \geq 0$.

Buyer's problem: Given that type-i buyers enjoy profit flow $\pi_i^B(s)$ when they are matched with s suppliers, such buyers choose their search intensity to solve:

$$V_i^B(s) = \max_{\sigma^B}\left\{\frac{\pi_i^B(s) - k^B\left(\sigma^B\right) + s\delta V_i^B(s-1) + \sigma^B\theta^B V_i^B(s+1)}{\rho + s\delta + \sigma^B\theta^B}\right\} \tag{12}$$

where ρ is the rate of time preference and $V_i^B(s)$ is the present value of a type$-i$ buyer that is currently matched with s sellers. Intuitively, the seller reaps profit flow $\pi_i^B(s) - k^B\left(\sigma^B\right)$

until the next event occurs. With hazard $s\delta$ this event is an exogenous termination of one of the s relationships, and with hazard $\sigma^B\theta^B$ it is a new match.

The optimal search policy for type-i buyers with s sellers, $\sigma_i^B(s)$, therefore satisfies

$$\frac{\partial k^B\left(\sigma^B, s\right)}{\partial \sigma^B} = \theta^B\left[V_i^B(s+1) - V_i^B(s)\right]. \tag{13}$$

Sellers' problem: Since sellers have constant marginal costs, the number of buyers they currently supply does not affect their expected returns from adding another one. On the other hand, the seller's payoff function from a particular match, $\tau_i(s)$, depends upon the buyer's type, i, and the buyer's current seller count, s, so ex post, it matters whom sellers match with. The value to any seller of matching with a type$-i$ buyer who has s suppliers is:[12]

$$V_{i,s}^S = \frac{\tau_i(s) + (s-1)\delta V_{i,s-1}^S + \sigma_i^B(s)\theta^B V_{i.s+1}^S}{\rho + s\delta + \sigma_i^B(s)\theta^B}. \tag{14}$$

Intuitively, a business relationship with a type-i buyer who has s suppliers will terminate with exogenous hazard δ, become a relationship with a type-i buyer who has $s-1$ suppliers with hazard $(s-1)\delta$. Analogously, it will become a relationship with a type-i buyer who has $s+1$ suppliers with hazard $\sigma_i^B(s)\theta^B$.

Taking expectations over the population of buyers that sellers might meet, the *ex ante* value of a new relationship is:

$$V^S = \sum_i \sum_{s=0}^{\infty} V_{i,s+1}^S P_i^B(s),$$

where $P_i^B(s) = H_i^B(s)/H^B$ is the relative visibility of buyers who are type$-i$ and have s

[12]The destruction hazard δ is weighted by $(s-1)$ to adjust for the fact that the seller's own relationhip with the buyer may die, in which case the continuation value of this relationship for this seller is zero. Of course V_s^S makes sense only if $s > 0$, as a seller can't have a connection with a buyer with zero sellers.

sellers. So the optimal search intensity for any seller with n buyers satisfies:

$$\frac{\partial k^S\left(\sigma^S, s\right)}{\partial \sigma^S} = \theta^S V^S. \tag{15}$$

3.3.3 Equilibria and Transition Dynamics

Equations of motion: Given that all relationships end with exogenous hazard δ, the equation of motion for the measure of buyers of type i with s sellers is:

$$\begin{aligned} \dot{M}_i^B(s) &= \sigma_i^B(s-1)\theta^B M_i^B(s-1) + \delta(s+1)M_i^B(s+1) \\ &\quad - \left(\sigma_i^B(s)\theta^B M_i^B(s) + \delta s M_i^B(s)\right). \\ s &= 1, ..., s_{\max};\ i = 1, ..., I \end{aligned} \tag{16}$$

This group gains a member whenever any of the $M_i^B(s-1)$ buyers with $s-1$ suppliers adds a supplier, and the hazard of this happening is $\sigma_i^B(s-1)\theta^B$. Similarly, it gains a member whenever any of the $M_i^B(s+1)$ buyers with $s+1$ suppliers loses a supplier because of exogenous attrition, and this occurs with hazard $\delta(s+1)$. By analogous logic, the group loses existing members that either successfully add a supplier (with hazard $\sigma_i^B(s)\theta^B$) or loses one (with hazard δ). Finally, the measure of buyers of type i with $s = 0$ sellers evolves according to:

$$\dot{M}_i^B(0) = \delta M_i^B(1) - \sigma_i^B(0)\theta^B M_i^B(0) \quad i = 1, ..., N_B \tag{17}$$

Replacing B with S and s with n in (16) and (17), the equations of motion for seller measures $M_j^S(n)$ obtain.

Steady state: To characterize the steady state of this system, we set $\dot{M}_i^B(s) = \dot{M}_j^S(n) = 0$ and solve the system of $I \cdot (s_{\max}+1) + J \cdot (n_{\max}+1)$ equations implied by both versions of (16) and (17)–for buyers and sellers. In doing so we, treat the measures of each type of intrinsic type as exogenous constants and impose the adding-up constraints:

$$M_i^B = \sum_{s=0}^{s_{\max}} M_i^B(s) \tag{18}$$

$$M_j^S = \sum_{n=0}^{n_{\max}} M_j^S(n), \tag{19}$$

Transition dynamics: Solving for transition dynamics is more involved. Suppose we wish to find the transition path from one market environment to a new one under perfect foresight. We begin by finding the steady distribution of buyers and sellers across types for the new regime, as well as the associated value functions. We then guess the trajectory of endogenous market-wide aggregates $\{\theta^B(t), \theta^S(t), P(t)\}$ from the initial state to this steady state, and solve for buyer and seller distribution functions using backward induction and finite differencing. Appendix C provides details.

3.4 Introducing assortative matching

Thus far, our model does not allow for the possibility that some retailers specialize in atheletic shoes, while others are more about dress shoes, and still others do both types of business. Nor it does it provide a mechanism through which assortative matching on the basis of product quality might be accomodated. These features of the model can be relaxed by introducing a compatibility function. This exercise is tangential to our purposes, so we relegate details of this extension to the appendix.

4 Fitting the model to data

In this section, we calibrate the model to the data and assess the quality of the fit.

4.1 Transfer function estimates

Our data allow us to infer annual payments from each Colombian footwear importer to each of its foreign suppliers. These bilateral payment records provide a means for estimating equation (7), which we re-state here in log form, adding time dummies d_t and a stochastic match-specific shock $\varepsilon_{j,i}$:

$$\ln\ r_{jit} = \left(\frac{\eta-1}{\alpha-1}\right)\ln h_{j|i,t} + \ln\left(\frac{\mu_i^B}{\tilde{c}_{ji}}\right)^{\eta-1} + d_t + \varepsilon_{jit} \tag{20}$$

The time dummies d_t are meant to capture the constant $\ln\left\{\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\beta}{\alpha-1}+\lambda\right]\right\}$ and variation in $\ln\frac{E}{P^{1-\eta}}$ over time.

In estimating this equation, we face several choices. First, we must decide how to handle the term $\ln\left(\frac{\mu_i^B}{\tilde{c}_{ji}}\right)^{\eta-1}$. One option is to absorb it with match-specific fixed effects; an alternative is to impose that buyer and seller effects on marginal costs are log-separable, so that separate buyer and seller fixed effects suffice. Second, we must decide whether to treat $\ln h_{j|i,t}$ as exogenous, and if not, what instrumenting strategy to use. Under the assumptions of the model, all variation in $h_{j|i}$ is driven by random matching patterns and no instruments are needed. However, to the extent that the data reflect covariation in $h_{j|i,t}$ and r_{jit} due to transitory demand or marginal cost shocks, we expect an upward bias in our estimate of $\left(\frac{\eta-1}{\alpha-1}\right)$ unless some type of IV estimator is used. Even without transitory shocks, we might prefer to use an IV approach because of measurement error. For example, if sellers' shares in marginal

	OLS-FE		IV-FE		OLS-FE	
	rubber	textiles	rubber	textiles	rubber	textiles
$\ln h_{i\|j}$	0.823 (0.019)	0.818 (0.019)	0.669 (0.062)	0.969 (0.071)	–	–
$\ln n_i$	–	–	–	–	−0.382 (0.088)	−0.289 (0.095)
match effects	yes	yes	yes	yes	no	no
buyer effects	no	no	no	no	yes	yes
year effects	yes	yes	yes	yes	yes	yes
R^2	0.937	0.855	0.326	0.291	0.425	0.454
obs.	3,445	3,245	2,859	2,775	3,445	3,245

Table 5: Estimation of transfer equation

costs (λ) were to vary across matches, then equation (6) would only holds approximately.

Table 5 reports estimates of the transfer equation for two of the larger footwear categories–rubber uppers and textile uppers. (Results for the leather uppers are similar; we leave them out to conserve space.) The first two columns are obtained by applying an OLS fixed effects to equation (20). The results imply that there are diminishing returns to adding additional sellers of any type, or put differently, the elasticity of substitution across varieties within a store (α) exceeds the elasticitiy of substitution across stores (η).

The next two columns report estimates of the same equation, except $h_{j|i,t}$ is treated as correlated with the error term and an IV fixed effects estimator is used. Here the instrument is a share-weighted average of number of buyers of the *other* sellers at buyer j, which should be correlated with $h_{j|i,t}$ to the extent that cost or product appeal shocks specific to these sellers will affect the revenue share of seller j. (Of course, this instrument is not motivated by the model, and in that sense is less than ideal.) the estimates of $\left(\frac{\eta-1}{\alpha-1}\right)$ are not systematically different from those in the first two columns.

The last two columns report OLS fixed effects estimates of (8), and thus emboy the assumption that $\tilde{c}_{ji} = \tilde{c}_i$. The explanatory variable is now the log of the total number of sellers, s, and the coefficient on this variable is $\frac{\alpha-\eta}{1-\alpha} = \left(\frac{\eta-1}{\alpha-1}\right) - 1$. Recognizing this relationship, we note that the estimates remain in the same ballpark.

4.2 A preliminary calibration

Using our estimates of the elasticity of buyer-seller payments with respect to the number of matches, $\left(\frac{\eta-1}{\alpha-1}\right)$ and our estimated distribution of fixed effects, $\ln\left(\frac{\mu_i^B}{\tilde{c}_{ji}}\right)^{\eta-1}$, we now move to a preliminary calibration of the dynamic structural model.[13] To keep the calculations simple, we shut down seller heterogeneity for now and assume that $\tilde{c}_{ji} = \tilde{c}$. Then the parameters we need to identify include the elasticity of substitution across products, α, the dispersion in the buyer types, $var(\mu)$, the search cost parameters $(k_0^B, k_0^S, \nu_V, \nu_S, \gamma^B, \gamma^S)$, the exogenous separation hazard, δ, and the discount rate, ρ.

Some of these parameters we fix *ex ante*. First, based on estimates in Hottman *et al.* (forthcoming), we set $\alpha = 4.35$. Next, following the macro literature, we assume a discount rate of $\rho = 0.05$. Finally, we impose symmetry across buyers and sellers in the search cost scalars, $k_0^B = k_0^S = k_0$, and we simply assume that both cost functions are quadratic in search intensity: $\nu_V = \nu_S = 2$.

Given our assumption that $\alpha = 4.35$, we can infer from $\left(\frac{\eta-1}{\alpha-1}\right) \approx 0.8$ that $\eta \approx 3.7$. Also, since we estimate $var[(\eta-1)\ln\mu] \approx 2.2$ from the OLS-FE regressions in columns 5 and 6 of Table 5, we calculate $var(\ln\mu) = 2.2/(3.7-1)^2 = 0.3$, and we discretize the associated distribution of buyer effects using the method suggested in Kennan (2006)

[13] A more careful estimation that allows for seller heterogeneity and exploits a much larger set of moments is in progress.

# sellers	6402 rubber	6403 leather	6404 textile	Model
1	0.560	0.608	0.600	0.512
2	0.163	0.161	0.158	0.165
3	0.075	0.072	0.074	0.078
4	0.049	0.046	0.049	0.045
5	0.039	0.027	0.029	0.030
6	0.025	0.021	0.025	0.021
7	0.022	0.016	0.020	0.016
8	0.017	0.014	0.012	0.012
9	0.010	0.011	0.007	0.008
10	0.008	0.005	0.005	0.006
Regression coef.	-2.005	-2.176	-2.216	-2.243

Table 6: Estimated Degree Distribution

	Data		Model	
	1	2	1	2
1	0.680	0.201	0.797	0.122
2	0.404	0.260	0.507	0.366
3	0.242	0.267	0.300	0.393
4	0.144	0.200	0.169	0.323
5	0.080	0.160	0.092	0.231
6	0.088	0.100	0.049	0.152
7	0.065	0.065	0.025	0.094
8	0.021	0.021	0.013	0.056
9	0.000	0.024	0.006	0.032
10	0.026	0.000	0.003	0.018

Table 7: Estimated Transition Matrix

It remains to discuss the search cost level parameter k_0, the network effects in search cost γ^B and γ^S, and the match death hazard δ. To calibrate these parameters we minimize the sum of differences between the model and 2009 data in the degree distribution of sellers per buyer and buyers per seller as well as the first several columns of the partner transition matrix. Loosely speaking, the degree distributions pin down the search cost parameters, and the transition matrix identifies the match death hazard.

This exercise yields a search cost level parameter $k_0 = 0.9$, and more interestingly, network

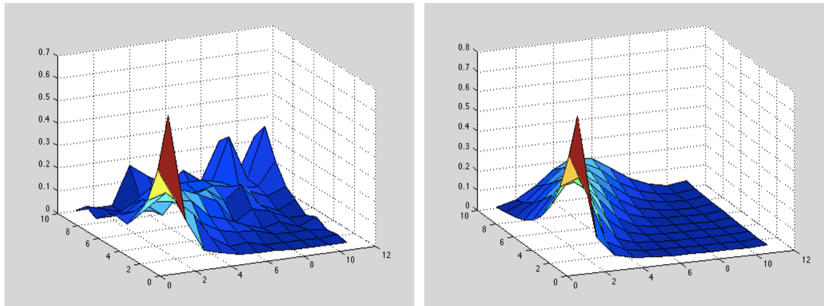

Figure 4: Transitions, sellers per buyer, data vs model

effects of $\gamma^S = 0.65$ and $\gamma^B = 0.7$. These parameters imply that network effects play an important role in the model's ability to match the data. In particular, as in Eaton *et al.* (2014), reductions in search costs due to high visibility allow the model to explain the very large firms that populate the right-hand tails of the client distributions (Figures 1 and 2). The calibration also generates a match death hazard of $\delta = 0.6$, implying that 45 percent of all matches die in the first year, and 70 of all matches die by their second year.

Aside from underestimating the number of buyers with one seller, the model replicates the data-based client distributions quite well (Table 6). It also captures the shape of the partner transition matrix (Figure 4), including the general tendency to lose clients over time. However, with only four free parameters, it fails to replicate the spikes that occur at high s values.

	2009	2013	2009	2013
# sellers	model	model	leather	leather
1	0.512	0.497	0.608	0.475
2	0.165	0.166	0.161	0.176
3	0.078	0.079	0.072	0.090
4	0.045	0.046	0.046	0.061
5	0.030	0.031	0.027	0.047
6	0.021	0.027	0.021	0.026
7	0.016	0.017	0.016	0.022
8	0.012	0.013	0.014	0.022
9	0.008	0.010	0.011	0.017
10	0.006	0.008	0.005	0.013
Regression coef.	-2.243	-1.781	-2.176	-1.986

Table 8: Degree distribution, doubling number of sellers

5 Putting the model to work

5.1 preliminary counterfactuals

In this section we run two experiments with the model. In the first experiment we double the mass of sellers as an approximation to the observed increase in sellers between 2009 and 2013 (Table 1). Our model predicts the steady state change in the degree distribution of sellers per buyer reported in the first two columns of Table 8, and summarized by the estimated power law coefficient in the last row. The model adjusts to the increase in sellers by increasing the fatness of the tail in the sellers per buyer distribution. This is exactly what we see in the data, reported in Table 8 only for leather shoes. This change leads to a non-trivial increase in consumer surplus of 5.51 percent, mainly because the new steady state delivers a richer menu of product varieties to consumers.

Our second experiment is to reduce the search cost parameter k_0 by 30 percent, scaling back costs proportionately at all levels of search intensity. Roughly speaking, we think of this exercise as approximating improvements in global communications, and perhaps also the effects of better access to intermediaries in Panama.[14] Again, we see that the tail of the degree distribution of sellers per buyer gets fatter (Figure 6).

For this experiment, we show the full transition from the estimated steady state to eight years after the shock (Figure 5). One implication is that it takes 6-7 years for the welfare benefits of lower costs to be realized. These amount to more that 10 percent per year. Here the gains are driven partly by the increase in the number of varieties available to consumers, and partly by the fact that varieties are spread across more retailers.

5.2 Interpreting the value functions

Exploiting the structure of our model, we can measure the intangible capital stocks that retailers accumulate as their build international business relationships. Figure 7 depicts $V_i^B(s)$ values for the different intrinsic buyer types, and shows how these values vary as firms add and lose clients. To reduce clutter, here have averaged the value functions across the firms with μ_i values in the lowest tercile (denoted "low μ"), the middle tercile (denoted "medium μ"), the upper tercile (denoted "high μ"). All values are normalized by the cross-importer average annual value of imports, US$ 7,665.

Several features of this figure merit note. First, $V_i^B(s)$ increases with the number of clients because each client adds value to the retailer by increasing the flow of rents. This is especially true at high-μ retailers, where relatively large sales volumes are generated per variety sold. Second, however, because of diminishing returns to varieties ($\left(\frac{\eta-1}{\alpha-1}\right) < 1$) and convex search

[14]**Note:** this experiment will be replaced by an exercise in which the reduction in k_0 is chosen to replicate the growth of trade flows, conditioned on the observed increase in the number of exporters.

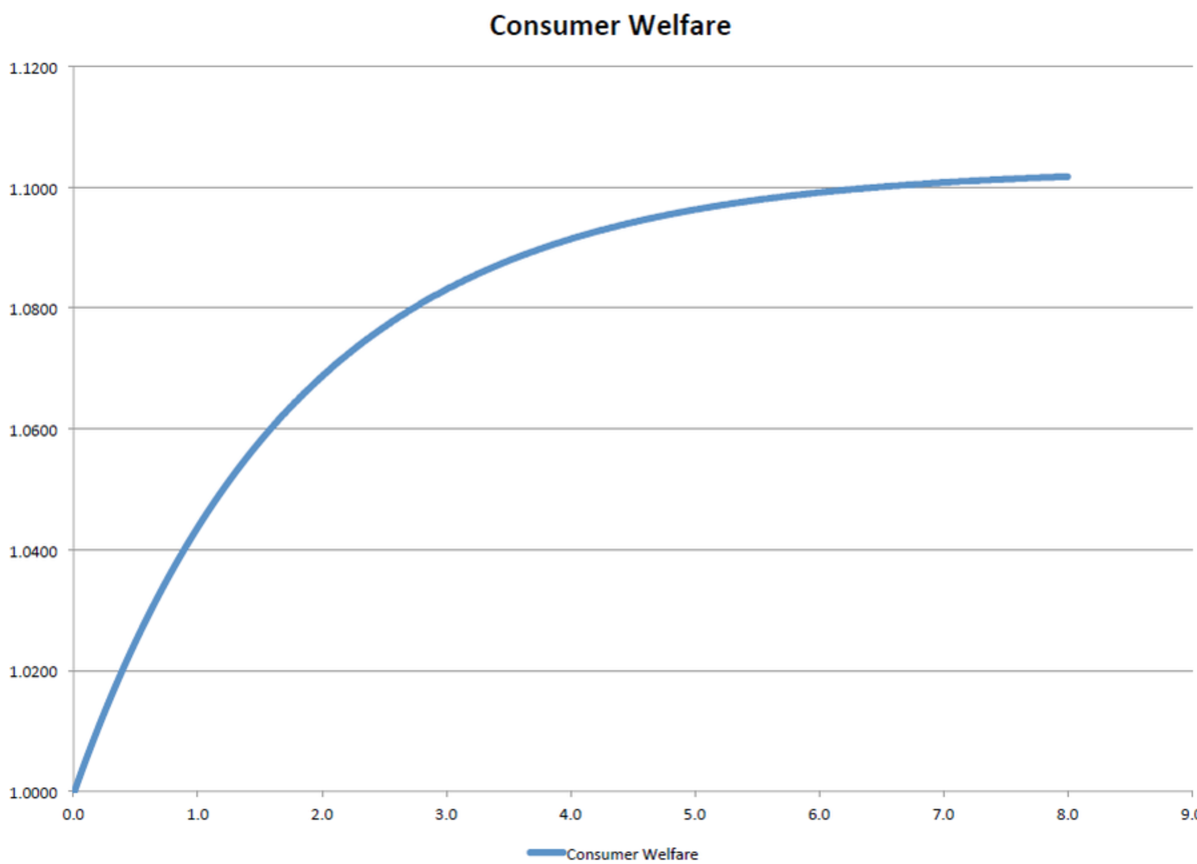

Figure 5: 30% reduction in search cost: Welfare

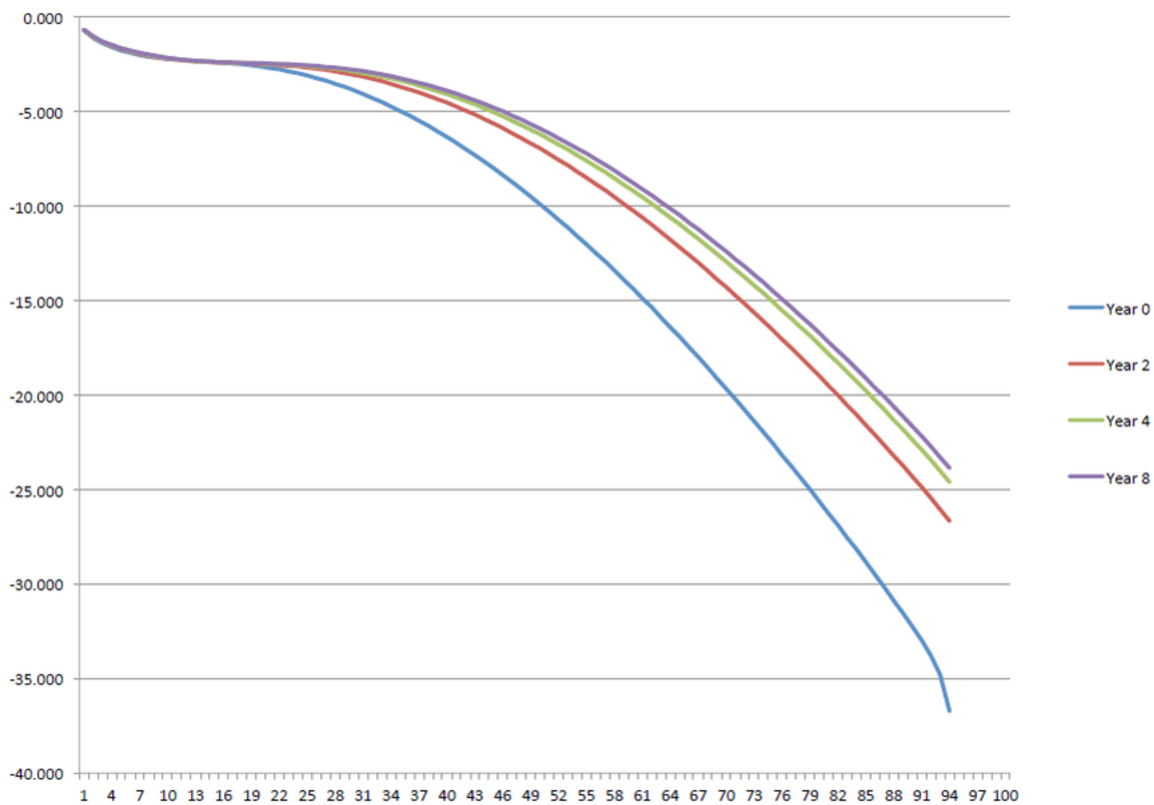

Figure 6: 30% reduction in search cost: Sellers per buyer

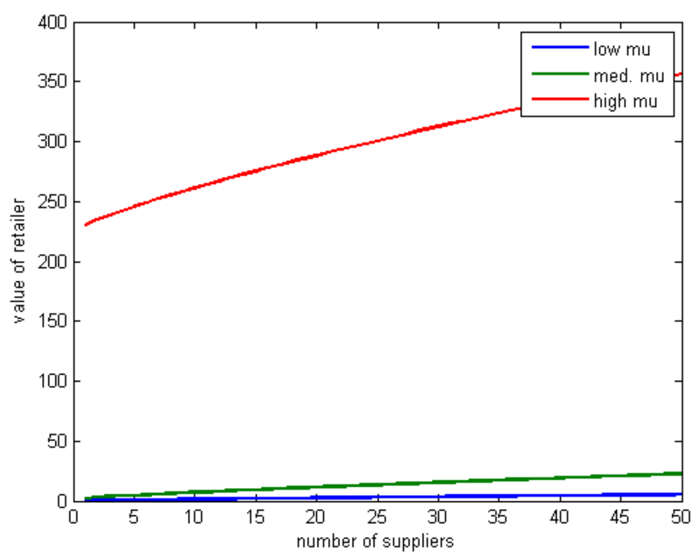

Figure 7: Retail firm values by type and state

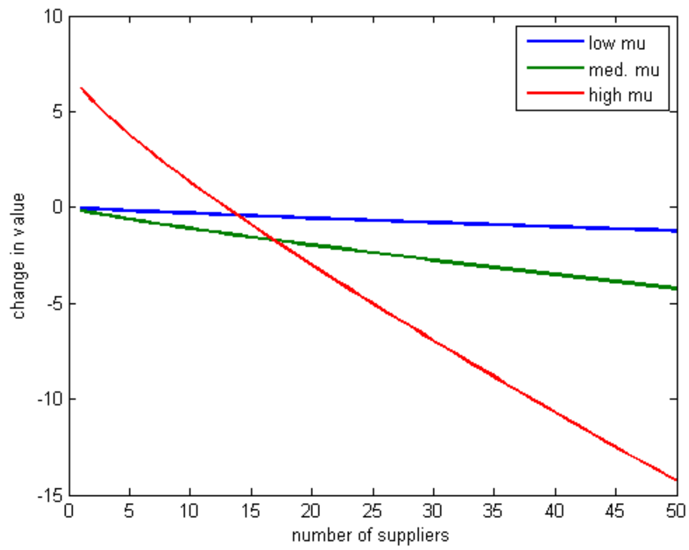

Figure 8: Capital gains for retailers with search cost reduction

costs, $V_i^B(s)$ is concave in s. Finally, the international business connections of high-μ shoe retailers are quite valuable. Consider, for example, a high-μ retailer with 30 suppliers. If we took these suppliers away but permitted it to search for replacements, its value would drop by $(300 - 230) \times \$7,665 \approx \$540,000$. Or, if we took these connections away and did not let it replace them, its value would drop by $300 \times \$7,665 \approx \$2,300,000$.

Whatever portfolio of sellers a retailer happens to have, the value of its international business relationships is sensitive to market conditions. Returning to our counterfactual experiments, we now ask how they adjust as search costs fall. Figure 8 depicts the change in value for each of our three classes classes of retailers as a function of the number of suppliers they have.

Two forces are in play in this graph. First, reductions in international search costs generate capital losses because the value of any business relationship is bounded by the costs of replacing it. Firms that have invested in building an extensive portfolio of foreign suppliers therefore lose more value than firms of the same intrinsic appeal that have not. Second, however, reductions in search costs make it less costly for firms to expand, and this is particularly important to high-μ firms that currently have just a few business partners. These firms are net beneficiaries of search cost reductions. Put differently, high-quality start-ups prefer a world with low search frictions, while established retailers would rather see their investments in suppliers maintain their values.

6 Summary

We have developed a dynamic model of international buyer-seller matching in which search intensities are optimally chosen on both sides of the markets, and we have shown that it nicely captures key cross-sectional and dynamic features of international business relationships. Counterfactual exercises based the model yield several basic messages. First, changes in the population of foreign suppliers–especially in China–led to substantial welfare improvements among Colombian consumers. Second, reductions in search frictions also have the potential to generate large welfare gains. Third, however, search frictions spread firms' adjustments to market shocks over subtantial periods, so that the full benefits of greater market participation by foreign suppliers may take 8-10 years to accrue. Finally, because of these search frictions, connections with foreign business partners are an important component of retailers' intangible capital stock. For the largest retailers, these can be worth millions of dollars.

The empirical application we report is preliminary. In future drafts we plan to incorporate seller-side heterogeneity and will exploit a larger set of moments in the estimation exercise.

We also hope to explore applications to other markets, including the U.S. market for apparel.

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Appendix

A Demand and Pricing

Using standard CES results, we begin by characterizing prices and market shares for a particular retailer b offering a particular subset of product varieties in the group, $x \in J_b$:

$$C_b = \left(\sum_{x \in J_b} \left(\xi_x C_b^j\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}, \qquad C = \left(\int_b \left(\mu_b C_b\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A-1}$$

$$P_b = \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{1/(1-\alpha)}, \quad P = \left[\int_b \left(\frac{P_b}{\mu_b}\right)^{(1-\eta)}\right]^{1/(1-\eta)} \tag{A-2}$$

$$h_b = \frac{\left(\frac{P_b}{\mu_b}\right)^{1-\eta}}{P^{1-\eta}}, \qquad h_{x|b} = \frac{\left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}}{P_b^{1-\alpha}}$$

These expressions imply the revenue generated by retail sales of product x at store b is:

$$\begin{aligned} R_{xb} &= P_{xb} q_{xb} \\ &= h_{x|b} h_b E \\ &= \mu_b^{\eta-1} \xi_x^{\alpha-1} P_{xb}^{1-\alpha} P_b^{\alpha-\eta} P^{\eta-1} E \end{aligned} \tag{A-3}$$

Since we assume a continuum of buyers, $\frac{\partial \ln P}{\partial \ln P_b} = 0$. Also,

$$\begin{aligned} \frac{\partial \ln P_b}{\partial \ln P_{ib}} &= \frac{\partial P_b}{\partial P_{ib}} \frac{P_{ib}}{P_b} \\ &= 1/(1-\alpha) \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{1/(1-\alpha)-1-1/(1-\alpha)} \left[(1-\alpha)\left(\frac{P_{ib}}{\xi_i}\right)^{1-\alpha}\right] \\ &= \left[\sum_{x \in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{-1} \left[\left(\frac{P_{ib}}{\xi_i}\right)^{1-\alpha}\right] = h_{i|b} \end{aligned}$$

Bertrand-Nash pricing therefore implies:

$$\begin{aligned} \frac{\partial \ln R_{xb}}{\partial \ln P_{xb}} &= (1-\alpha) + h_{x|b}\,(\alpha - \eta) \\ \frac{\partial \ln R_{xb}}{\partial \ln P_{x'b}} &= h_{x'|b}\,(\alpha - \eta) \qquad \forall x' \neq x \end{aligned}$$

Plugging these expressions into the first-order conditions for pricing,

$$q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b,$$

we obtain:

$$\begin{aligned} \frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{E} (\frac{p_{xb} - c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{x'b}}{E} (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \\ \frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{1}{q_{xb}} \left(\frac{p_{xb} q_{xb}}{E}\right) (\frac{p_{xb} - c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{1}{q_{x'b}} \left(\frac{q_{x'b} p_{x'b}}{E}\right) (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \\ \frac{p_{xb} q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{q_{xb}} \left(\frac{p_{xb} q_{xb}}{E}\right) (\frac{p_{xb} - c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{xb}}{q_{x'b}} \left(\frac{q_{x'b} p_{x'b}}{E}\right) (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \\ h_{xb} + \frac{\partial q_{xb}}{\partial p_{xb}} \frac{p_{xb}}{q_{xb}} (h_{xb}) (\frac{p_{xb} - c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}} \frac{p_{xb}}{q_{x'b}} h_{jb} (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \\ h_{xb} + \left(-\alpha + (\alpha - \eta) h_{x|b}\right) (h_{xb}) (\frac{p_{xb} - c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \left((\alpha - \eta) h_{x'|b}\right) h_{xb} (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \\ 1 - \alpha(\frac{p_{xb} - c_{xb}}{p_{xb}}) + (\alpha - \eta) \sum_{x' \in J_b} h_{x'|b} (\frac{p_{x'b} - c_{x'b}}{p_{x'b}}) &= 0 \end{aligned}$$

where $h_{xb} = h_{x|b} h_b = \frac{p_{xb} q_{xb}}{E}$. From this we can infer $\epsilon^b_{xx} = \frac{\partial \ln h_{xb}}{\partial \ln P_{xb}} - 1 = -\alpha + h_{x|b}(\alpha - \eta)$, and by analogous logic, $\epsilon^b_{xx'} = (\alpha - \eta) h_{x|b}$.

Next, plugging these expressions into the first-order conditions for pricing, we obtain:

$$\begin{aligned} q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \quad \forall j \in J_b, \\ q_{xb} + \sum_{x' \in J_b} \epsilon^b_{xx'} \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \\ q_{xb} + \sum_{\substack{x' \neq x \\ x' \in J_b}} \left[h_{x|b}(\alpha - \eta)\right] \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) + \left[(\alpha - \eta) h_{x|b}\right] \frac{q_{xb}}{p_{xb}} (p_{xb} - c_{xb}) &= 0 \end{aligned}$$

Since this relationship holds for all $x \in J_b$, the mark-up for each product must be the same. Call it $m = \frac{p_{xb} - c_{xb}}{p_{xb}}$ and reduce this equation to $1 - \alpha m + (\alpha - \eta)m = 0$, or

$$m = \frac{1}{\eta}.$$

Essentially the same result can be found in Atkeson and Burstein (2008) and Hottman *et al.* (forthcoming).

B Value functions with heterogenous buyers

Let $\mathbf{s} = \{s_1, s_2, ..., s_J\}$ be a vector of counts of the number of sellers of each type $j \in \{1, 2, ...J\}$ who are attached to a particular buyer, and let $\mathbf{s}_{-j} = \{s_1, s_2, .s_{j-1}, s_{j+1,}.., s_J\}$ be the same vector without its j^{th} element, so that $(s_j, \mathbf{s}_{-j})$ is one way to indicate that a seller is in state $\mathbf{s}$.

The buyer-to-seller transfer function $\tau^S_{ji}(\mathbf{s})$ and the type-i buyer payoff function $\pi^B_i(\mathbf{s})$ are chosen to satisfy the surplus sharing rule

$$(1-\beta)V^S_i(s_j, \mathbf{s}_{j-1}) = \beta \left[V^B_i(s_j, \mathbf{s}_{-j}) - V^B_i(s_j - 1, \mathbf{s}_{-j})\right], \; s_j \in \{1, 2, ..., s^{\max}\}, \; i \in \{1, 2, ..., N^B\}, \tag{A-4}$$

We now derive closed-form expressions for the surplus shares implied by (A-4). The logic is similar to that found in Bertola and Garibaldi (2001), though it is adapted to our discrete state space.

Suppressing buyer-type indices, the flow value of a type-i buyer who is currently in state

$\mathbf{s}$ is:

$$\rho V_i^B(\mathbf{s}) \;=\; \pi_i^B(\mathbf{s}) - k_s^B(\sigma_i^B(\mathbf{s})) + \sigma_i^B(\mathbf{s}) \sum_j^J \theta_j^B \left[V_i^B(s_j+1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})\right] + \delta \sum_j^J s_j \left[V_i^B(s_j-1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})\right] \tag{A-5}$$

Likewise the value to a type-j seller of being matched with a type-i buyer in state $\mathbf{s}$ is:

$$\rho V_{ji}^S(\mathbf{s}) \;=\; \tau_{ji}(\mathbf{s}) + \sigma_i^B(\mathbf{s}) \sum_k^J \theta_k^B \left[V_{ji}^S(s_k+1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})\right] + \delta \sum_{k=1}^J (s_k - \mathbf{1}_{k=j}) \left[V_{ji}^S(s_k-1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})\right] \tag{A-6}$$

Finally, the ex ante expected value of a new business relationship for a type-j seller is:

$$V_j^S = \sum_i \sum_{s \in S} P_i^B(\mathbf{s}) V_{ji}^S(\mathbf{s})$$

where $P_i^B(\mathbf{s}) = H_i^B(\mathbf{s})/H^B$ is the relative visibility of type-i buyers in state $\mathbf{s}$.

C Bargaining

Differencing the buyer's value function and suppressing buyer type i, we have:

$$\begin{aligned}
& \rho(V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j-1, \mathbf{s}_{-j})) \\
= \;& [\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j-1, \mathbf{s}_{-j})] - [k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j-1, \mathbf{s}_{-j})] \\
& + \sigma^B(s_j, \mathbf{s}_{-j})[\sum_{k \neq j} \theta_k^B V^B(s_j, s_k+1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j+1, \mathbf{s}_{-j}) - \theta^B V^B(s_j, \mathbf{s}_{-j})] \\
& - \sigma^B(s_j-1, \mathbf{s}_{-j})[\sum_{k \neq j} \theta_k^B V^B(s_j-1, s_k+1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j-1, \mathbf{s}_{-j})] \\
& + \delta[\sum_{k \neq j} s_k V^B(s_j, s_k-1, \mathbf{s}_{-j,k}) + s_j V^B(s_j-1, \mathbf{s}_{-j}) - \bar{s} V^B(s_j, \mathbf{s}_{-j})] \\
& - \delta[\sum_{k \neq j} s_k V^B(s_j-1, s_k-1, \mathbf{s}_{-j,k}) + (s_j-1) V^B(s_j-2, \mathbf{s}_{-j}) - (\bar{s}-1) V^B(s_j-1, \mathbf{s}_{-j})]
\end{aligned}$$

Now we simplify this equation in two steps. First apply a discrete approximation of the first order condition at $(s_j - 1, s_{-j})$ for the buyer search:

$$\begin{aligned}
[k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j - 1, \mathbf{s}_{-j})] \;&\approx\; (\sigma^B(s_j, \mathbf{s}_{-j}) - \sigma^B(s_j - 1, \mathbf{s}_{-j}))[\sum_{k \neq j} \theta_k^B V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k}) \\
&\quad + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j - 1, \mathbf{s}_{-j})]
\end{aligned}$$

Using the above, we can simplify

$$\begin{aligned}
&-[k^B(s_j, \mathbf{s}_{-j}) - k^B(s_j - 1, \mathbf{s}_{-j})] \\
&+\sigma^B(s_j, \mathbf{s}_{-j})[\sum_{k \neq j} \theta_k^B V^B(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j + 1, \mathbf{s}_{-j}) - \theta^B V^B(s_j, \mathbf{s}_{-j})] \\
&-\sigma^B(s_j - 1, \mathbf{s}_{-j})[\sum_{k \neq j} \theta_k^B V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j, \mathbf{s}_{-j}) - \theta^B V^B(s_j - 1, \mathbf{s}_{-j})] \\
= \;& \sigma(s_j, \mathbf{s}_{-j})[\sum_{k \neq j} \theta_k^B (V^B(s_j, s_k + 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k})) \\
&+\theta_j^B (V^B(s_j + 1, \mathbf{s}_{-j}) - V^B(s_j, \mathbf{s}_{-j})) - \theta^B (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j}))]
\end{aligned}$$

Second, we can also simplify the destruction side using

$$\begin{aligned}
&\delta[\sum_{k \neq j} s_k V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) + s_j V^B(s_j - 1, \mathbf{s}_{-j}) - \bar{s} V^B(s_j, \mathbf{s}_{-j})] \\
&-\delta[\sum_{k \neq j} s_k V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k}) + (s_j - 1) V^B(s_j - 2, \mathbf{s}_{-j}) - (\bar{s} - 1) V^B(s_j - 1, \mathbf{s}_{-j})] \\
= \;& \delta[\sum_{k \neq j} s_k (V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k})) \\
&+(s_j - 1)(V^B(s_j - 1, \mathbf{s}_{-j}) - V^B(s_j - 2, \mathbf{s}_{-j})) - \bar{s}(V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j}))]
\end{aligned}$$

To summarize, the above gives us:

$$
\begin{aligned}
& \rho[V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j})] \\
= \; & [\pi^B(s_j,\mathbf{s}_{-j}) - \pi^B(s_j-1,\mathbf{s}_{-j})] \\
& +\sigma^B(s)[\sum_{k\neq j}\theta_k^B(V^B(s_j,s_k+1,\mathbf{s}_{-j,k}) - V^B(s_j-1,s_k+1,\mathbf{s}_{-j,k})) \\
& +\theta_j^B(V^B(s_j+1,\mathbf{s}_{-j}) - V^B(s_j,\mathbf{s}_{-j})) - \theta^B(V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j}))] \\
& +\delta[\sum_{k\neq j}s_k(V^B(s_j,s_k-1,\mathbf{s}_{-j,k}) - V^B(s_j-1,s_k-1,\mathbf{s}_{-j,k})) \\
& +(s_j-1)(V^B(s_j-1,\mathbf{s}_{-j}) - V^B(s_j-2,\mathbf{s}_{-j})) - \bar{s}(V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j}))]
\end{aligned} \tag{A-7}
$$

By the definition of the type j seller's value function, we have

$$
\begin{aligned}
\rho V_j^S(\mathbf{s}) \; = \; & \tau^j(\mathbf{s}) + \sigma^B(s)[\sum_{k\neq j}\theta_k^B V_j^S(s_j,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j+1,\mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s})] + \\
& \delta[(\sum_{k\neq j}s_k V_j^S(s_k-1,\mathbf{s}_{-k}) + (s_j-1)V_j^S(s_j-1,\mathbf{s}_{-j})) - \bar{s}V_j^S(\mathbf{s})]
\end{aligned} \tag{A-8}
$$

Finally, using equations (A-7), (A-8) and (A-4), we have

$$
\begin{aligned}
& \beta\rho[V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j})] - (1-\beta)\rho V_j^S(s_j,\mathbf{s}_{-j}) \\
= \; & \beta[\pi^B(s_j,\mathbf{s}_{-j}) - \pi^B(s_j-1,\mathbf{s}_{-j})] + \\
& (1-\beta)\sigma^B(s)\left[\sum_{k\neq j}\theta_k^B(V_j^S(s_j,s_k+1,\mathbf{s}_{-j,k})) + \theta_j^B V^S(s_j+1,\mathbf{s}_{-j})] - \theta^B V^S(s_j,\mathbf{s}_{-j})\right] + \\
& \delta(1-\beta)\left[\sum_{k\neq j}s_k(V^S(s_j,s_k-1,\mathbf{s}_{-j,k})] + (s_j-1)(V^S(s_j-1,\mathbf{s}_{-j}) - \bar{s}(V^S(s_j,\mathbf{s}_{-j})\right] \\
& -(1-\beta)\left[\tau^j(\mathbf{s}) + \sigma^B(s)[\sum_{k\neq j}\theta_k^B V_j^S(s_j,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j+1,\mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s})]\right] \\
& -(1-\beta)\delta\left[[(\sum_{k\neq j}s_k V_j^S(s_k-1,\mathbf{s}_{-k}) + (s_j-1)V_j^S(s_j-1,\mathbf{s}_{-j})) - \bar{s}V_j^S(\mathbf{s})]\right] \\
= \; & 0
\end{aligned}
$$

Or, cancelling terms and re-arranging, the flow transfer to a type$-j$ seller by a type-i buyer in state $\mathbf{s}$ is share β of the total flow surplus generated by their match:

$$\tau^j(\mathbf{s}) = \beta[\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j}) + \tau^j(\mathbf{s})] \tag{A-9}$$

The total flow surplus created by the marginal match between a type$-j$ seller by a type-i buyer in state $\mathbf{s}$ must equal the sum of the flow surpluses reaped by the buyer and the seller:

$$\pi_i^B(s_j, \mathbf{s}_{-j}) - \pi_i^B(s_j - 1, \mathbf{s}_{-j}) + \tau^j(\mathbf{s}) = \pi_i^T(\mathbf{s}) - \pi_i^T(s_j - 1, \mathbf{s}_{-j})$$

So we can re-state (A-9) as:

$$\tau_{ji}(\mathbf{s}) = \beta\left[\pi_i^T(\mathbf{s}) - \pi_i^T(s_j - 1, \mathbf{s}_{-j})\right] \tag{A-10}$$

D Transition dynamics

Details to come

E Adding assortative matching

It is straightforward to modify the model so that particular sellers tend to specialize in particular types of goods. To do so, continue to assume that buyers and sellers encounter each other through an undirected search process. But now suppose that shipments only take place between *compatible* buyers and sellers who meet, and let any randomly selected pair of type$-i$ buyer and type$-j$ seller be compatible with probability $d_{ij} \in [0, 1]$. Finally, assume that buyers and sellers know these probabilities and choose their search intensities accordingly. With these additional assumptions, we are able keep the random search aspects of the model while accomodating the fact that we observe particular types of businesses doing business with one other with greater or lesser frequency than pure randomness would imply.

Success rates: For type-i buyers, the expected share of encounters that result in business partnerships is now:

$$a_i^B = \frac{\sum_j \sum_{n=0}^{n_{\max}} d_{ij}\sigma_j^S(n)P_j^S(n)}{\sum_j \sum_{n=0}^{n_{\max}} \sigma_j^S(n)P_j^S(n)} \tag{A-11}$$

where $P_j^S(n) = H_j^S(n)/H^S$ is the share of matches that involve type$-j$ sellers with n buyers. Similarly, for type j sellers, the expected share of meetings that result in business partnerships is:

$$a_j^S = \frac{\sum_i \sum_{s=0}^{s_{\max}} d_{ij}\sigma_i^B(s)P_s^B(i)}{\sum_i \sum_{s=0}^{s_{\max}} \sigma_i^B(s)P_s^B(i)} \tag{A-12}$$

where, recall, $P_i^B(s) = H_i^B(s)/H^B$ is the share of matches that involv type-i buyers who have s sellers. Thus, for a type-i buyer with s suppliers, the hazard of finding another compatible seller is $\sigma_i^B(s)a_i^B\theta^B$. Likewise, for a type-$j$ seller with n buyers, the hazard of finding another compatible buyer is $\sigma_j^S(n)a_j^S\theta^S$.

Policy functions: Incorporating compatibility, the programming problem for a type-i buyer with s sellers becomes:

$$V_i^B(s) = \max_{\sigma_i^B(s)} \left\{ \frac{\pi_i^B(s) - c_B\left(\sigma^B\right) + s\delta V_i^B(s-1) + \sigma_i^B(s)a_i^B\theta^B V_i^B(s+1)}{\rho + s\delta + \sigma_i^B(s)a_i^B\theta^B} \right\} \tag{A-13}$$

Accordingly, the new buyer policy functions, $\sigma_s^B(i)$, solve the first order conditions:

$$c_B'\left(\sigma_i^B(s)\right) = a_i^B\theta^B\left[V_i^B(s+1) - V_i^B(s)\right]. \tag{A-14}$$

Similar modifications apply on the sellers' side. The value to a seller of an existing compatible relationship with a type$-i$ buyer in state s now depends on a_i^B. This is because the hazard of this buyer adding another seller depends upon her compatibility:

$$V_{i,s}^S = \frac{\tau_i(s) + (s-1)\delta V_{i,s}^S(s-1) + \sigma_i^B(s) a_i^B \theta^B V_{i,s}^S(s+1)}{\rho + s\delta + \sigma_i^B(s) a_i^B \theta^B} \tag{A-15}$$

And the ex-ante potential value of a new relationship with a compatible buyer is:

$$V_j^S = \sum_{i=1}^{I} \sum_{s=0}^{s_{\max}} V_i^S(s+1) \frac{d_{ij} P_i^B(s)}{\sum_{i,s} d_{ij} P_i^B(s)}$$

The associated seller policy functions, $\sigma_j^S(n)$, therefore solve:

$$c_S'\left(\sigma_j^S(n)\right) = a_j^S \theta^S V_j^S. \tag{A-16}$$

Empirical implementation: The d_{ij} 's can be solved for using observed shares of different product categories at different firms, so this extension adds no new parameters to identify. (Details to come.)